CS 517: Problem Set 1

1. Prove that the following problem is undecidable.

   Given $2 \times 2$ matrices $A_1, \ldots, A_k$ and $B_1, \ldots, B_k$, each with integer entries, decide whether there exist $n > 0$ and indices $i_1, \ldots, i_n \in \{1, \ldots, k\}$ such that

   $A_{i_1} \times A_{i_2} \times \cdots \times A_{i_n} = B_{i_1} \times B_{i_2} \times \cdots \times B_{i_n}$.

2. A Turing printer (TP) is a Turing machine with a work tape, a special "print" tape, and a special "print" state. The TM starts with an empty work tape. Every time it enters the "print" state, we consider the current contents of the print-tape to be "printed." If $P$ is a TP, then we write $L(P)$ to denote the set of strings that $P$ eventually prints.

   (a) Prove that a language $L$ is Turing-recognizable if and only if $L = L(P)$ for some TP $P$.

   (b) Prove that a language $L$ is Turing-decidable if and only if $L = L(P)$ for some TP $P$ that prints strings in lexicographic order.

   (c) Prove that every infinite Turing-recognizable language $L$ contains an infinite subset that is Turing-decidable.

3. (a) Show that the following language is Turing-decidable:

   $L = \{\langle M, x \rangle \mid M \text{ eventually changes a space character to a non-space character on input } x\}$

   (b) Let # be a special tape symbol (distinct from 0, 1, or the blank symbol). Show that the following language is not Turing-decidable:

   $L = \{\langle M, x \rangle \mid M \text{ eventually changes a space character to } # \text{ on input } x\}$