CS 321: Theory of Computation

CS 321: Homework #5

Due: Friday Nov 4 at 6pm, on Canvas. Homeworks must be typed.

**Def:** \(\text{num}(x, y)\) means the number of times \(x\) appears as a substring in \(y\).

1. Using the pumping lemma technique, prove that the following languages are not regular. For reference, here is the pumping lemma game:

   1. Adversary picks a positive integer \(p\)
   2. You pick \(w \in A\) with \(|w| \geq p\)
   3. Adversary splits up \(w\) into \(w = xyz\) with \(|xy| \leq p\) and \(|y| > 0\)
   4. You pick a nonnegative integer \(i\)

   You win the game if \(xy^iz \notin A\). If you can describe a strategy in which you always win, then \(A\) is not regular.

   (a) \(\{w \in \{a, b\}^* \mid \text{num}(aa, w) = \text{num}(bbb, w)\}\)

   Note: Be careful in your counting: \(bbbb\) contains the substring \(bbb\) two times.

   (b) \(\{w \in \{a, b\}^* \mid |w|\text{ is a square number}\}\)

   Hint: choose \(i\) so you can argue that the length of the resulting string is strictly between adjacent squares, i.e., \(n^2 < |xy^iz| < (n+1)^2\) for some \(n\).

2. Describe a CFG for the following languages:

   (a) \(\{w \in \{0, 1\}^* \mid \overline{w} = \text{rev}(w)\}\)

   Note: \(\overline{w}\) denotes flipping every bit in the string \(w\), for example: \(00101 = 11010\).

   (b) \(\{w \in \{a, b\}^* \mid \text{num}(a, w) = \text{num}(b, w)\}\)

   Hint: Consider the following example where we graph \(\text{num}(a, w) - \text{num}(b, w)\) for all prefixes of \(w\):

   ![Graph Example]

   Imagine breaking up the string into pieces every time the graph crosses the \(x\)-axis. Look at the first and last character of each such piece and think recursively.

3. Describe a PDA for the following language: \(\{w \in \{a, b\}^* \mid \text{num}(aaa, w) = \text{num}(bb, w)\}\)