CS 321: Homework #0

This homework is for review / warmup only. Do these problems to make sure you are prepared for the course. You may optionally submit solutions, and the TAs & I will gladly provide feedback.

It’s OK if you need to review some concepts to solve these problems (especially after summer break). However, it is a red flag if you find these problems impossible even after some light review.

1. Let \( F(n) \) denote the \( n \)th Fibonacci number, defined as follows:

\[
F(0) = 0; \quad F(1) = 1; \quad F(n) = F(n - 1) + F(n - 2), \text{ for } n \geq 2
\]

Using this definition of the Fibonacci sequence, give a formal, inductive proof of the following identity:

\[
\sum_{i=0}^{n} F(i) = F(n + 2) - 1
\]

2. Let \( A = \{1, 2, 3\} \), \( B = \{1, \{2, 3\}\} \), and \( C = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid x \neq y\} \).

   (a) What is \( A \times B \)?
   
   (b) What is \( P(A) \setminus B \)?
   
   (c) What is \( P(A) \setminus P(B) \)?
   
   (d) What is \( (A \times A) \setminus C \)?

“\( \times \)” is the cartesian product operation, “\( P(X) \)” is the powerset of a set \( X \), “\( \setminus \)” is the set difference operation, and \( \mathbb{Z} \) is the set of integers \{\ldots, -1, 0, 1, \ldots\}.

3. Four cards are on a table. Each card has a number (from 1 to 10) on one side and a pattern (checkerboard or solid) on the other side. The cards are currently in the following state:

   ![](image)

   Someone has made a rule about cards:

   “If a card has an even number on one face, then it must have a solid pattern on the opposite face.”

   Which of the four cards (maybe none, maybe just one, maybe more than one) must be turned over to determine whether the rule is being followed? Why?

4. Write an equivalent logical statement without using any negation operations (\( \neg \)):

\[
\neg \forall x : \exists y : ([\neg f(x, y)] \land g(y))
\]

Hint: the final answer may involve an implication \( a \Rightarrow b \)

\( ^1 \)It sounds like click-bait, but literally less than 10% of people get this question right (it’s a famous psychology experiment about deductive reasoning). Subjects do much better (75% success) when you replace “card” with “person”; “even number” with “drinks beer”; “solid pattern” with “age over 21”; and ask them to verify whether the drinking age rule is being obeyed.