ChE445 Skip Class Notes - Thursday 11/06/14

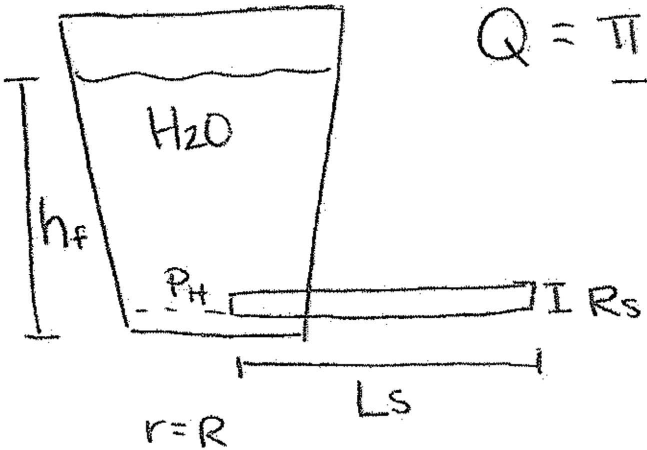
Fluid Mechanics & Viscosity

Fundamental Dimensions	SI (mks)	(Dr. Skip's) cgs	Engineering
Mass (M)	kg	g	lb mass
Length (L)	m	cm	ft
time (t)	s	s	s
Temperature (T) (K) Absolute	°C (K)	°C (K)	°F (R)
Density H_2O (ρ_f)	$1000 \frac{kg}{m^3}$	$1.0 \frac{g}{cm^3}$	$62.4 \frac{lb_m}{ft^3}$
Viscosity H_2O (μ_f)	$1 mPa\text{-}s = 10^{-3} Pa\text{-}s$	$1 cP = 10^{-2}$ Poise	

(Q) = Volumetric Flow Rate

$Re < 2100$

$$Q = \frac{\pi R_s^4 (\rho_f g h_f)}{8 \boxed{\mu_F} L_s}$$



$$Q = \int_{r=0}^{r=R} v_x(y)\, dy$$

Unit Conversions:

454 g/lbm

2.2 kg/lbm

2.54 cm/in

$$\left(\frac{1\,g}{cm^3}\right)\left(\frac{2.2\,lb_m}{kg}\right)\left(\frac{kg}{10^2 g}\right)\left(\frac{2.54\,cm}{in}\right)^3 \Big($$

$$= 62.4 \frac{lb_m}{ft^3}$$

$$\left(\frac{1\,g}{cm^3}\right)\left(\frac{kg}{1000\,g}\right)\left(\frac{100\,cm}{m}\right)^3$$

$$= 1000 \frac{kg}{m^3}$$

Fluid Mechanics of Showering:

Q [=] Volumetric Flow Rate $= \frac{\text{volume}}{\text{time}} \left(\frac{L^3}{t}\right)$

v [=] velocity $\left(\frac{L}{t}\right) = \frac{Q}{A_x} \frac{\left(\frac{L^3}{t}\right)}{\left(L^2\right)}$

Density (ρ) [=] $\left(\frac{M}{L^3}\right)$

Viscosity $(\mu_f) = \frac{\text{shear stress}}{\text{shear rate}}$

Shear stress: $\frac{\text{force}}{\text{area}} = \frac{ma}{\text{area}}$ [=] $\frac{ML/t^2}{L^2}$

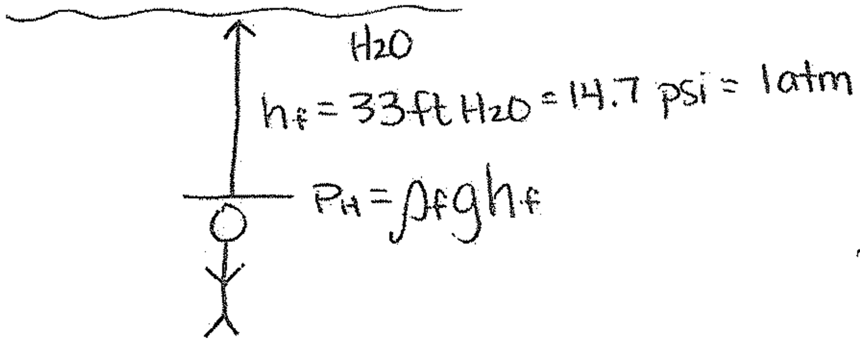
pressure = stress [=] $\left(\frac{M}{Lt^2}\right)$

