CS 321: CFL practice: CFGs

1. What is the language of this grammar? $S \rightarrow aSb \mid aS \mid \epsilon$

Design a CFG for each of the following:

2. $\{a^n b^m \mid n \neq m\}$
   \textit{Hint: }$n \neq m \iff (n < m \text{ or } n > m)$

3. $\{a^n b^m \mid m = 2n\}$

4. $\{a^n b^m \mid m > 2n\}$

5. $\{a^n b^m \mid 2n \geq m \geq n\}$
   \textit{Hint: }for every $b$, generate either 1 or 2 corresponding $a$'s

6. $\{a^n b^m a^m b^n \mid m, n \in \mathbb{N}\}$

7. $\{a^n b^n a^m b^m \mid m, n \in \mathbb{N}\}$

8. $\{a^n b^m c^{m+n} \mid m, n \in \mathbb{N}\}$

9. $\{w \in \{a, b\}^* \mid w = \text{rev}(w)\}$

10. $\{w \in \{a, b\}^* \mid w \text{ has same number of } a\text{'s } \& b\text{'s}\}$
    \textit{Hint: }Graph the function $#a's$ minus $#b's$. 

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CS 321: CFL practice: PDAs

Design a PDA for each of the following:

1. \( \{w \in \{a,b\}^* \mid \text{w has same number of } a\text{'s } & b\text{'s} \} \)

2. \( \{w \in \{a,b\}^* \mid \text{w has more } a\text{'s than } b\text{'s} \} \)

3. \( \{w \in \{a,b\}^* \mid \text{w has exactly twice as many } a\text{'s as } b\text{'s} \} \)

4. \( \{w \in \{a,b\}^* \mid \text{w has at most twice as many } a\text{'s as } b\text{'s} \} \)

5. \( \{w \in \{a,b\}^* \mid \text{number of } a\text{'s in } w \text{ is at least the number of } b\text{'s, but at most twice the number of } b\text{'s} \} \)

6. \( \{a^m b^m c^{m+n} \mid m, n \in \mathbb{N} \} \)