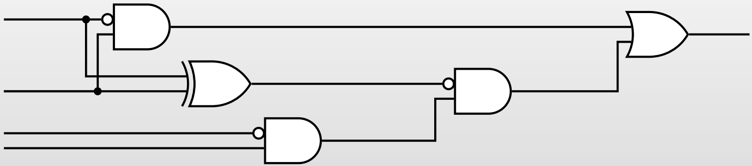
# Practical Garbled Circuit Optimizations

**Mike Rosulek**
Oregon State University OSU

Collaborators: David Evans / Vlad Kolesnikov / Payman Mohassel / Samee Zahur

# Garbled circuit framework [Yao86]



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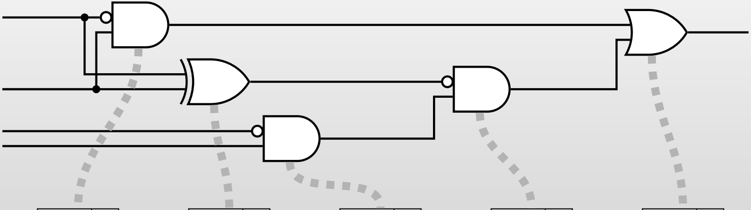

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

| | | |
|---|---|---|
| 0 | 0 | 0 |
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| | | |
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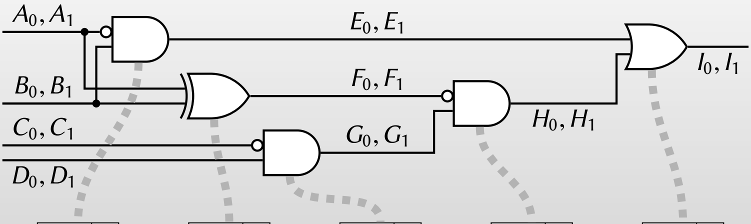


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- Pick random **labels** $W_0, W_1$ on each wire

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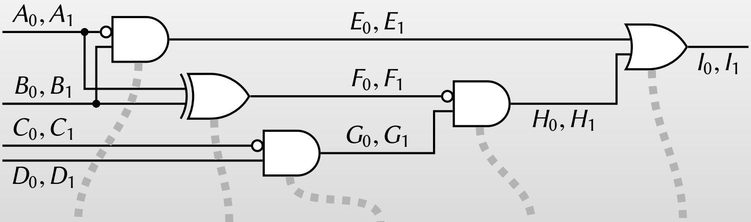


| | | |
|---|---|---|
| $A_0$ | $B_0$ | $E_0$ |
| $A_0$ | $B_1$ | $E_1$ |
| $A_1$ | $B_0$ | $E_0$ |
| $A_1$ | $B_1$ | $E_0$ |

| | | |
|---|---|---|
| $A_0$ | $B_0$ | $F_0$ |
| $A_0$ | $B_1$ | $F_1$ |
| $A_1$ | $B_0$ | $F_1$ |
| $A_1$ | $B_1$ | $F_0$ |

| | | |
|---|---|---|
| $C_0$ | $D_0$ | $G_0$ |
| $C_0$ | $D_1$ | $G_1$ |
| $C_1$ | $D_0$ | $G_0$ |
| $C_1$ | $D_1$ | $G_0$ |

| | | |
|---|---|---|
| $F_0$ | $G_0$ | $H_0$ |
| $F_0$ | $G_1$ | $H_1$ |
| $F_1$ | $G_0$ | $H_0$ |
| $F_1$ | $G_1$ | $H_0$ |

| | | |
|---|---|---|
| $E_0$ | $H_0$ | $I_0$ |
| $E_0$ | $H_1$ | $I_1$ |
| $E_1$ | $H_0$ | $I_1$ |
| $E_1$ | $H_1$ | $I_1$ |

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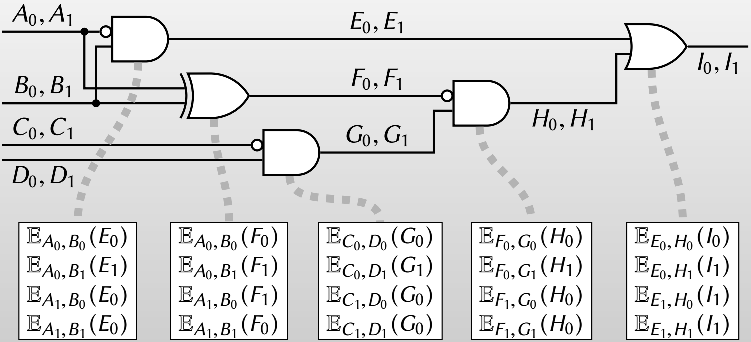


