CS 517: Problem Set 3

1. Construct a language $B$ such that $\text{NP}^B \neq \text{coNP}^B$.

   Hint: You can construct a $B$ such that $L_B = \{ x \mid \exists y \in B : |x| = |y| \} \in \text{NP}^B \setminus \text{coNP}^B$.

2. Let $\Delta = \text{NP} \cap \text{coNP}$. Show that $\Delta = \text{P}^\Delta$.

   Note: Think about what it means to be in $\text{NP} \cap \text{coNP}$. If someone gives you an arbitrary problem $L \in \text{NP} \cap \text{coNP}$, it means they are giving you both an “NP way” and a “coNP way” of writing the same problem $L$.

3. Show that $\Sigma_k = \text{NP}^{\Sigma_k \cap \Pi_k}$, for all $k$.

   Note: Similar to above, if you have an oracle for $L \in \Sigma_k \cap \Pi_k$, then $L$ has a “$\Sigma_k$ way” and a “$\Pi_k$ way” of being written, and you can choose whichever is more preferable to you.