Pascal [=] $\frac{Kg}{m \cdot S^2}$

Shear rate: $\frac{dv_x(y)}{dy} = \frac{\Delta v_x}{\Delta y}$ [=] $\frac{L/t}{L}$ [=] t^{-1}

Viscosity [=] $\frac{\frac{M}{Lt^2}}{t^{-1}}$ Pa-S (mks) [=] $\frac{M}{L \cdot t} * \frac{Kg}{m \cdot S}$ Poise (cgs) [=] $\frac{M}{L \cdot t} = \frac{g}{cm \cdot s}$

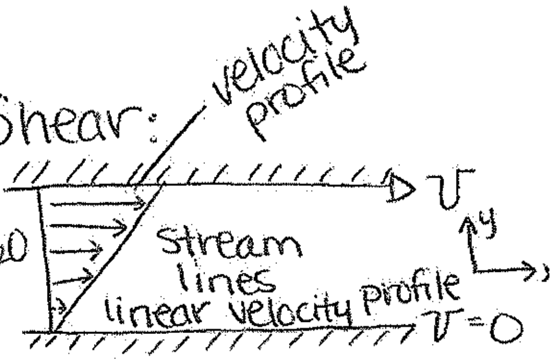
Pressure Head:



$h_f = 33 \text{ ft } H_2O = 14.7 \text{ psi} = 1 \text{ atm}$

$P_H = \rho_f g h_f$

Shear:



drag flow

Boundry Conditions:

@ BC#1 @ $X = 0$ $v_x = 0$ nosli

BC#2 @ $X = Y$ $v_x = U$ no slip

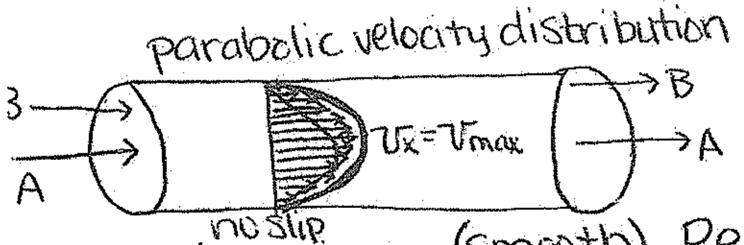
Specific Gravity (S.G)

$S.G. = \frac{\rho_f}{\rho_{H_2O} @ 4°C}$

$S.G.(Hg) = 13.6$

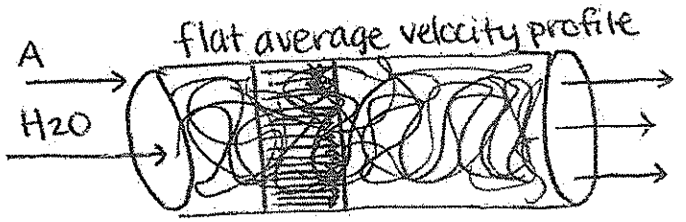
Dimensionless Group

Fluid Flow in Pipes:



- A exists before B
- maintain relative positions.

Laminar (Smooth) $Re < 2100$



- exit at the same time
- position in unknown
- mixing flow

Turbulent (Rough) $Re > 2300$

Reynolds Number: (Dimensionless Group)

