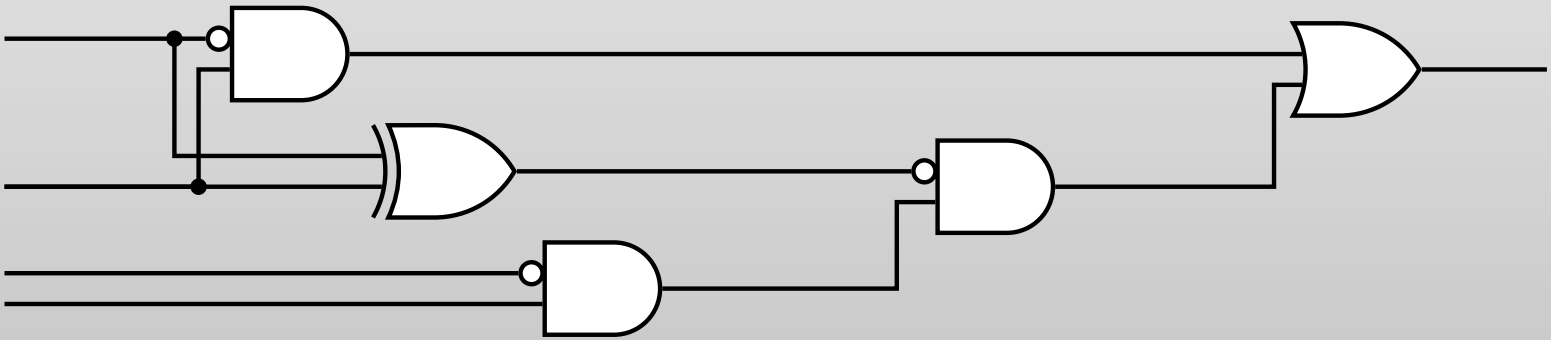
Garbled Circuits: New Results for Arithmetic and High Fan-In Computations

Mike Rosulek

Oregon State University OSU



Joint work with Marshall Ball & Tal Malkin

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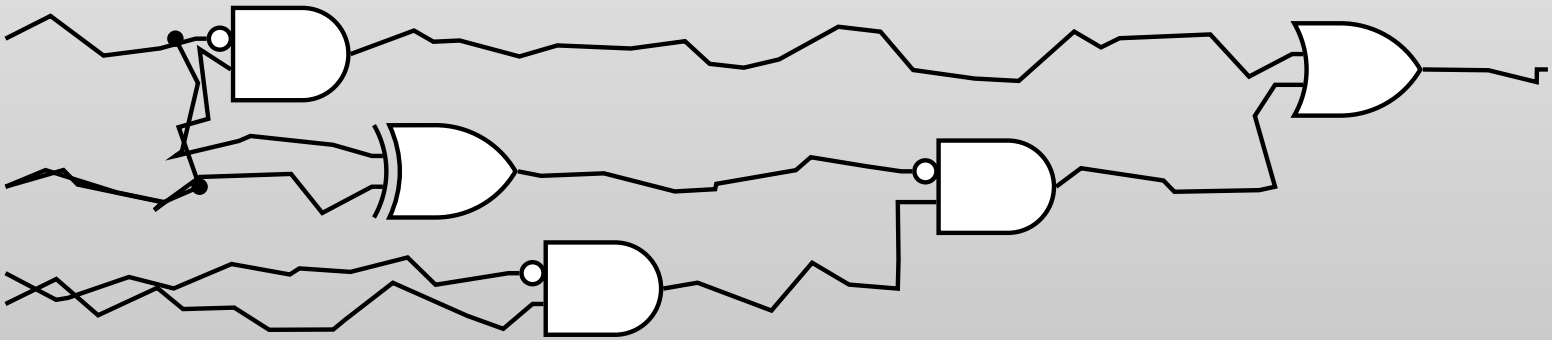

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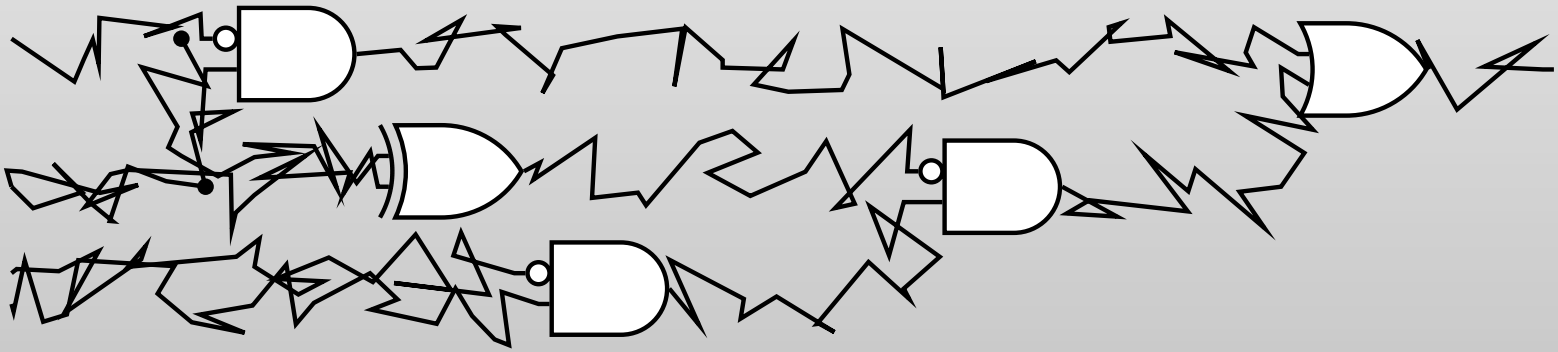

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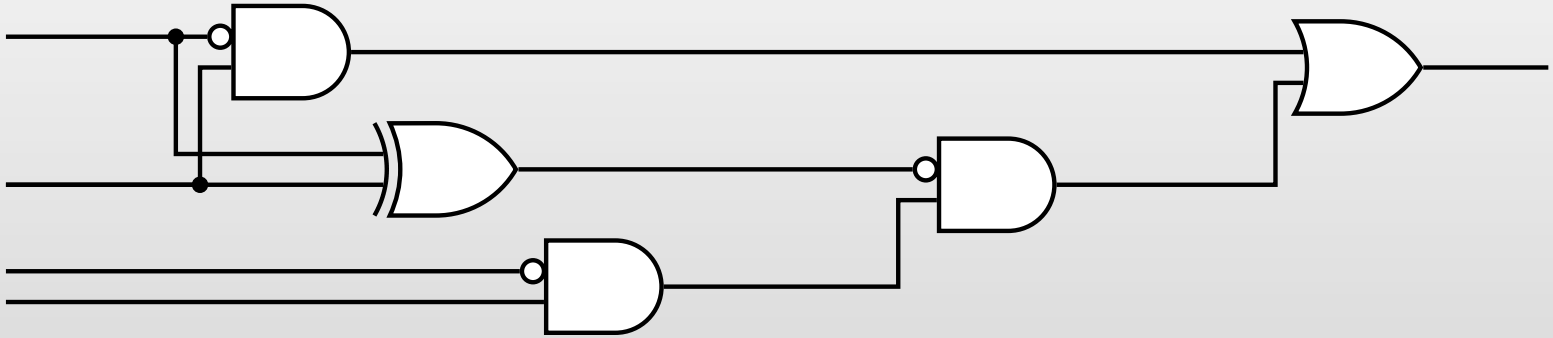
Garbled circuit framework [Yao86]



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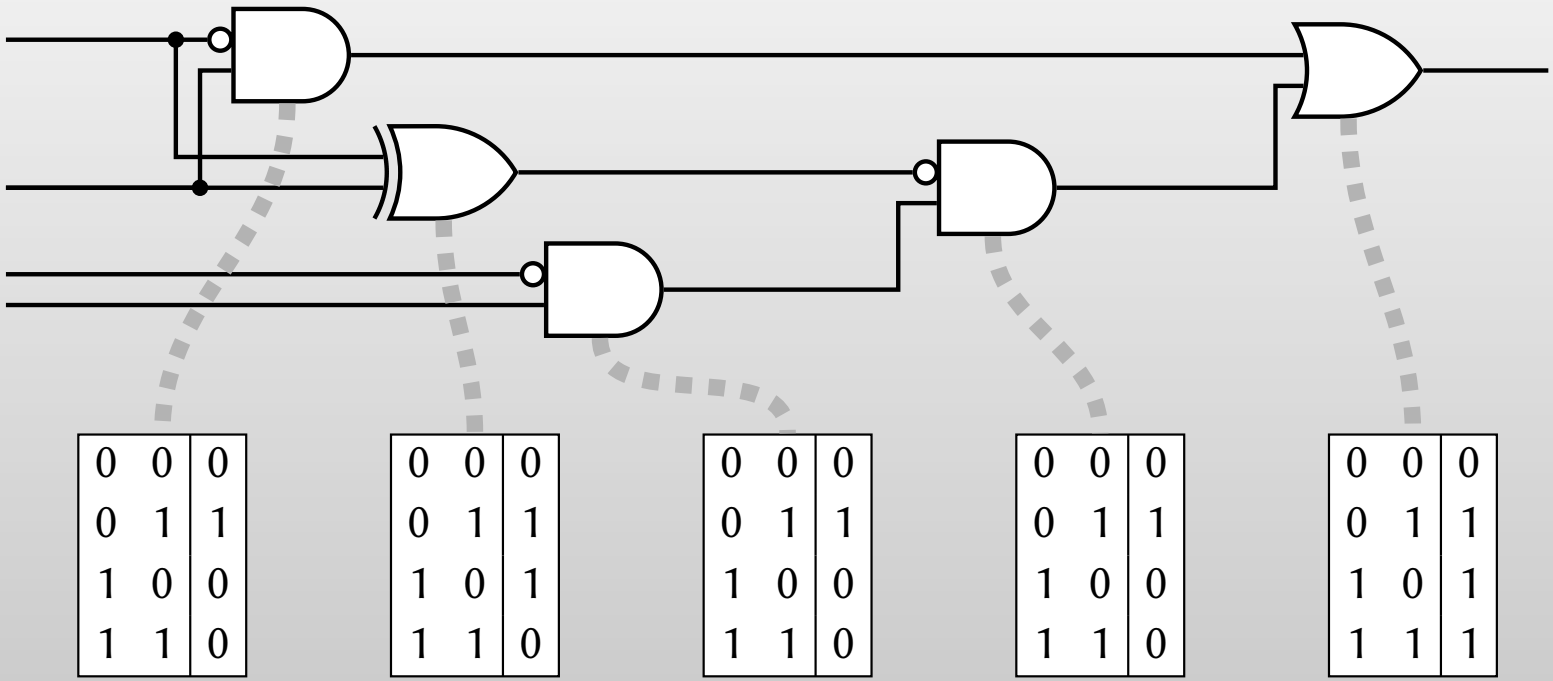

0	0	0
0	1	1
1	0	0
1	1	0

0	0	0
0	1	1
1	0	1
1	1	0

0	0	0
0	1	1
1	0	0
1	1	0

0	0	0
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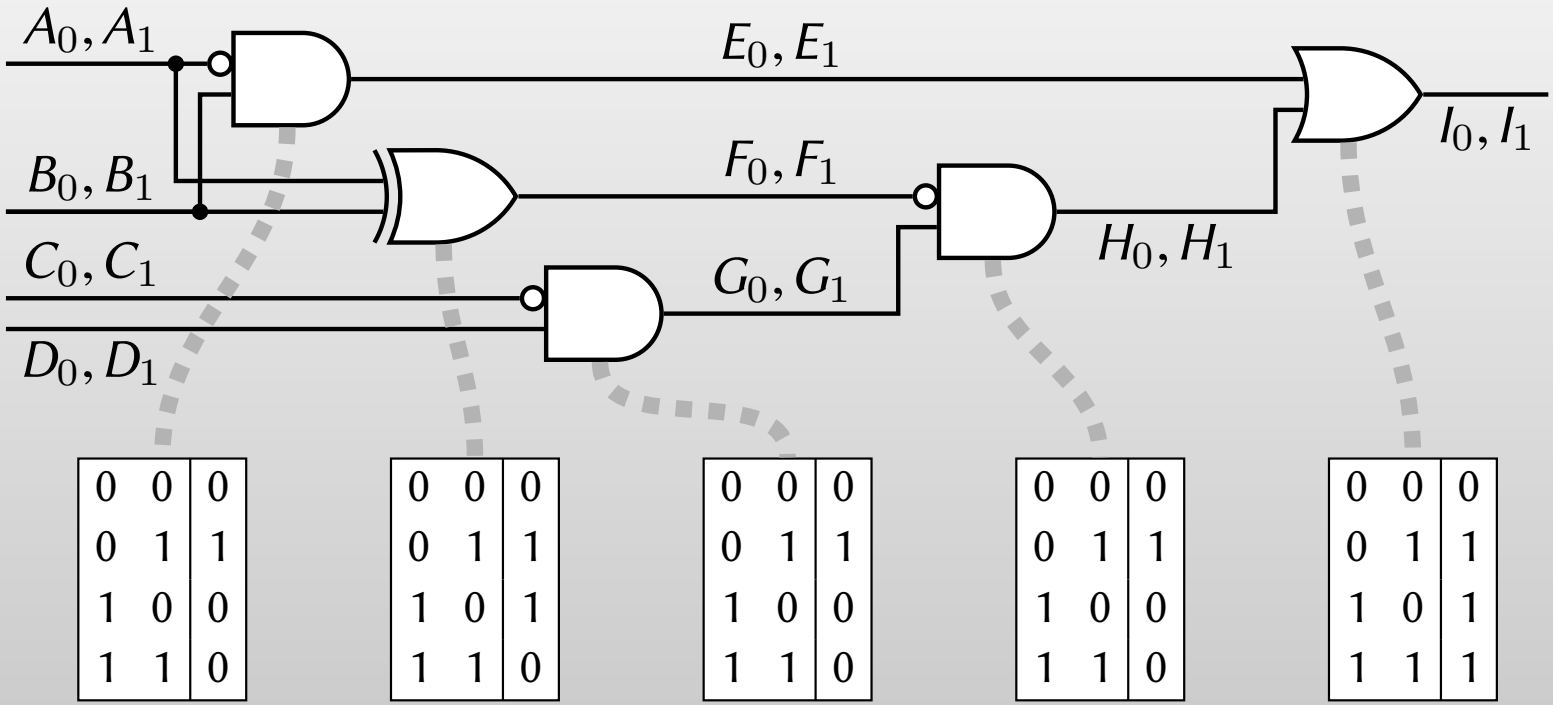

