CS 321: Homework #3

Due: Friday Oct 14 at 6pm, on Canvas. Homeworks must be typed.

1. Give regular expressions for the following languages:
   (a) \{w \in \{a, b\}^* | w \text{ has an even number of } b\text{'s}\}
   (b) \{w \in \{a, b\}^* | w \text{ does not contain the substring } ab\}
   (c) \{w \in \{a, b\}^* | w \text{ contains substring } ab \text{ an even number of times}\}
   (d) \{w \in \{a, b\}^* | w \text{ has an even number of } a\text{'s and even number of } b\text{'s}\}

   To help the grader out (and to increase chance of partial credit), if you have a long regular expression, please identify small conceptual parts and explain what each part does.

2. You probably know the trick that a number is a multiple of 9 if and only if the sum of its decimal digits is a multiple of 9. Here is a similar trick for multiples of 3 in binary.

   Prove that a number is a multiple of 3 if and only if writing that number backwards in binary is also a multiple of 3.

   Example: 0011_{bin} = 3_{dec} is a multiple of 3, and so is 1100_{bin} = 12_{dec}. However, 100101_{bin} = 37_{dec} is not a multiple of 3, and neither is 101001_{bin} = 41_{dec}.

   Hint: First transform this to a statement about regular languages over alphabet \{0, 1\}.

3. If \(A\) is a language over alphabet \(\Sigma\), define:

   \[
   \text{undouble}(A) \overset{\text{def}}{=} \{w \in \Sigma^* | \text{ww} \in A\}.
   \]

   Show that if \(A\) is regular, then so is \(\text{undouble}(A)\).

   Example: if \(A = \{\varepsilon, 0, 11, 0010, 0101\}\) then \(\text{undouble}(A) = \{\varepsilon, 1, 01\}\).

   Warmup / partial credit: Suppose \(M = (Q, \Sigma, \delta, s, F)\) is a DFA for \(A\). Show that the following language over alphabet \(\Sigma \cup Q\) is regular:

   \[
   \{qw | q \in Q \text{ and } w \in \Sigma^* \text{ and } \delta^*(s, w) = q \text{ and } \delta^*(q, w) \in F\}
   \]

   Just to be clear, strings in this language (for the warm-up question only) consist of one character from \(Q\) and then any number of characters from \(\Sigma\).