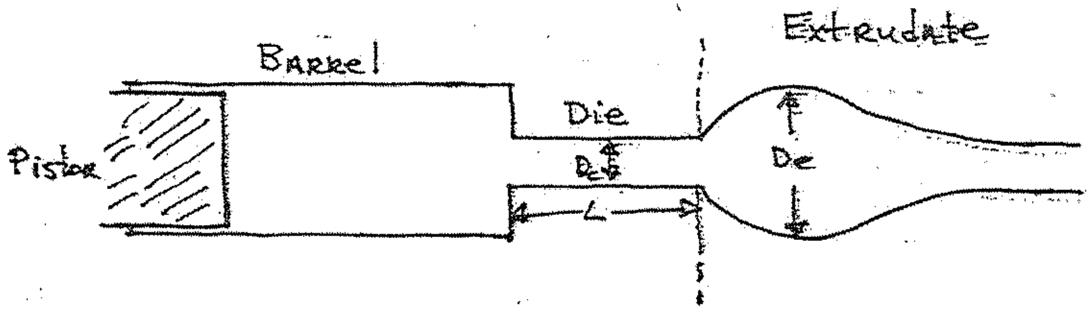
$$Re \,[=]\, \frac{(\text{density})(\text{velocity})(\text{diameter})}{(\text{viscosity})} \,[=]\, \frac{\rho_f v_x D_p}{\mu_f} \,[=]\, \frac{\left(\frac{M}{L^3}\right)\left(\frac{L}{t}\right)(L)}{\left(\frac{M}{L \cdot t}\right)}$$

What flow rate should the pump be set at in order to reach Turbulent Flow?

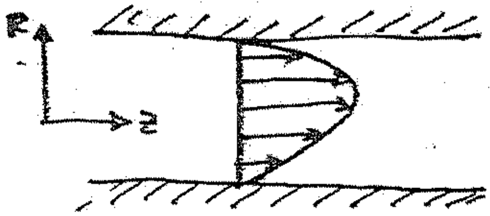
SUBJECT

Capillary Flow – Polymer Melts

JOB NO.	SHEET OF	FILE	BY	DATE



Poiseuille Law for Capillary Flow



parabolic velocity distribution
(Newtonian fluid)

$V_z = 0$ at the wall ("no slip")

$V_z = \text{max}$ at centerline

Apparent wall shear stress $\sigma_{App} = \dfrac{\Delta P R}{2L}$

Apparent wall shear rate $\dot{\gamma}_{App} = \dfrac{4Q}{\pi R^3}$

Apparent shear viscosity $\eta_{App} = \sigma/\dot{\gamma}$

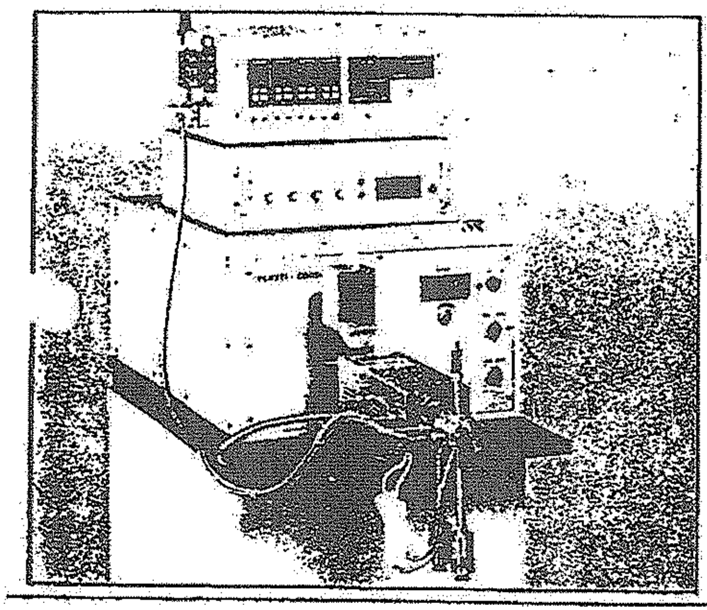
Extrudate Swell ("Die Swell") = Elasticity

$$\boxed{De/D \propto \text{Normal Stress}}$$

$$De/D = f(L/D)$$

The ultimate
tool for research
in polymer rheology . . .

Rheometric Capillary Die



The field of polymer melt rheology is an area of increasing concern to not only the researcher, but also to the producer and user of plastic materials. Rheological data define processing parameters such as extruder output, pressure drop, elasticity and melt fracture thus enabling the processor to program his equipment for maximum output and product quality. However, rheological data are also indicative of molecular properties such as chemical structure, average molecular weight and molecular weight distribution. These data are of specific interest to the plastic producer. Polymer melt rheology, then, serves a broad field and is intimately related to processing and molecular properties of polymers.