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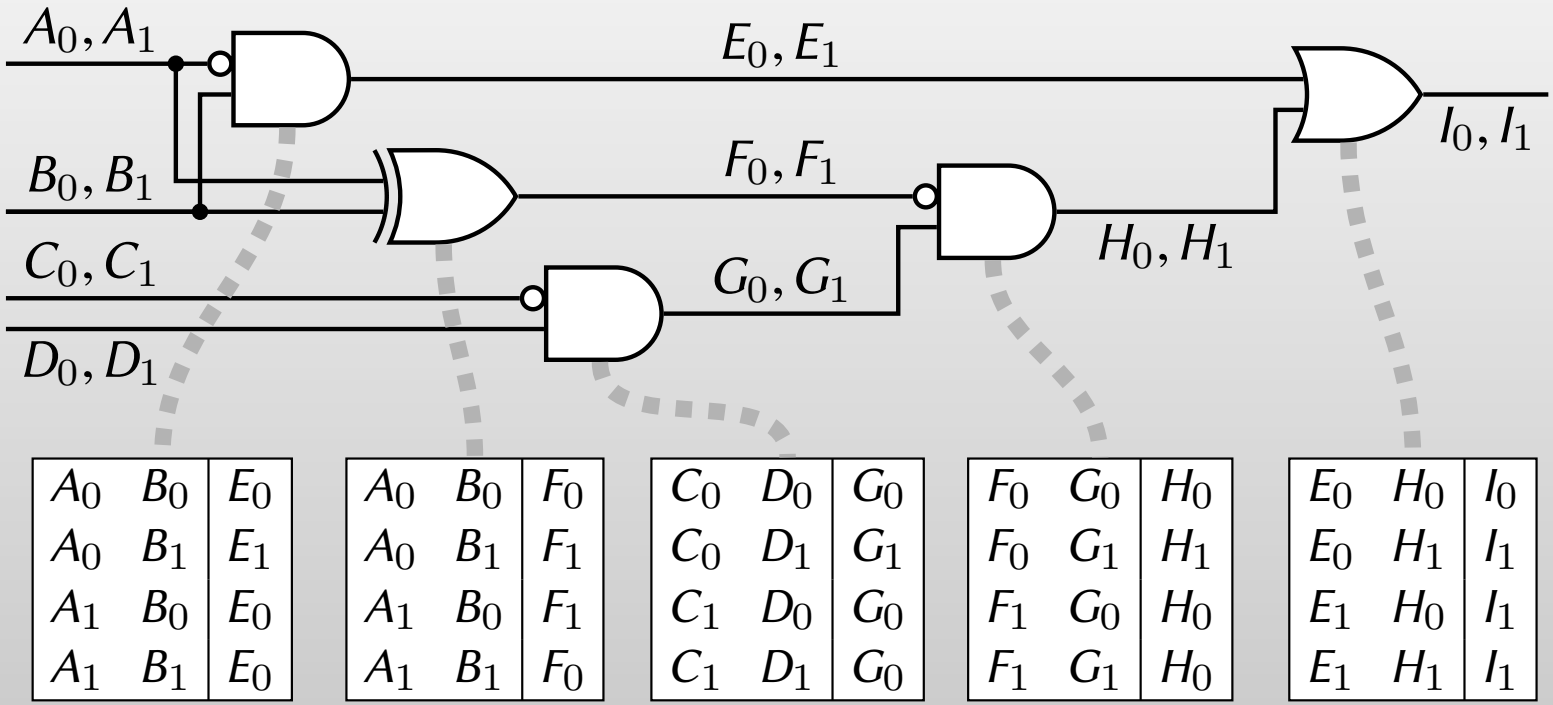

A_0	B_0	E_0
A_0	B_1	E_1
A_1	B_0	E_0
A_1	B_1	E_0

A_0	B_0	F_0
A_0	B_1	F_1
A_1	B_0	F_1
A_1	B_1	F_0

C_0	D_0	G_0
C_0	D_1	G_1
C_1	D_0	G_0
C_1	D_1	G_0

F_0	G_0	H_0
F_0	G_1	H_1
F_1	G_0	H_0
F_1	G_1	H_0

E_0	H_0	I_0
E_0	H_1	I_1
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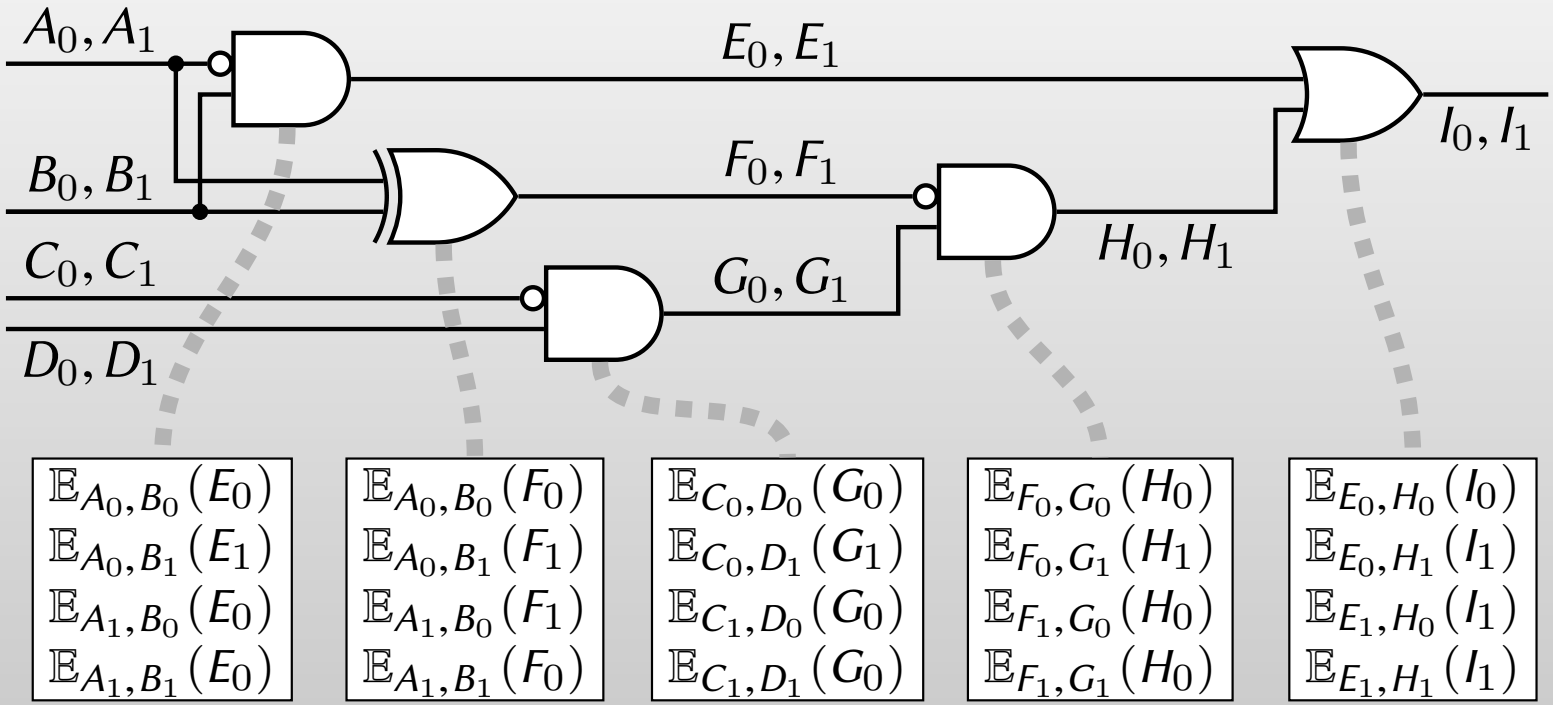

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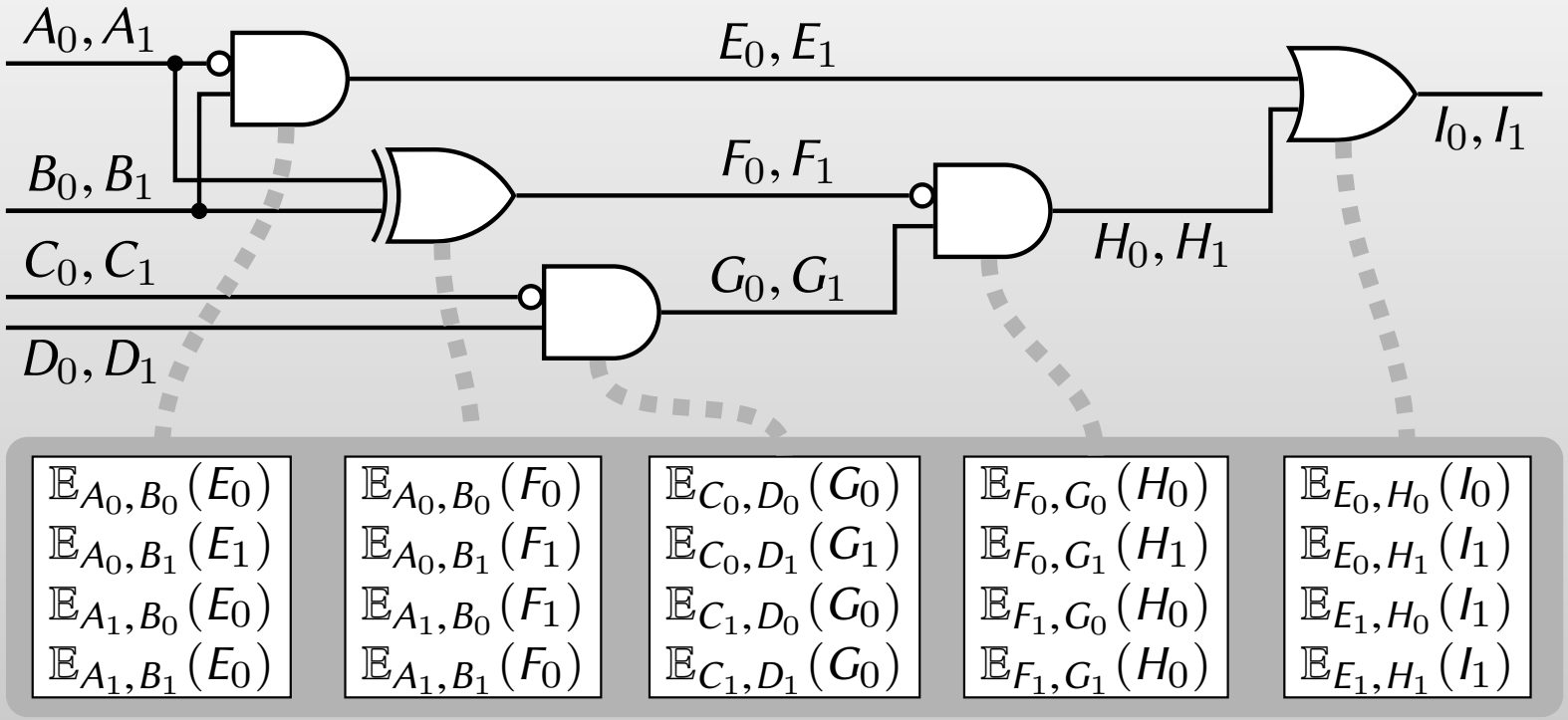

