CS 517: Homework #1 Hints

1. Imagine a standard Turing machine $M$ whose computation begins in the following way:

$\begin{array}{c}
\text{0 1 1 0 1 \ldots} \\
\text{q_{start}}
\end{array}$

$\begin{array}{c}
\text{1 1 1 0 1 \ldots} \\
\text{q_{11}}
\end{array}$

$\begin{array}{c}
\text{1 1 1 0 1 \ldots} \\
\text{q_{5}}
\end{array}$

An ETM can simulate this computation by remembering $M$'s internal state in its own internal state, and writing the sequence of configurations on its tape:

$\begin{array}{c}
\hat{0}1101\ldots, \\
\hat{1}1101\ldots, \\
1\hat{1}01\ldots
\end{array}$

Here $\hat{0}$ and $\hat{1}$ are just additional tape symbols different from 0 & 1. Suppose an ETM has such a TM configuration written somewhere on its tape. Show how it can write down the next TM configuration in sequence elsewhere on its tape, while obeying the restrictions of an ETM. You have to be careful that your solution works no matter what kind of transition $M$ takes.

Additional technical note: In this problem, you are showing that for every standard TM $M$, there exists an ETM $M'$ with the same behavior (i.e., $M'$ accepts whenever $M$ accepts, $M'$ rejects whenever $M$ rejects). So the design of $M'$ can depend on $M$. In particular, you do not need to design a universal ETM that can simulate every TM. This is why it’s safe in this problem to store $M$'s internal state in $M'$’s internal state — the number of states in $M'$ can depend on the number of states in $M$. 
