CS 321: Homework #7

Due: Friday Dec 1 at 11:59pm, on Canvas

Homeworks should be typed.

1. We have assumed that a Turing machine can detect the left end of its tape. One way this can be done (without adding it as a “feature” of the TM) is to insert a special left-end-of-tape marker symbol.

Formally describe a TM that does the following. It starts with some string \( w \in \{a, b\}^* \) on its input tape. It should halt with “\( \triangleright \) \( w \)” on its tape and its head on the first character of \( w \). Here \( \triangleright \) is just a new tape symbol. The idea is that this TM could be used as a convenient “initialization subroutine,” after which you can always detect the left end of the tape from the \( \triangleright \) symbol.

\[
\begin{array}{cccccccc}
& a & b & b & a & b & \bigcirc & \cdots & \rightarrow & a & b & b & a & b & \bigcirc & \cdots \\
\downarrow & & & & & & & & & & \downarrow & & & & & & & & \downarrow & \uparrow \\
q_{\text{start}} & & & & & & & & & & q_{\text{halt}} & & & & & & & & \end{array}
\]

Describe your TM using a complete transition table, like the one used in Nov 17 lecture. Give your states human-readable names. Of course you should not assume that you can detect the left end of the tape (chicken & egg problem).

2. Describe a (single-tape) Turing machine for each of the following languages. Please don’t give a complete description of the transition table. Instead, give an algorithm at the same level of detail as in Nov 20 lecture. Clearly describe the tape movement and modifications to the tape cells.

(a) \( \{w \in \{a, b\}^* \mid w \text{ has odd length and its middle character is } a\} \)

(b) \( \{a^nb^mc^k \mid k = nm\} \)

(c) \( \{a^n \mid n \text{ is a power of } 2\} \)