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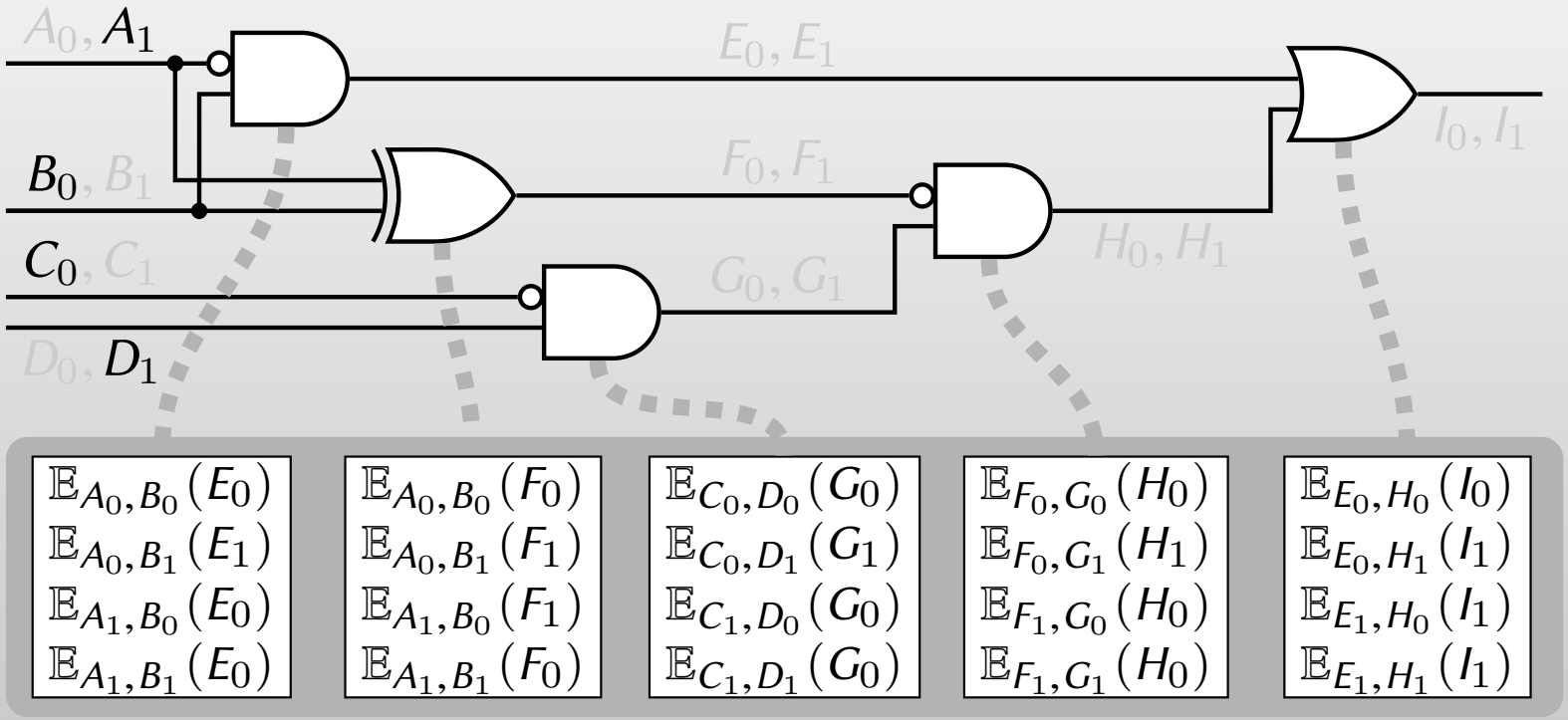

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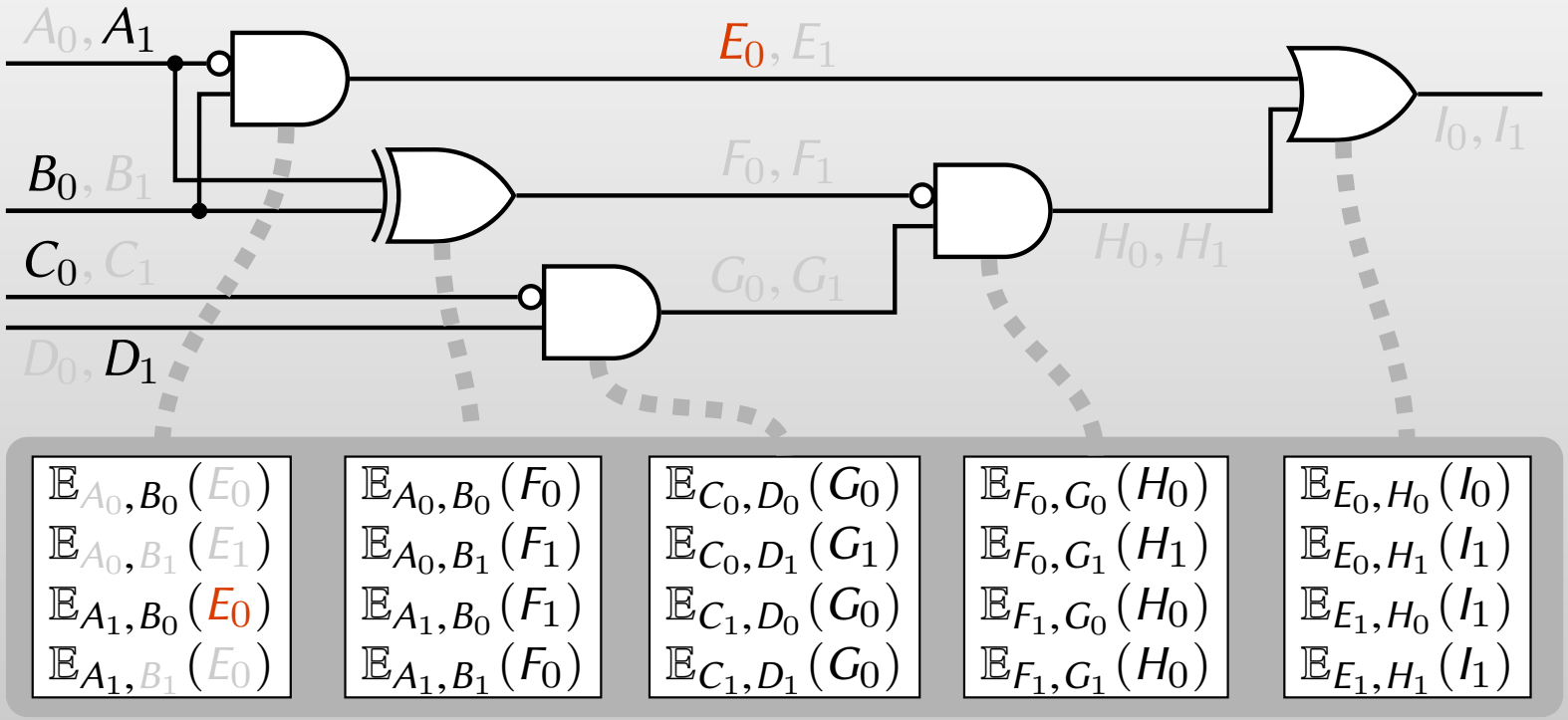

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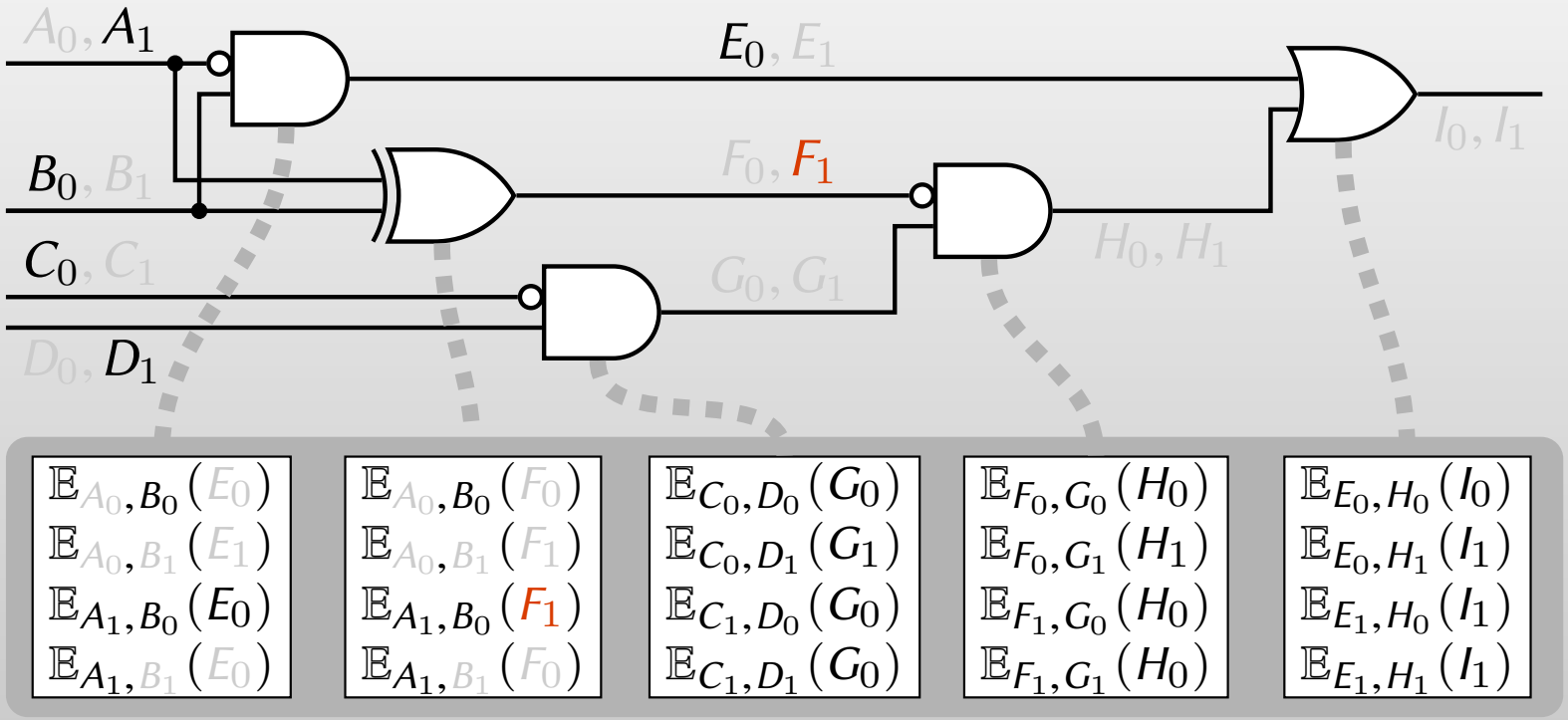

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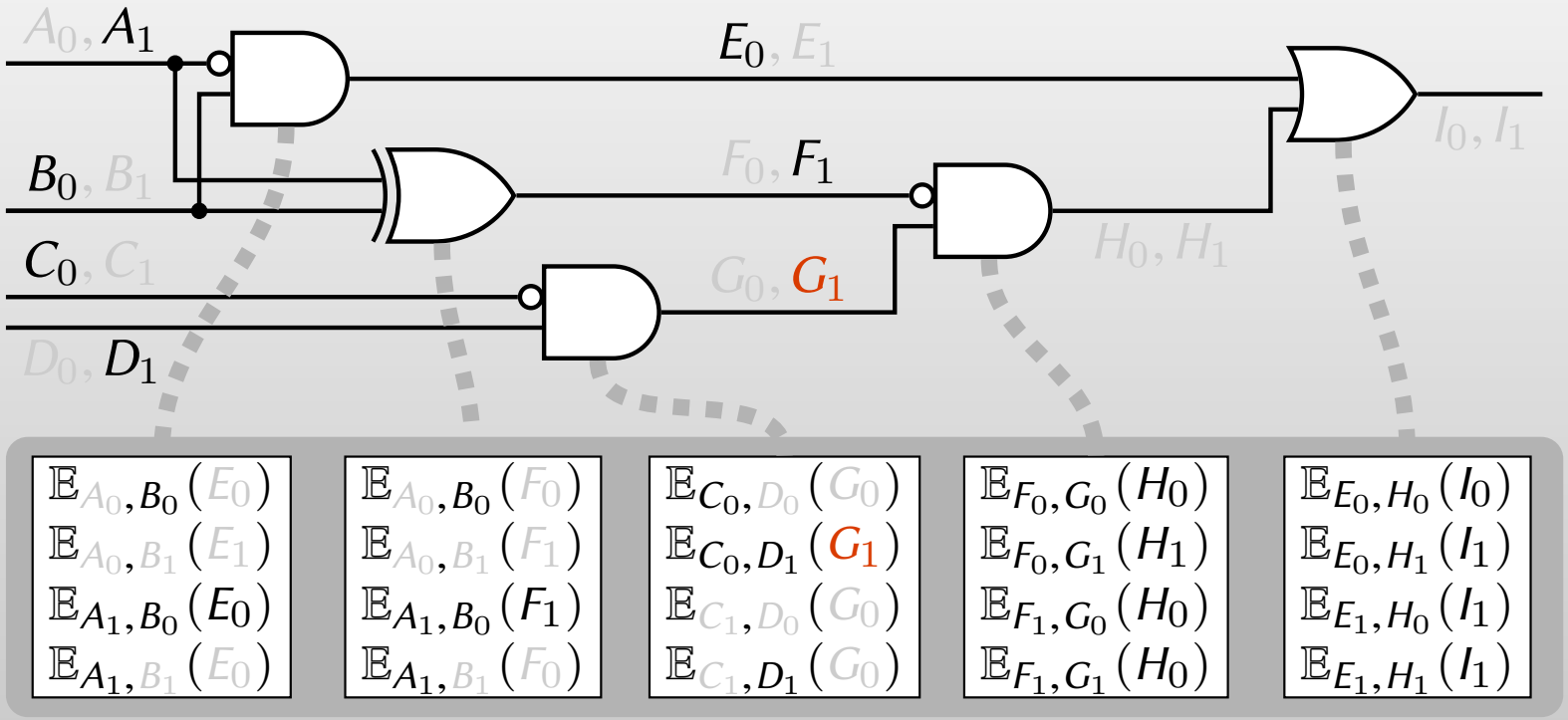