The C. W. Brabender Rheometric Capillary Die is attachable to any of the C. W. Brabender Extruders and functions as a continuous high pressure capillary rheometer. This die is designed to provide a high degree of versatility into a precision laboratory instrument. By applying the appropriate equations for pressure drop in laminar flow of a fluid through a cylindrical tube to the rheometric capillary die data, flow is expressed in terms of classical rheological dimensions.

C. W. Brabender Extruders transform the polymer into a homogeneous melt and deliver it to the capillary die at a uniform rate. When the polymer encounters the restriction of the capillary orifice, a build-up in pressure results. The amount of pressure is proportional to the screw speed, and can be varied by the Plasti-Corder® Drive. Pressure and stock temperature are measured at the capillary entrance by a combined Pressure-Temperature probe. The pressure data at a constant screw speed are used to compute Shear Stress in dynes/cm^2. From the volumetric flow rate through the capillary at constant pressure, the Shear Rate in sec^{-1} is calculated. These data enable the researcher to investigate polymer rheology using the classical capillary viscometry techniques.

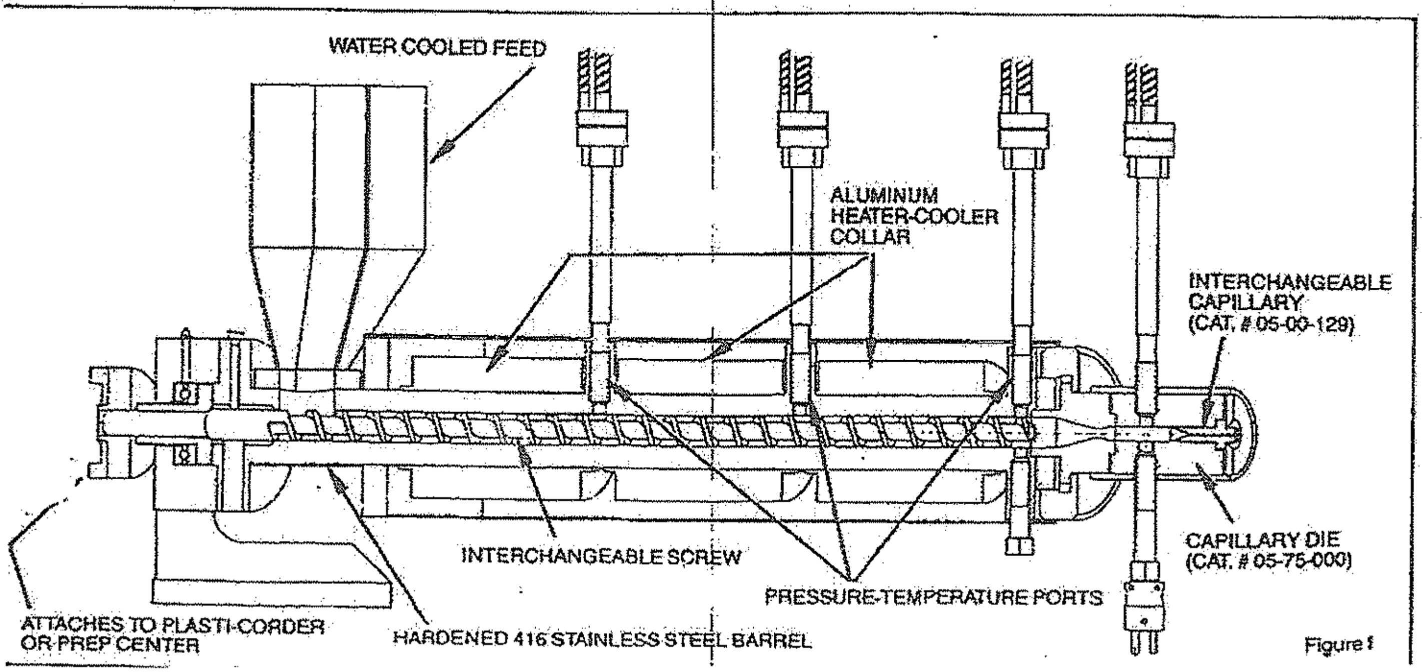


Figure 1

A complete system (Fig. I, below left) for measuring the pressure and temperature in the various zones of the extruder can be provided. As the polymer is transported through the barrel, the operator can continuously monitor the pressure and temperature changes encountered as the polymer is transformed into a homogeneous melt. Valuable data relating to feed, point of fusion, lubricity, effect of additives, influences of screw and die geometry, etc., on the extrusion process can be characterized. The data can also be used to develop mathematical models for the extrusion process.

