Final Problems 1

Due Friday Sep 29

1. Show that the following language is regular (which may not involve explicitly constructing a DFA):

\[ \{ w \in \{a, b\}^* \mid \text{first 3 characters of } w \text{ are equal to the last 3 characters of } w \}\]

2. Give a DFA for the following language:

\[ \{ w \in \{a, b\}^* \mid \text{the last character of } w \text{ appears at least 2 times in } w \}\]

Explain in plain language the meaning of every state.

3. Consider the alphabet \( \Sigma = \{[000], [001], \ldots, [111]\} \). We can consider strings in this alphabet to consist of a “top, middle, and bottom row.” Let \( A \) be the set of strings whose top, middle, and bottom rows encode binary numbers \( x, y, z \), respectively, where \( x+y = z \). For example,

\[
\begin{bmatrix}
[0] & [1] & [0] \\
[0] & [0] & [1] \\
[1] & [0] & [0] \\
\end{bmatrix} \in A \quad \text{but} \quad \begin{bmatrix}
[0] & [0] & [0] \\
\end{bmatrix} \notin A
\]

Show that \( A \) is regular.