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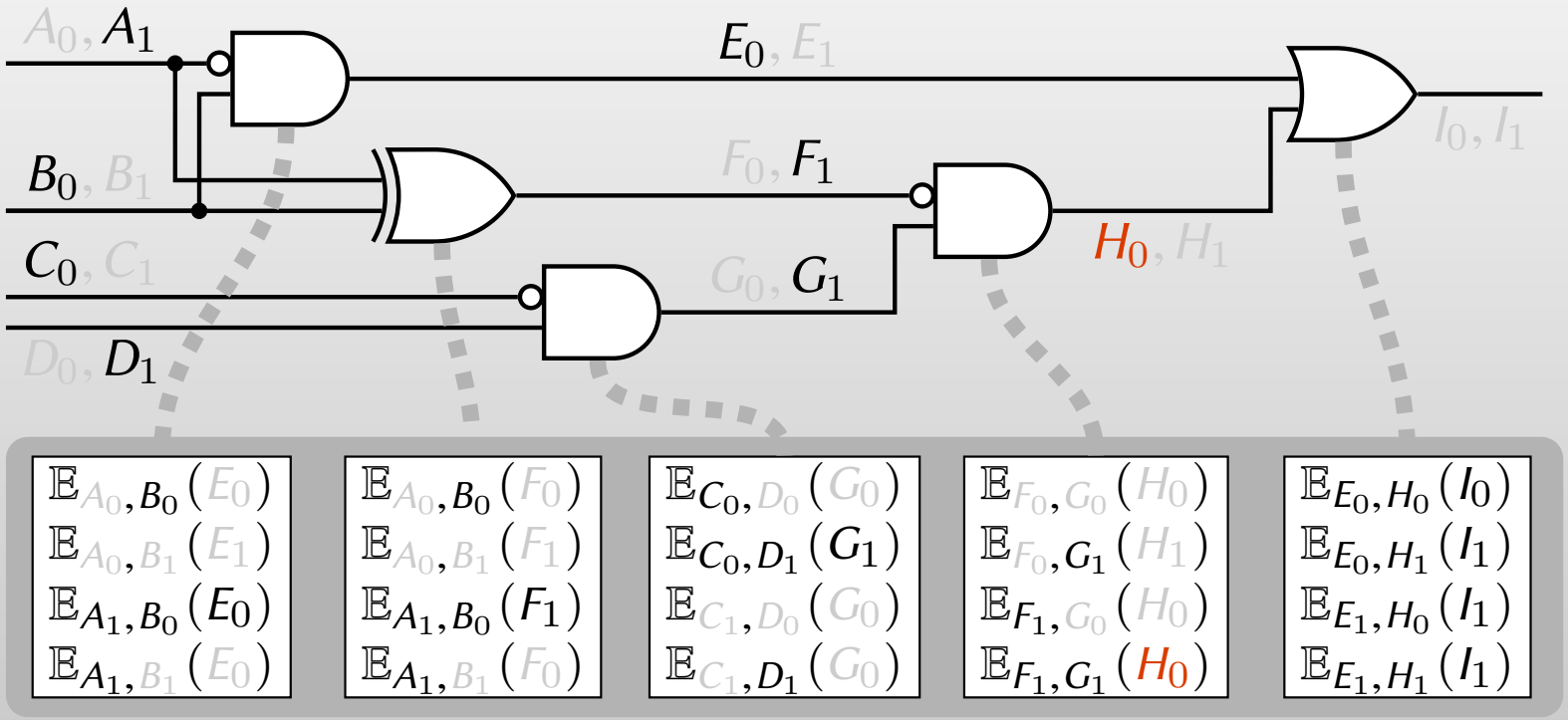

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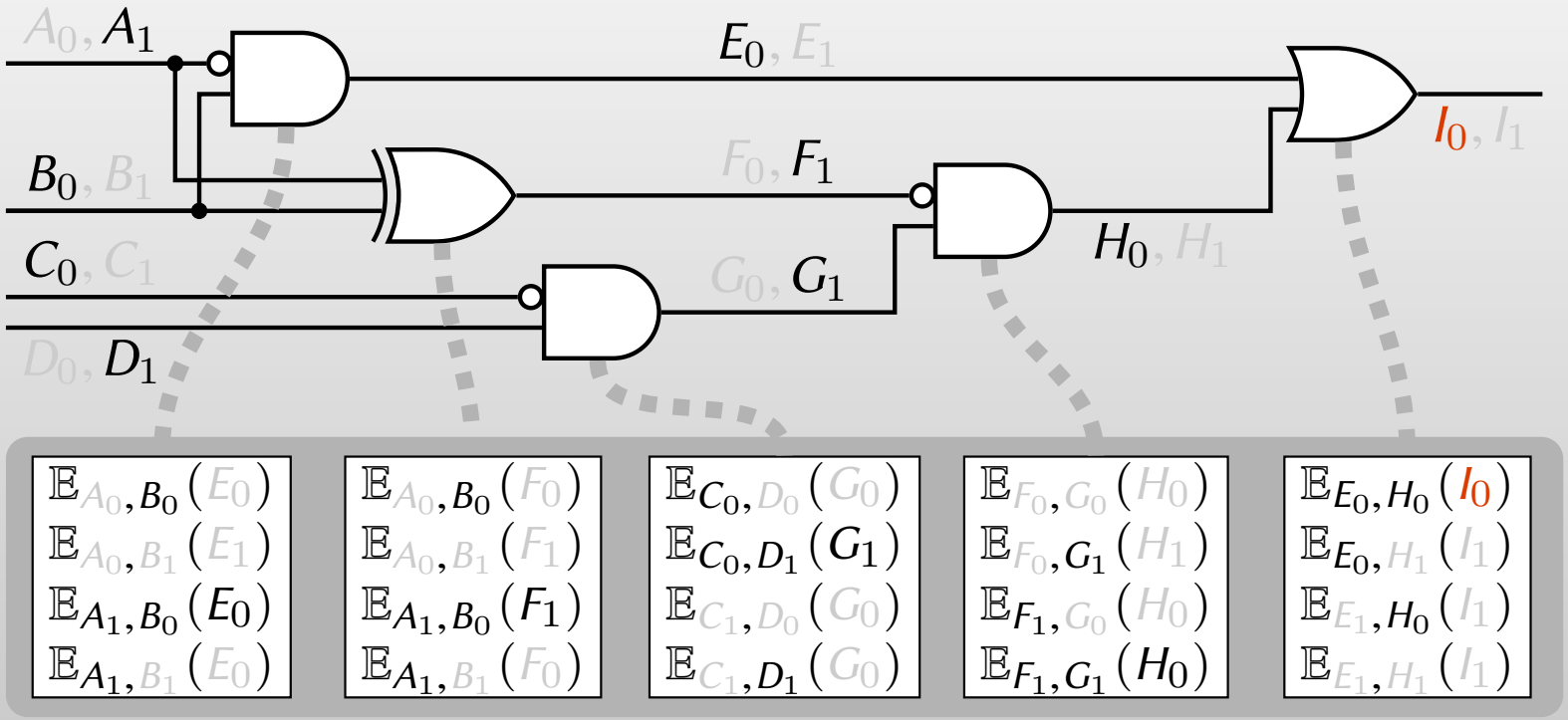

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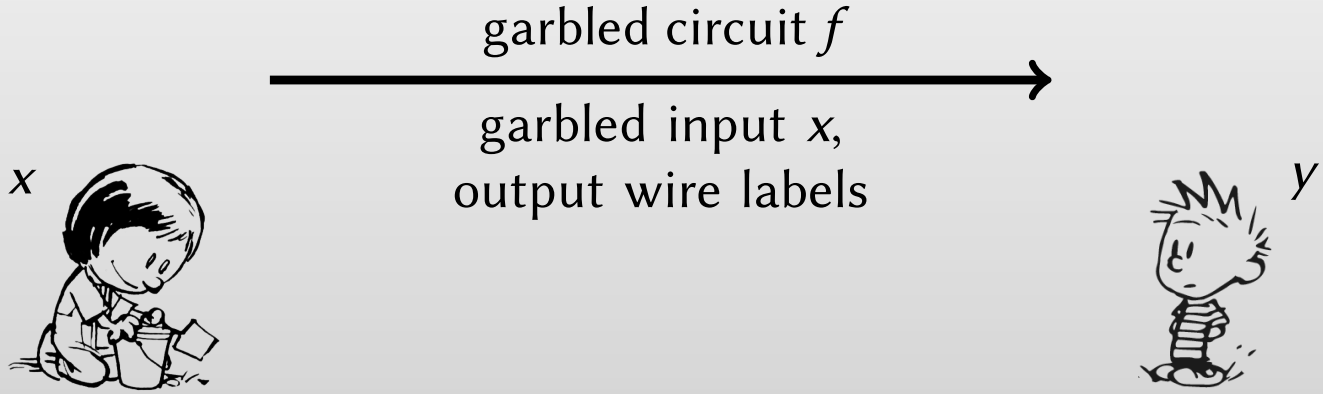

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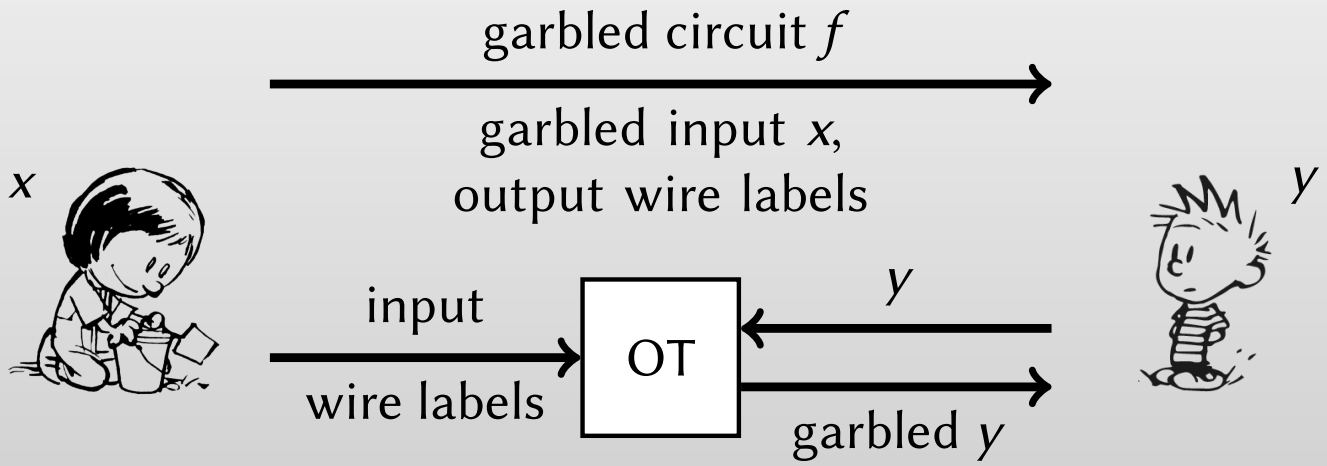