A range of pressure values at the capillary entrance is achieved by varying the screw speed from the Plasti-Corder Drive. Output is measured at each pressure value. Figure II describes a typical plot of output vs. pressure and shows the effect of capillary L/D ratio on the output curves.

Poiseuille's equation for pressure drop in laminar flow through a capillary can be applied to the data in Figure II.

$$\Delta P = \frac{8Q\eta l}{\pi r^4}$$

where:
Q = Volumetric Flow Rate (cm^3/sec)
η = Viscosity (poise)
r = Radius of capillary
l = Length of capillary

solving Poiseuille's equation for viscosity:

$$\eta = \frac{\Delta P r}{2l} \bigg/ \frac{4Q}{\pi r^3} = \frac{\tau}{\gamma} \begin{array}{l}\text{(Shear Stress)}\\ \text{(Shear Rate)}\end{array}$$

Thus the Shear Stress, τ is: $\Delta Pr/2l$ and the Shear Rate, γ is $4Q/\pi r^3$

A plot of Shear Stress vs. Shear Rate for a typical pseudoplastic polymer melt (non-Newtonian) is shown in Figure III.

The Shear Rate γ expressed by the equation $\gamma = \frac{4Q}{\pi r^3}$ is a linear relationship between flow rate and capillary radius and applies to Newtonian flow. For non-Newtonian fluids, a correction is necessary to include the pressure loss when entering into the capillary since the relationship of Shear Stress and Shear Rate is not linear.

The dimensions for Shear Rate are in sec^{-1} and for Shear Stress dynes/cm^2. The conversions necessary for converting rheometric capillary die data to the appropriate dimensions are:

For Shear Stress, observed pressure in psig must be converted to dynes/cm^2.
For Shear Rate, mass output must be converted to volumetric output.

When Shear Stress and Shear Rate are expressed in these dimensions, the apparent viscosity (Shear Stress/Shear Rate) is expressed in poise values.

$$\frac{\tau\ (\text{dynes/cm}^2)}{\gamma\ (\text{Sec}^{-1})} = \eta\ (\text{poise})$$

Evaluation of Rheometric Capillary Flow Data . . .

