CS 321: Homework #4

Due: Monday Oct 23 at 9am, on Canvas

Homeworks should be typed. You can describe a DFA by giving its transition table (don’t forget to indicate start state and accept states), or by drawing a state diagram. You can easily draw state diagrams using this web-based tool: http://madebyevan.com/fsm/.

For reference, here is the pumping lemma game (for language A):

1. Adversary picks a number \( p \geq 0 \).
2. You pick a string \( w \in A \), such that \(|w| \geq p\).
3. Adversary breaks \( w \) into \( w = xyz \), such that \(|xy| \leq p\) and \( y \neq \epsilon \).
4. You pick a number \( i \geq 0 \). If \( xy^iz \not\in A \), then you win.

If you can describe a strategy in which you always win, then \( A \) is not regular.

1. Show that the following languages are not regular. You can use the pumping lemma game, or you can use closure properties (or both).
   
   (a) \( \{w \in \{a, b, c\}^* \mid \text{num}(a, w) = \text{num}(b, w) + \text{num}(c, w)\} \)
      In this problem \( \text{num}(a, w) \) means the number of \( a \) characters in the string \( w \).
   
   (b) \( \{a^n b^m c^k \mid n = m \text{ or } m = k\} \)

   (c) \( \{w \in \{a, b\}^* \mid \text{the length of } w \text{ is a square number}\} \)
      This language contains all strings of length 1, 4, 9, 16, etc. \textit{Hint:} after you pump, you’ll want to show that the length of the resulting string is \textit{not} a square. The best way to do this is to show that its length is \textit{strictly} between consecutive squares \( n^2 \) and \( (n + 1)^2 \) for some appropriate \( n \).

   (d) \( \{w \in \{a, b\}^* \mid w \neq \text{rev}(w)\} \).
      In this problem \( \text{rev}(w) \) denotes the reverse of \( w \) (i.e., characters put in opposite order).