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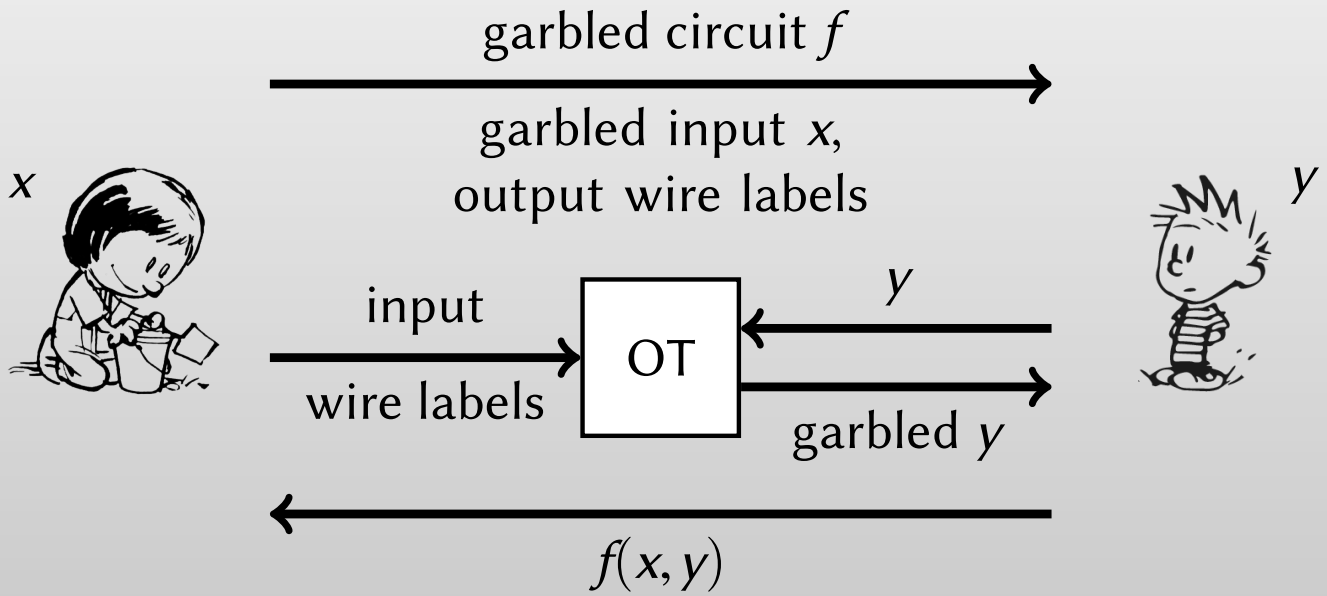

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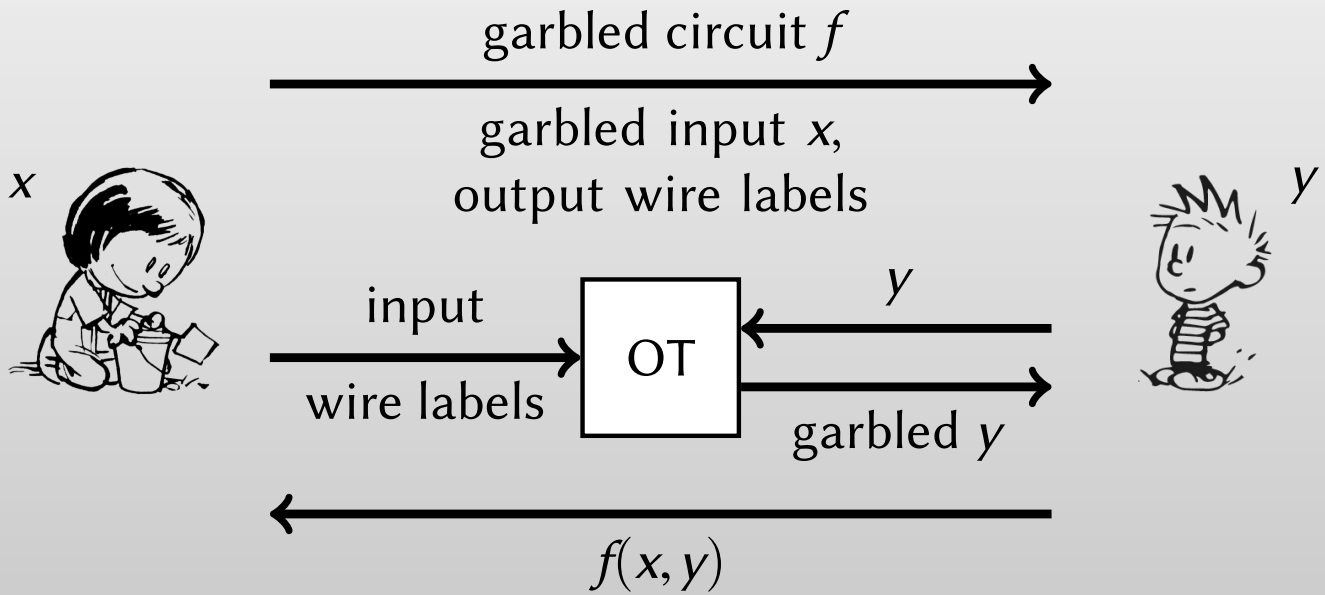

Other applications: Private function evaluation, zero-knowledge proofs, encryption with key-dependent message security, randomized encodings, secure outsourcing, one-time programs, . . .

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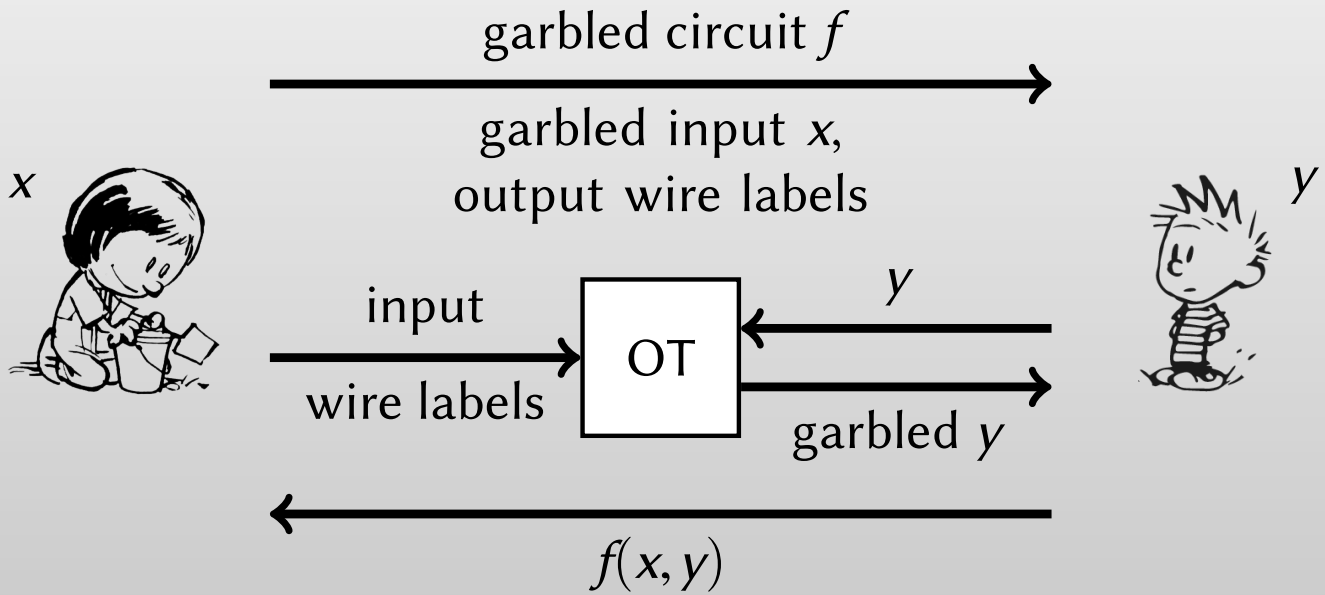

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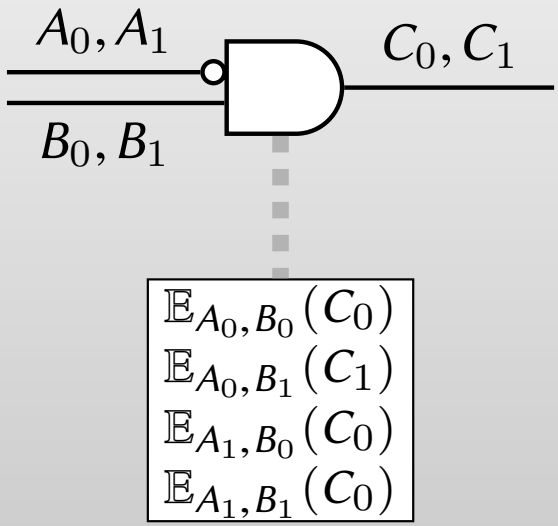
Consistent bottleneck: **size** of garbled circuits

Making garbled circuits smaller

Ciphertext expansion [Yao86]



Position in this list leaks semantic value!

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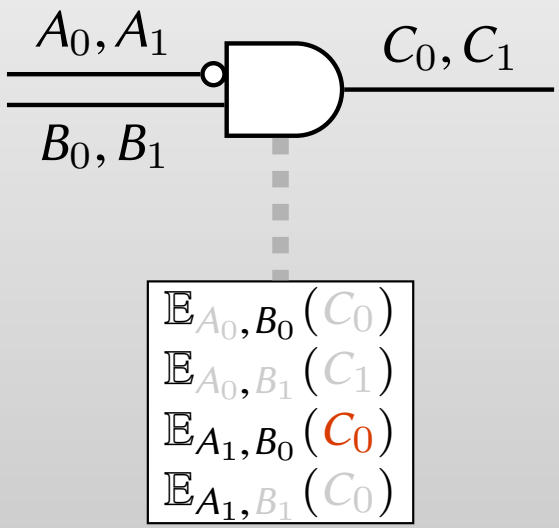

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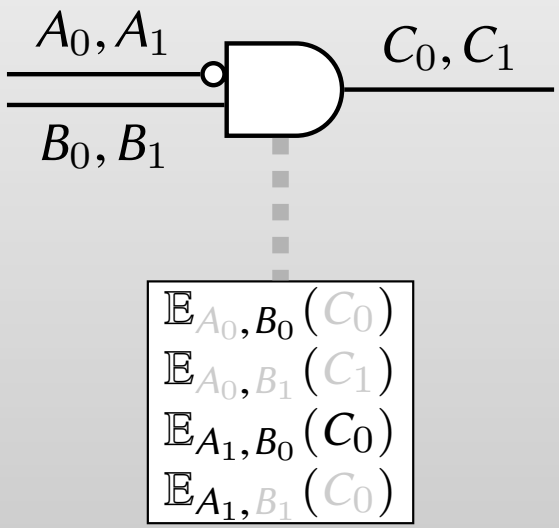

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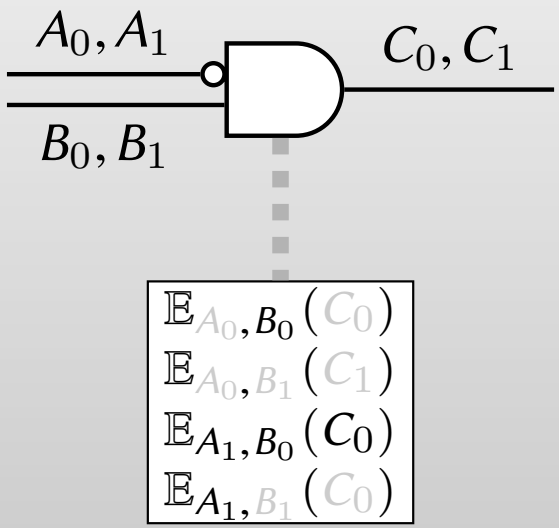

Position in this list leaks semantic value!