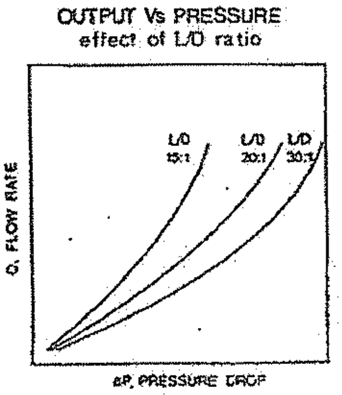


Figure II

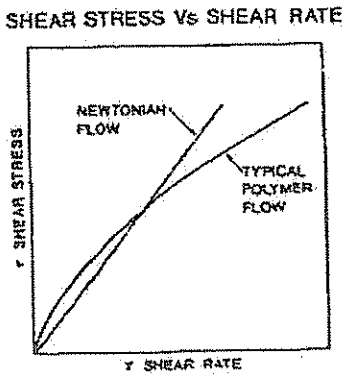


Figure III

DIE NOZZLES

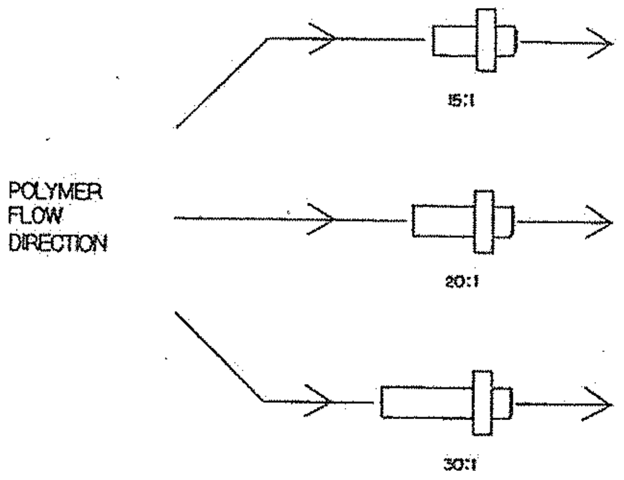


Figure 2

2.4 Internal Layout

A schematic diagram depicting the internal design of your round capillary die is illustrated in Figure 3. Notice that the flow of polymer through the die is from left to right.

ROUND CAPILLARY DIE (INTERNAL VIEW)

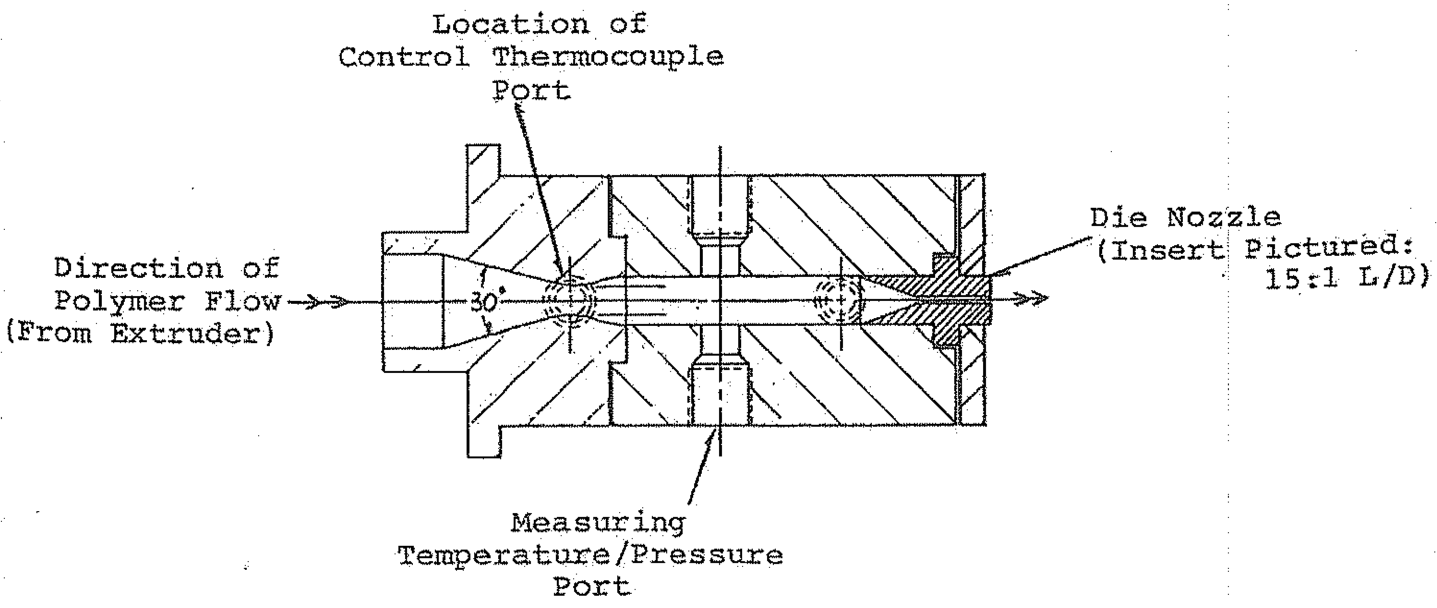


Figure 3