CS 321: Homework #3

Due: Monday Oct 16 at 9am, on Canvas

Homeworks should be typed. You can describe a DFA by giving its transition table (don’t forget to indicate start state and accept states), or by drawing a state diagram. You can easily draw state diagrams using this web-based tool: http://madebyevan.com/fsm/.

1. Give regular expressions for the following languages:
   (a) \{w \in \{a, b\}^* \mid w \text{ contains substring } ab \text{ an even number of times}\}
   (b) \{w \in \{a, b\}^* \mid w \text{ has an even number of } a’s \text{ and even number of } b’s\}
       Note: the a’s and b’s can come in any order, so strings like \text{abbbab} should be accepted.

   To help the grader out (and to increase chance of partial credit), if you have a long regular expression, please identify small conceptual parts and explain what each part does.

2. Give an NFA for the set of strings matching \((0 + 1(01^*0)^*1)^*\)

3. Give a regular expression equivalent to the following NFA:

4. Let \(M = (Q, \Sigma, \delta, s, F)\) be a DFA.
   (a) Show that for any \(q \in Q\) and \(P \subseteq Q\), the following language is regular:

\[ \{w \in \Sigma^* \mid \delta^*(q, w) \in P\} \]

   Clearly describe a procedure to construct a DFA for this language, in terms of \(M\).
   (b) If \(A\) is a language over alphabet \(\Sigma\), define:

\[ \text{undouble}(A) \overset{\text{def}}{=} \{w \in \Sigma^* \mid ww \in A\}. \]

   Show that if \(A\) is regular, then so is \(\text{undouble}(A)\).
   \text{Example: if } A = \{\varepsilon, 0, 11, 0010, 0101\} \text{ then } \text{undouble}(A) = \{\varepsilon, 1, 01\}.
   \text{Hint: First consider a simpler version where I fix the “middle” state } q, \text{ so:}

\[ \{w \in \Sigma^* \mid ww \in A \text{ and } \delta^*(s, w) = q\} \]

   In the “real” version of the problem, the “middle state” is not fixed.