- $\Rightarrow$ Need to randomly permute ciphertexts
- $\Rightarrow$ Need to **detect** [in]correct decryption

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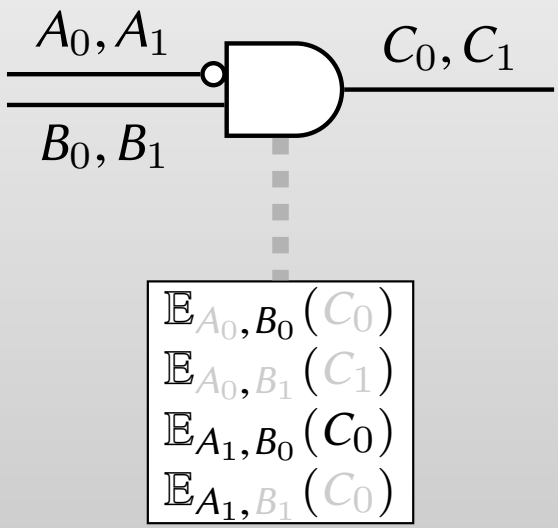
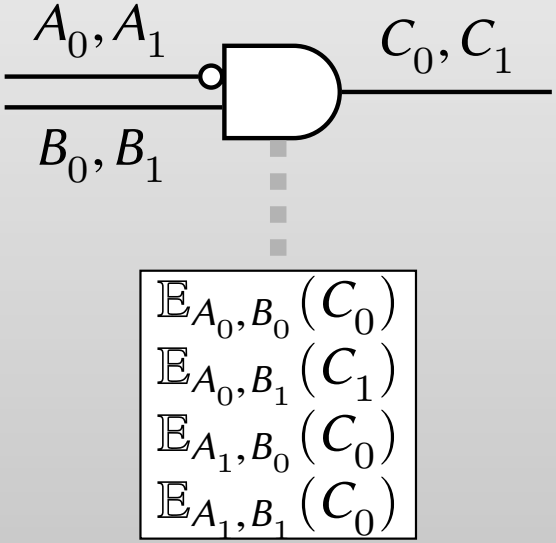


Position in this list leaks semantic value!

- ⇒ Need to randomly permute ciphertexts
- ⇒ Need to **detect** [in]correct decryption
- ⇒ Need encryption scheme with *ciphertext expansion* (size doubles)

Permute-and-Point [BeaverMicaliRogaway90]



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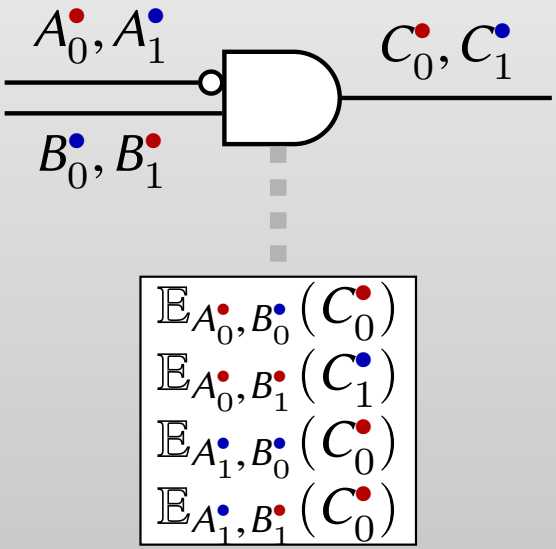

- Randomly assign $(\bullet, \bullet)$ or $(\bullet, \bullet)$ to each pair of wire labels
- Include color in the wire label (e.g., as last bit)

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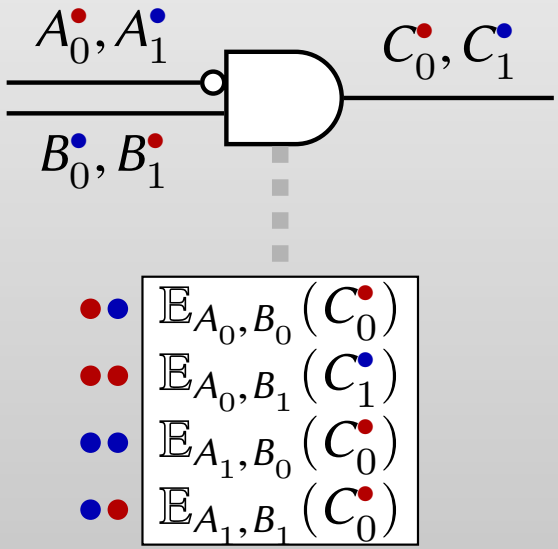

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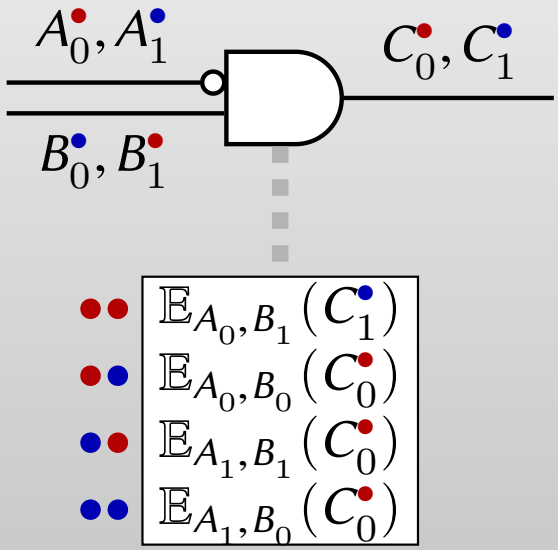

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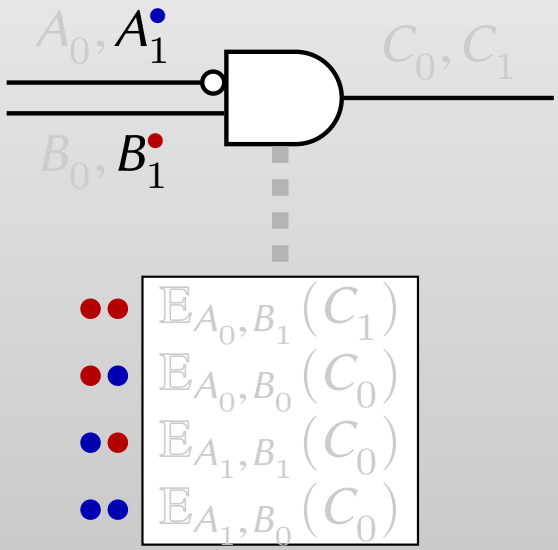

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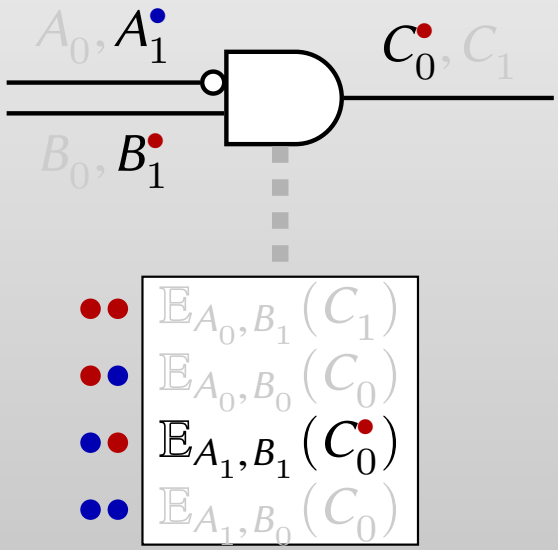

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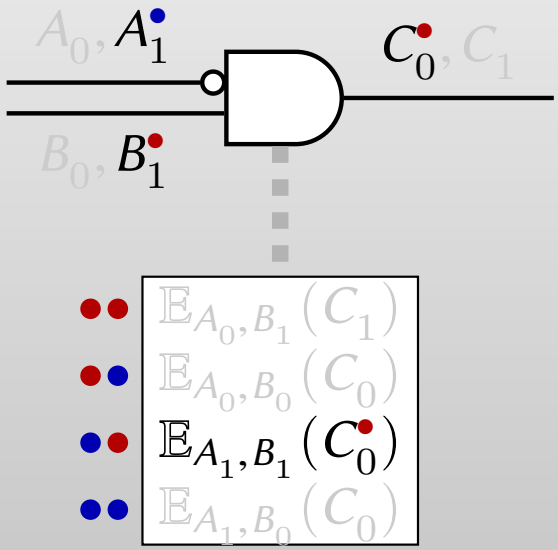

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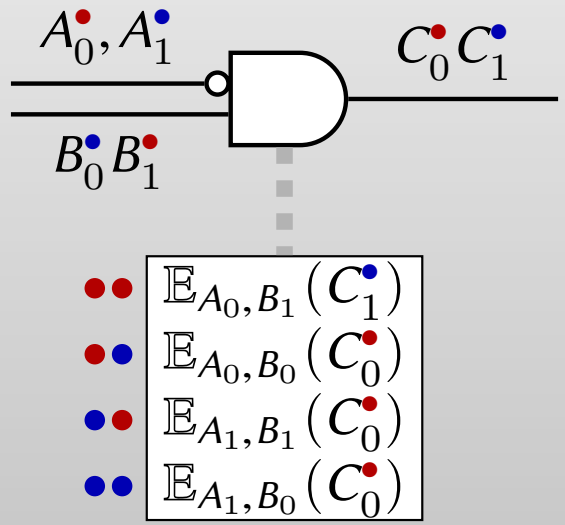
Can use **one-time-secure** encryption (no ciphertext expansion!)

Garbled Row Reduction [NaorPinkasSumner99]



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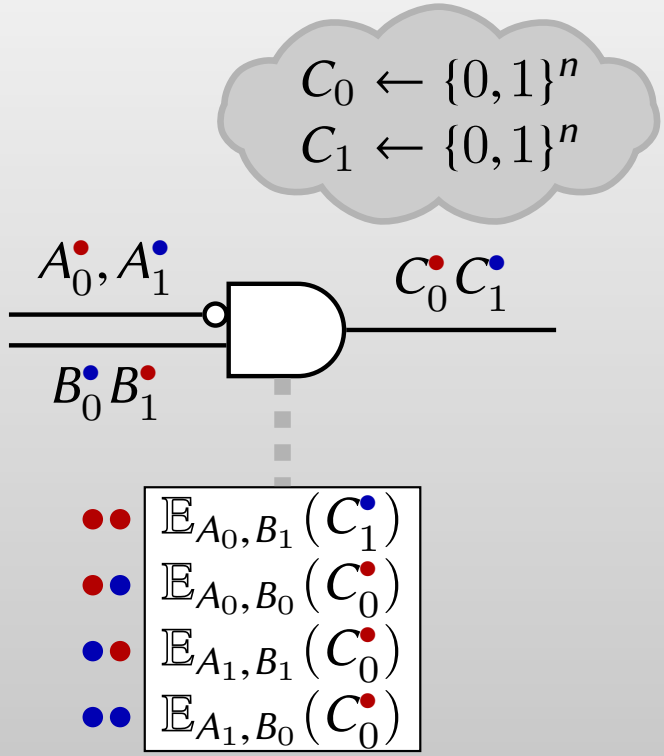

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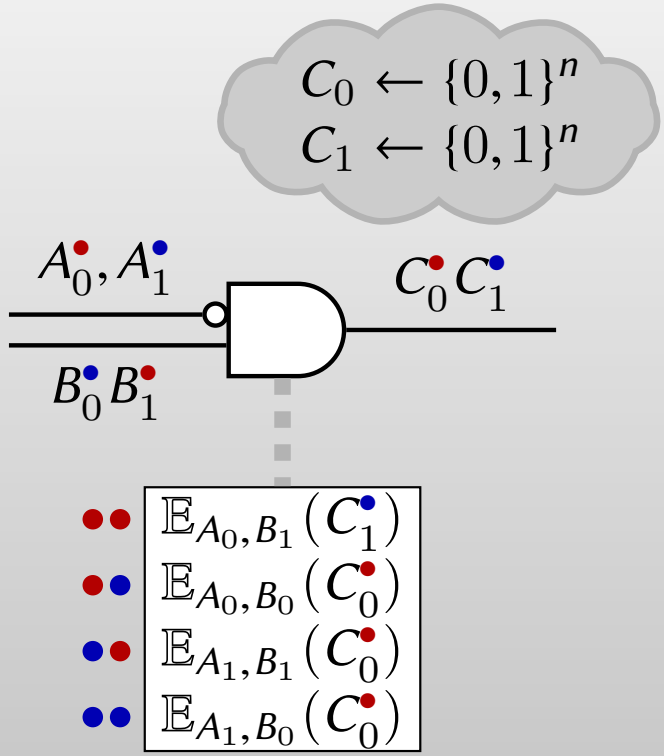

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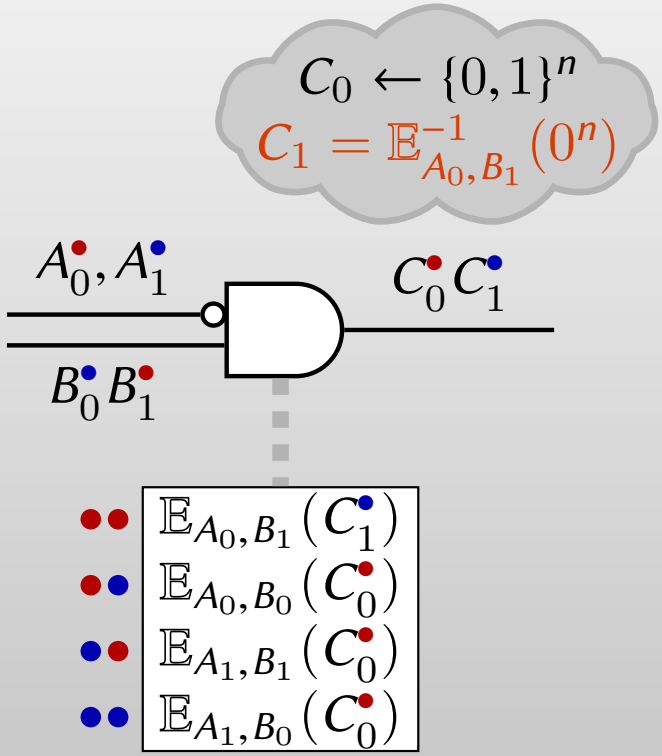

