CS 517: Problem Set 1

1. Prove that the following problem is undecidable.

   Given 2×2 matrices $A_1, \ldots, A_k$ and $B_1, \ldots, B_k$, each with *integer entries*, decide whether there exist $n > 0$ and indices $i_1, \ldots, i_n \in \{1, \ldots, k\}$ such that

   $$A_{i_1} \times A_{i_2} \times \cdots \times A_{i_n} = B_{i_1} \times B_{i_2} \times \cdots \times B_{i_n}.$$

2. A Turing printer (TP) is a Turing machine with a work tape, a special "print" tape, and a special "print" state. The TM starts with an empty work tape. Every time it enters the "print" state, we consider the current contents of the print-tape to be "printed." If P is a TP, then we write $L(P)$ to denote the set of strings that P eventually prints.

   (a) Prove that a language L is Turing-recognizable **if and only if** $L = L(P)$ for some TP P.

   (b) Prove that a language L is Turing-decidable **if and only if** $L = L(P)$ for some TP P that prints strings in lexicographic order.

   (c) Prove that every *infinite* Turing-recognizable language L contains an *infinite* subset that is Turing-decidable.

3. (a) Show that the following language is Turing-**decidable**:

   $$L = \{\langle M, x\rangle \mid M \text{ eventually changes a space character to a non-space character on input } x\}$$

   (b) Let # be a special tape symbol (distinct from 0, 1, or the blank symbol). Show that the following language is **not** Turing-decidable:

   $$L = \{\langle M, x\rangle \mid M \text{ eventually changes a space character to \# on input } x\}$$