

**CS 517: Problem Set 3**

1. Construct a language  $B$  such that  $\text{NP}^B \neq \text{coNP}^B$ .

*Hint:* You can construct a  $B$  such that  $L_B = \{x \mid \exists y \in B : |x| = |y|\} \in \text{NP}^B \setminus \text{coNP}^B$ .

2. Let  $\Delta = \text{NP} \cap \text{coNP}$ . Show that  $\Delta = \text{P}^\Delta$ .

*Note:* Think about what it means to be in  $\text{NP} \cap \text{coNP}$ . If someone gives you an arbitrary problem  $L \in \text{NP} \cap \text{coNP}$ , it means they are giving you both an “NP way” and a “coNP way” of writing the same problem  $L$ .

3. Show that  $\Sigma_k = \text{NP}^{\Sigma_k \cap \Pi_k}$ , for all  $k$ .

*Note:* Similar to above, if you have an oracle for  $L \in \Sigma_k \cap \Pi_k$ , then  $L$  has a “ $\Sigma_k$  way” and a “ $\Pi_k$  way” of being written, and you can choose whichever is more preferable to you.