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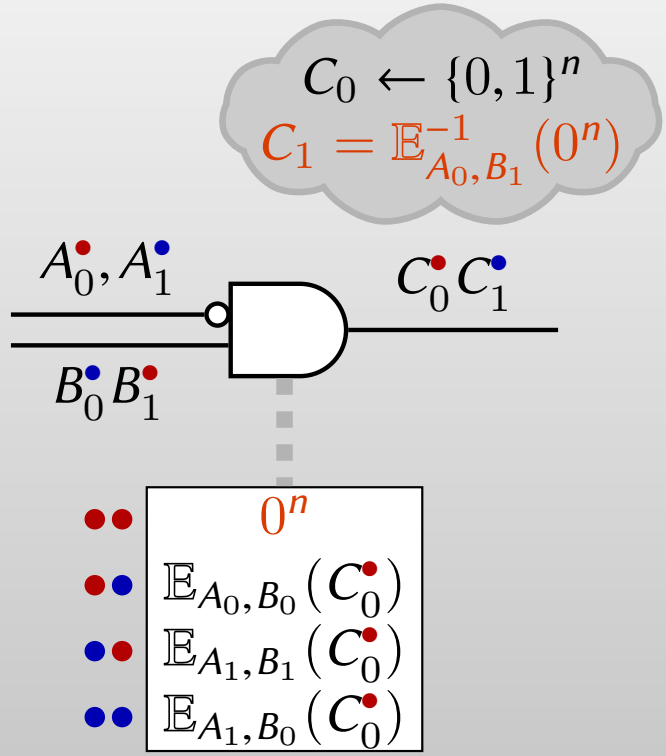

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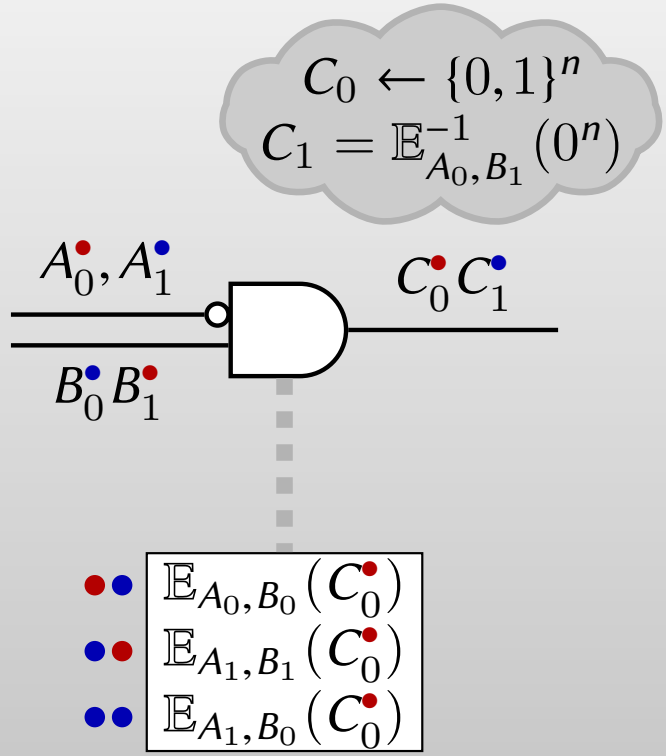

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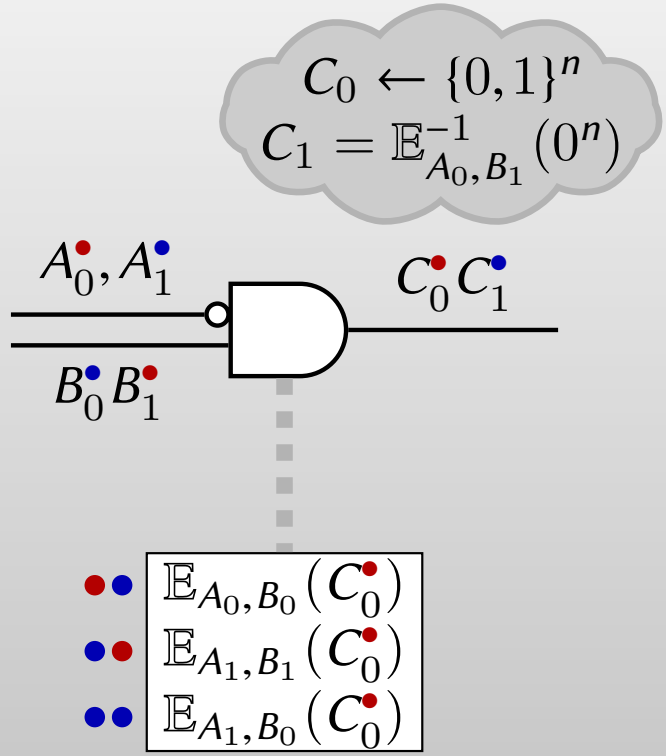

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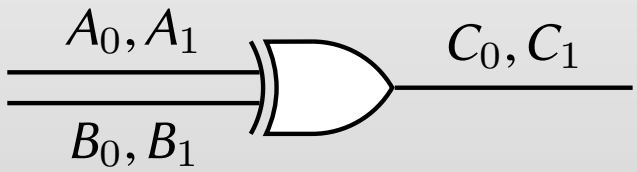
- Choose that label so that 1st ciphertext is 0^n
- No need to include 1st ciphertext in garbled gate
- Evaluate as before, but imagine ciphertext 0^n if you got ●●.

Scoreboard

	size ($\times\lambda$)	garble cost	eval cost
Classical [Yao86,GMW87]	8	4	2.5
P&P [BeaverMicaliRogaway90]	4	4	1
GRR3 [NaorPinkasSumner99]	**3**	**4**	**1**

Free XOR [KolesnikovSchneider08]



- Wire's **offset** ≡ XOR of its two labels

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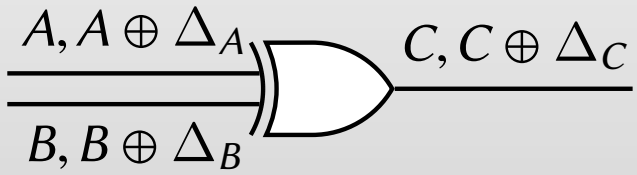

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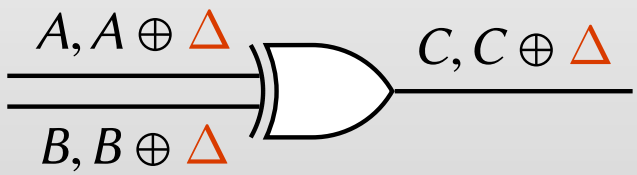
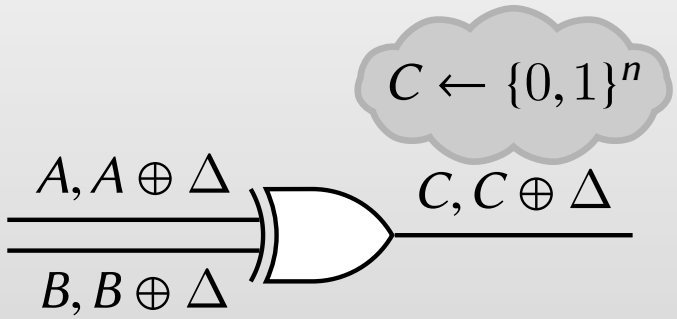


- Wire's **offset** ≡ XOR of its two labels
- Choose all wires in circuit to have same (secret) offset Δ

Free XOR [KolesnikovSchneider08]



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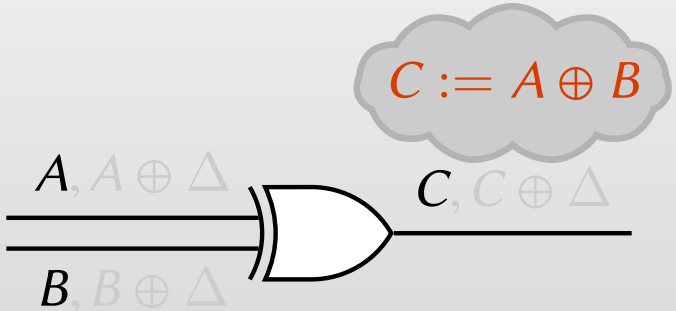

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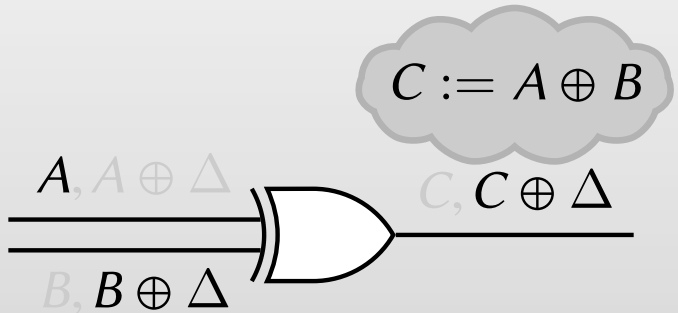

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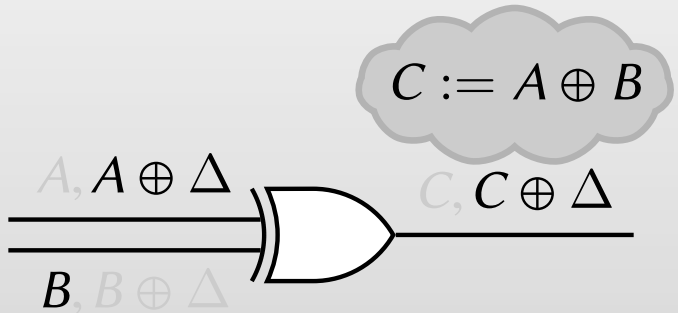

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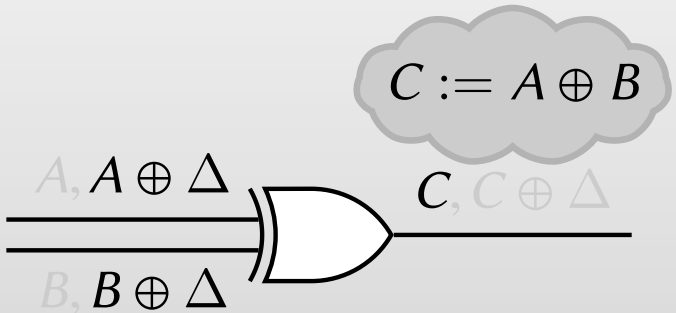

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Scoreboard

	size ($\times\lambda$)		garble cost		eval cost	
	XOR	AND	XOR	AND	XOR	AND
Classical [Yao86,GMW87]	8	8	4	4	2.5	2.5
P&P [BeaverMicaliRogaway90]	4	4	4	4	1	1
GRR3 [NaorPinkasSumner99]	3	3	4	4	1	1
Free XOR [KolesnikovSchneider08]	**0**	**3**	**0**	**4**	**0**	**1**

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"Half gates" [ZahurRosulekEvans15]	0	2	0	4	0	2

Beyond Boolean Circuits

What about all the interesting things that are clunky to write as a boolean circuit?

Garbling Gadgets

Garbling Gadgets for Boolean and Arithmetic Circuits (ACM CCS 2016)

- Marshall Ball, Tal Malkin, Mike Rosulek

1. New (simple!) garbled circuit building blocks
2. Applications to arithmetic computations
3. Applications to high-fan-in boolean computations

Generalized Free XOR

Free XOR:

Wire carries a *truth value* from $\{0,1\}$

Wire labels are bit strings $\{0,1\}^\lambda$.

