1. Consider the following method to convert a DFA to a CFG. Let $M = (Q, \Sigma, \delta, s, F)$ be a DFA. Construct a CFG $G$ with:

- nonterminals $\{A_q \mid q \in Q\}$,
- starting nonterminal $A_s$,
- production rules $\{A_p \rightarrow cA_q \mid \delta(p, c) = q\} \cup \{A_p \rightarrow \epsilon \mid p \in F\}$

(a) Using induction on $|w|$, prove that $\delta^*(s, w) = q$ in the DFA if and only if $A_s \Rightarrow_w A_q$ in the CFG. Then prove that $L(M) = L(G)$, hence the conversion from DFA to CFG is correct.

(b) Draw the DFA that identifies multiples of 3 in binary. Give names to the states, and then show the CFG that you get when converting this DFA using the approach above.

Note: You may wish to recall the recursive definition of $\delta^*$:

\[
\delta^*(q, c) \overset{\text{def}}{=} q \\
\delta^*(q, wb) \overset{\text{def}}{=} \delta(\delta^*(q, w), b) \quad \text{when } b \text{ is a single character}
\]

and the recursive definition of $\Rightarrow^*$:

- $\alpha \Rightarrow^* \alpha$, for all $\alpha$
- if $\alpha \Rightarrow^* \beta X Y$, and $X \Rightarrow \delta$ is a production rule, then $\alpha \Rightarrow^* \beta \delta Y$

2. Convert the following CFG to Chomsky normal form (show steps for better partial credit):

\[
S \rightarrow aSdd | T \\
T \rightarrow bTdd | R \\
R \rightarrow cR | \epsilon
\]

3. Show that the following languages are context-free:

(a) $\{a^k b^m c^n \mid k, m, n \text{ not all equal}\}$

(b) $\{x(y \mid x, y \in \{a, b\}^* \text{ and } \text{rev}(x) \text{ is a substring of } y\}$

*In other words*: strings that contain a single $c$ character, where the part of the string occurring before $c$ appears reversed somewhere after the $c$.

*Example*: $aaabbcababbbbaabbaaaabba$ is in the language.