CS 321: Homework #1

Due: Monday Oct 2 at 9am, on Canvas

Homeworks should be typed. You can describe a DFA by giving its transition table (don’t forget to indicate start state and accept states), or by drawing a state diagram. You can easily draw state diagrams using this web-based tool: http://madebyevan.com/fsm/.

For problems 1-3, describe a DFA that accepts the given language. Describe in plain language what is the purpose of each state. It will probably help to give thoughtful names to the states.

1. \( \{ x \in \{a, b\}^* \mid \text{last 5 characters of } x \text{ are not } ababb \} \).

2. \( \{ x \in \{0, 1\}^* \mid \text{bin}(x) \text{ is 1 more than a multiple of 5} \} \).

   \text{bin}(x) \text{ is the function from lecture, defined recursively as:}
   \begin{align*}
   \text{bin}(\epsilon) &= 0 \\
   \text{bin}(wc) &= 2 \cdot \text{bin}(w) + c, \quad \text{for all } w \in \{0, 1\}^* \text{ and } c \in \{0, 1\}
   \end{align*}

3. \( \{ x \in \{a, b\}^* \mid x \text{ contains at least 3 occurrences of the substring } aba \} \).

   Overlapping of substrings is allowed. For example, the string \textit{abababa} should be accepted.

4. In lecture, we defined the extended transition function \( \delta^* : Q \times \Sigma^* \rightarrow Q \) recursively in terms of \( \delta \), via:

   \begin{align*}
   \delta^*(q, \epsilon) &= q \\
   \delta^*(q, wc) &= \delta(\delta^*(q, w), c), \quad \text{for all } w \in \Sigma^* \text{ and } c \in \Sigma
   \end{align*}

   Using this definition, prove that:

   \( \delta^*(q, xy) = \delta^*(\delta^*(q, x), y), \quad \text{for all } x, y \in \Sigma^* \)

   In other words, the state that you get to by starting at \( q \) and reading \( xy \), is the state that you get to by starting at \( q \), reading \( x \), then reading \( y \).

   \text{Hint: Use induction on the length of } y. \text{ You can follow the examples for induction on strings from Erickson’s notes section 1. However, in this case you should do induction by adding a character to the end of } y \text{ (not beginning of } y \text{ as in the Erickson examples). This makes things match the recursive definition } \delta^* \text{ better.}