Global wire-label-offset $\Delta \in \{0,1\}^\lambda$

FALSE wire label is A
TRUE wire label is $A \oplus \Delta$

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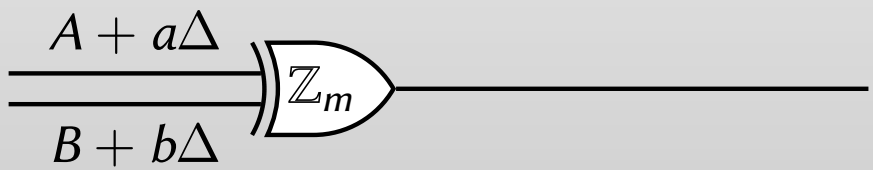

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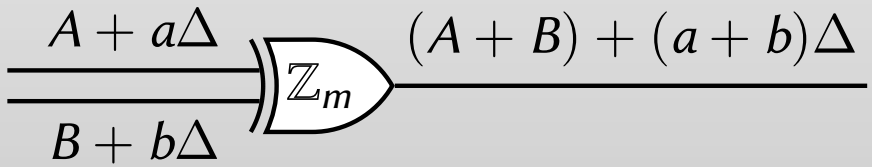

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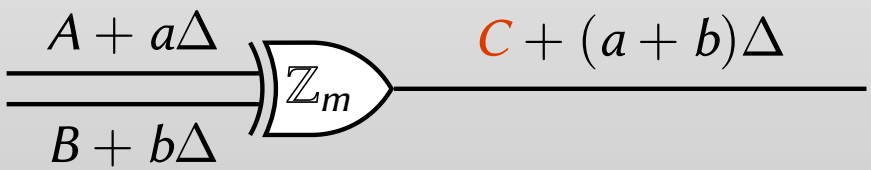

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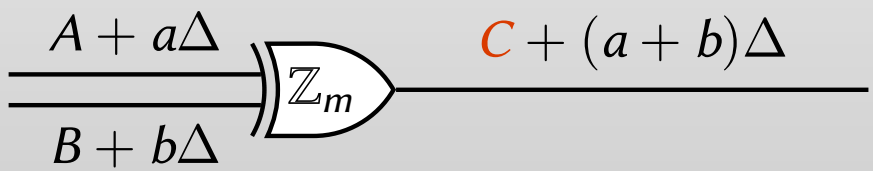

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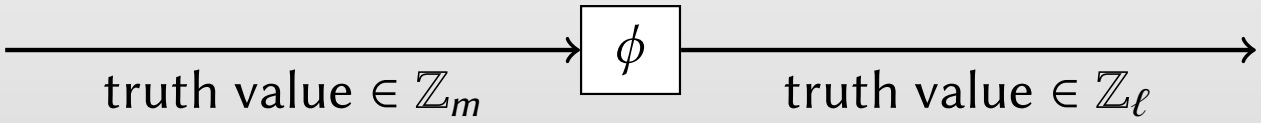
- Free multiplication by public constant c, if $\gcd(c, m) = 1$

Unary gates



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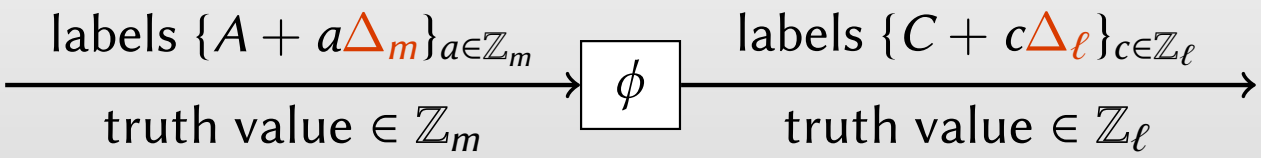

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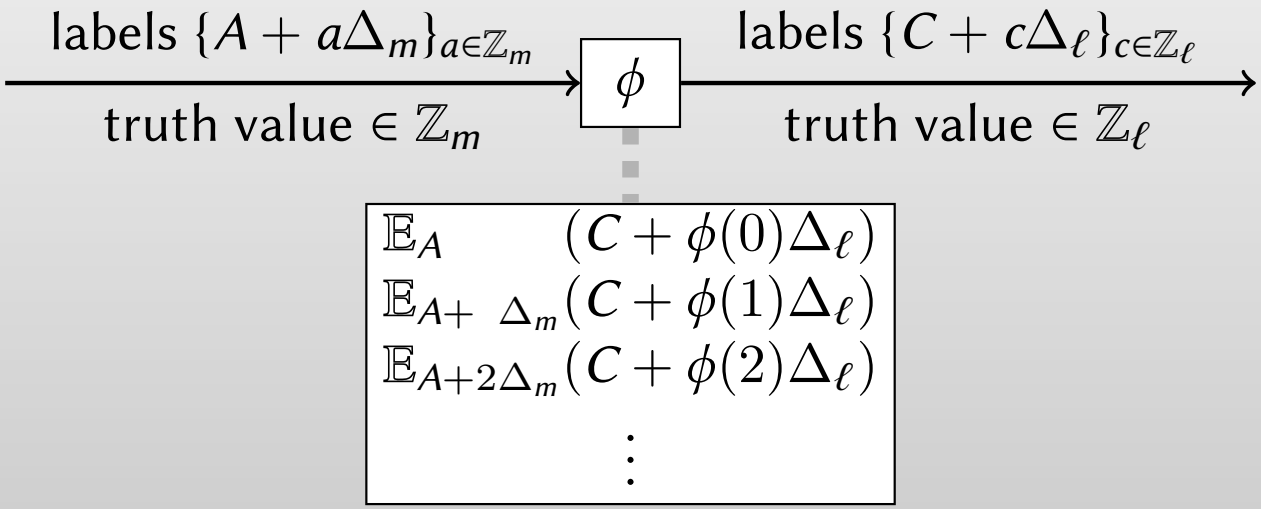

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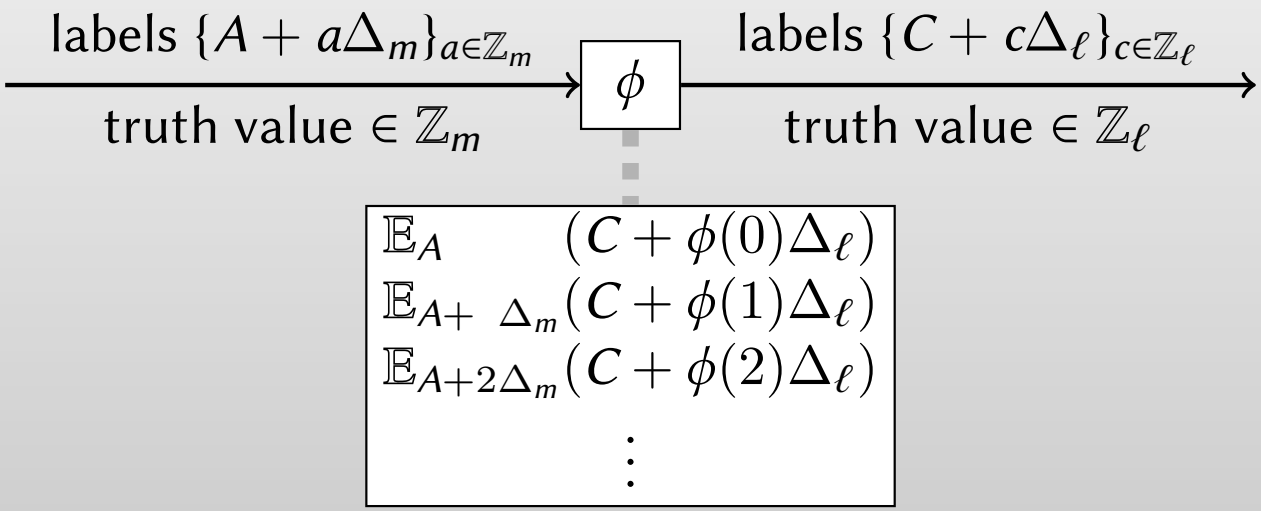

- Different "preferred modulus" on each wire $\Rightarrow$ different offsets Δ
- Cost: m ciphertexts
- $m - 1$ using simple generalization of permute-and-point technique

Summary of tools

We can efficiently garble any computation/circuit where:

- Each wire has a preferred modulus $\mathbb{Z}_m$
  - $\Rightarrow$ Wire-label-offset Δ_m global to all $\mathbb{Z}_m$-wires
- Addition gates: all wires touching gate have same modulus
  - $\Rightarrow$ Garbling cost: **free**
- Mult-by-constant gates: input/output wires have same modulus
  - $\Rightarrow$ Garbling cost: **free**
- Unary gates: $\mathbb{Z}_m$ input and $\mathbb{Z}_\ell$ output
  - $\Rightarrow$ Garbling cost: $m - 1$ ciphertexts

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Scenario

Securely compute linear optimization problem on 32-bit values.

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	cost (# ciphertexts)
addition	62
multiplication by public constant	758
multiplication	1200
squaring, cubing, etc	1864

Arithmetic computations

Insight: take advantage of **free addition**

- Represent 32-bit integers directly in $\mathbb{Z}_{2^{32}}$
- Do all arithmetic modulo 2^{32}
- In our new scheme, garbled addition is **free**

What about multiplication?

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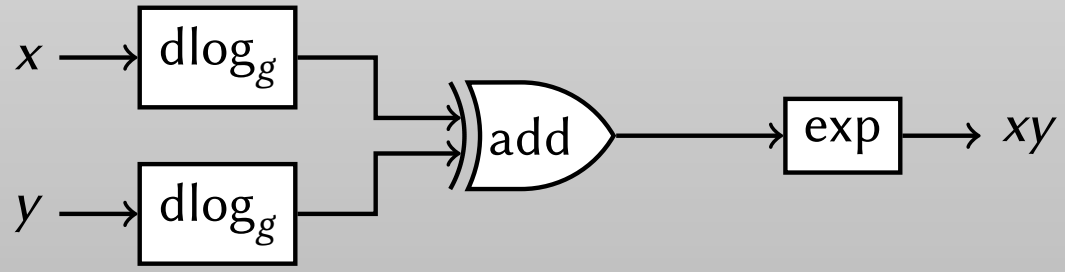
What about multiplication? Our approach: **log-transform**



(plus extra edge cases when $0 \in \{x, y\}$*)*

Total cost: $\sim 6m$ for garbled multiplication mod m

Arithmetic computations

	standard	ours
addition	62	0
multiplication by public constant	758	0
multiplication	1200	25769803776
squaring, cubing, etc	1864	4294967296

Arithmetic computations

$$\mathbb{Z}_{4294967296}$$

$$\downarrow$$

$$\mathbb{Z}_{6469693230}$$

Arithmetic computations

$$\mathbb{Z}_{4294967296} = \mathbb{Z}_{2^{32}}$$
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- For each *logical* value, circuit includes mod-2 wire, mod-3 wire, ...
- For 32-bit integers, first 10 primes suffice: $2 \cdot 3 \cdot 5 \cdots 29 > 2^{32}$

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  - $\Rightarrow$ garbling cost = $6(2 + 3 + 5 + \cdots + 29)$
- To raise to public power, use unary gate $x \mapsto x^c$ in each CRT residue
  - $\Rightarrow$ garbling cost = $(2 - 1) + (3 - 1) + (5 - 1) + \cdots + (29 - 1)$

Arithmetic computations

	standard	awful	**CRT**
addition	62	0	**0**
multiplication by public constant	758	0	**0**
multiplication	1200	25769803776	**724**
squaring, cubing, etc	1864	4294967296	**119**

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High fan-in computations

Scenario

Securely compute boolean circuit with **high-fan-in threshold gates.**

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	cost (# ciphertexts)
AND/OR	198
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Same logic for $\mathrm{MAJ}(x_1, \ldots, x_{100}) = [\sum_i x_i \stackrel{?}{>} 50]$

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$$\mathbb{Z}_{101}$$
$$\downarrow$$
$$\mathbb{Z}_{210}$$

High fan-in computations

$$\mathbb{Z}_{101}$$
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$$\mathbb{Z}_{210} = \mathbb{Z}_{2 \cdot 3 \cdot 5 \cdot 7}$$

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Insight: take advantage of **multiple moduli**

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	standard	better	**best**
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majority	948	100	

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Summarizing

For **arithmetic operations on bounded integers:**

- Represent in primorial modulus + CRT (10-20 primes)
- **Free** addition & multiplication by constant!
- Multiplication concretely better than boolean
- Exponentiation concretely+asymptotically better

Summarizing

For **arithmetic operations on bounded integers:**

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For **high-fan-in computations on bits:**

- Represent bits in primorial modulus + CRT (3-5 primes)
- Threshold gates (incl. AND, OR) **exponentially better** than boolean

Challenges

How to compute $x < y$ for 32-bit numbers?

Our approach: represent x, y in $\mathbb{Z}_2 \times \mathbb{Z}_3 \times \cdots \times \mathbb{Z}_{29}$

- Our best cost for garbled comparison = 2541

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Stndard approach: represent x, y in binary:

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Stndard approach: represent x, y in binary:

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How to efficiently convert CRT $\leftrightarrow$ binary?

Example: perform arithmetic operations, then compute a **cryptographic hash** of result

Challenges

"Free" stuff is not free!

In our garbling scheme:

- Addition gates treat wire labels as elements of $(\mathbb{Z}_p)^\lambda$
- Projection gates treat wire labels as **bits** (used as encryption scheme)

Challenges

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In our garbling scheme:

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- Conversion contributes significantly to cost in practice!

the end!