Final Problems 2

Due Friday October 6

1. (a) If $M$ is an NFA, show that the following languages are regular:

\[
\text{MultiPath}(M) = \{ x | \text{there is more than one accepting path for } M \text{ on } x \} \\
\text{SinglePath}(M) = \{ x | \text{there is exactly one accepting path for } M \text{ on } x \}
\]

Note: Two distinct accepting paths need not end in different states — they could diverge and later converge.

Hint: Modify the subset construction to keep track of some extra information.

(b) Let $A$ be a regular language. Show that the following language is also regular:

\[
\text{UnAmbig}(A) = \{ x | \text{there is exactly one way to write } x = x_1 \cdots x_k \text{ where each } x_i \in A \}
\]

For instance, if $A$ is the set of all English words, then \textit{theoryofcomputation} is in the language (there’s only one way to break it apart into English words) but \textit{bestart} is not (it can be written as \textit{best}·\textit{art} or \textit{be}·\textit{start}).

2. Let $A$ be an arbitrary regular language over alphabet $\Sigma = \{a, b\}$. Show that the following languages are also regular:

\[
\text{SwapOne}(A) = \{ x | \exists y \in A : x \text{ and } y \text{ differ in exactly one character} \}
\]

\[
\text{DropOne}(A) = \{ uv | \exists c \in \Sigma : ucv \in A \}
\]

\[
= \{ x | \exists y \in A : x \text{ is the result of removing exactly 1 character from } y \}
\]

\[
\text{AddOne}(A) = \{ ucv | c \in \Sigma \text{ and } uv \in A \}
\]

\[
= \{ x | \exists y \in A : x \text{ is the result of adding exactly 1 character from } y \}
\]

3. Let $M = (Q, \Sigma, \delta, s, F)$ be a DFA.

(a) For all $p, q \in Q$, show that the following language is regular:

\[
A_{p,q} \overset{\text{def}}{=} \{ w | \widehat{\delta}(s, q) = p \}
\]

(b) Remember the puzzle from the first lecture? A \textit{combobulation string} for $M$ is any string such that after reading that string there is no doubt about which state $M$ is in, regardless of which state you started from. Show that the set of combobulation strings for $M$ is regular.

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1For a real-world application, see https://www.boredpanda.com/worst-domain-names/.