

CS 517: Problem Set 4

1. The standard complexity classes PP, BPP, etc., are defined in terms of **strict** polynomial-time.

Define ePP to be the same as PP, except we allow the Turing machine to be **expected**-polynomial-time. That is, $L \in \text{ePP}$ if there exists an expected-poly-time probabilistic TM M with $L = \{x \mid \Pr[M(x) = 1] > 1/2\}$.

Recall that EXP are the problems that can be solved deterministically in time $2^{n^{O(1)}}$. Show that $\text{EXP} \subseteq \text{ePP}$.

2. Let L be a problem in RP, so that:

$$x \in L \Rightarrow \Pr[M(x) = 1] \geq 1/2$$

$$x \notin L \Rightarrow \Pr[M(x) = 0] = 1$$

(for some PPT M) Show that for each n , there exists a set R_n of random tapes for M with $|R_n| = n$ and satisfying the following property:

$$x \in L \iff \exists r \in R_{|x|} : M(x; r) = 1$$

Show how this proves $\text{RP} \subseteq \text{P/poly}$.

Hint: Show that there exists a single r that works for at least half of the length- n inputs, then repeat.