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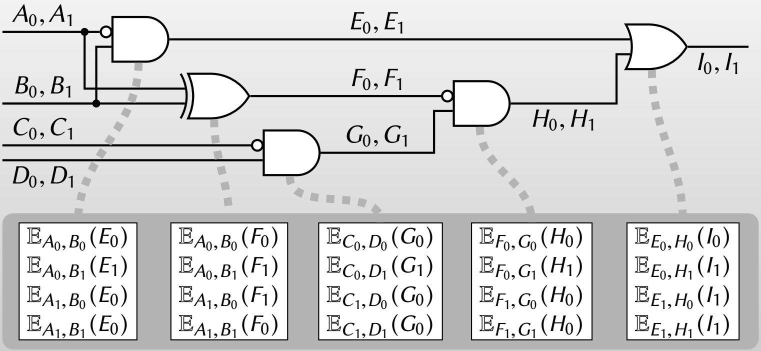


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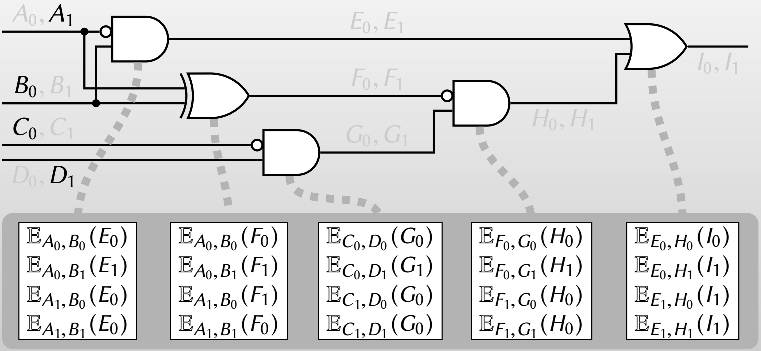


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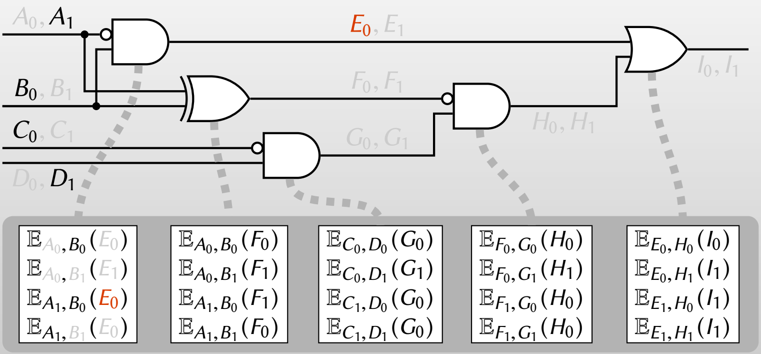


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- Only one ciphertext per gate is decryptable

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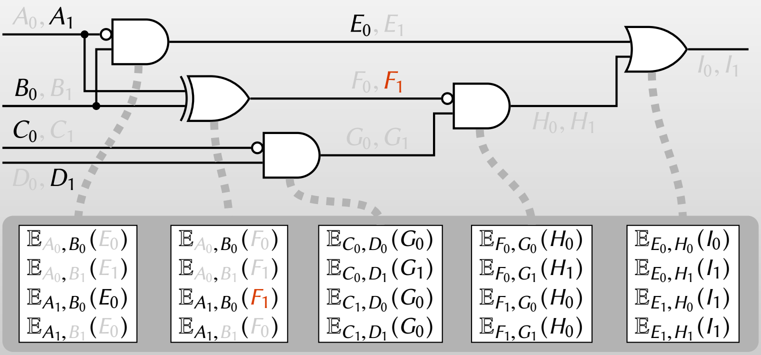


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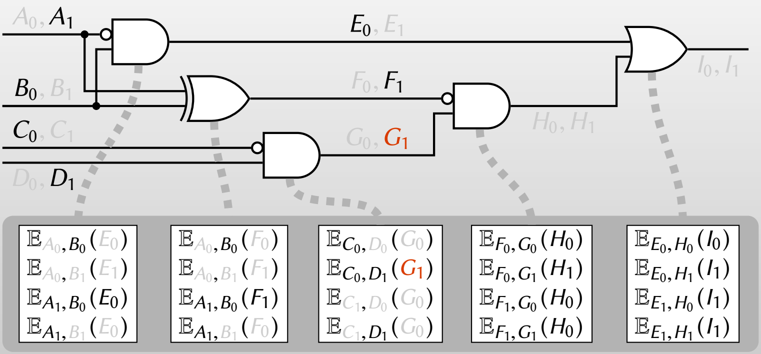


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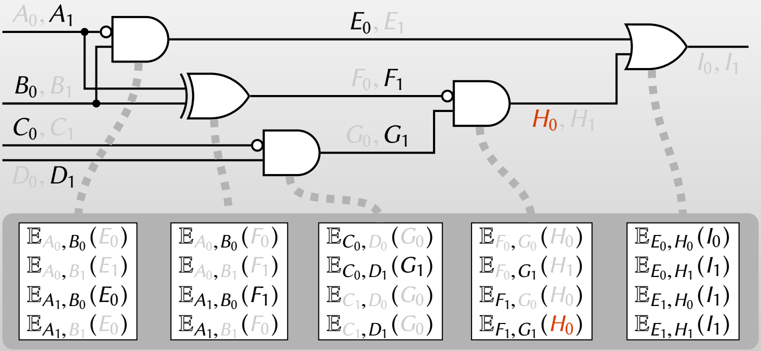


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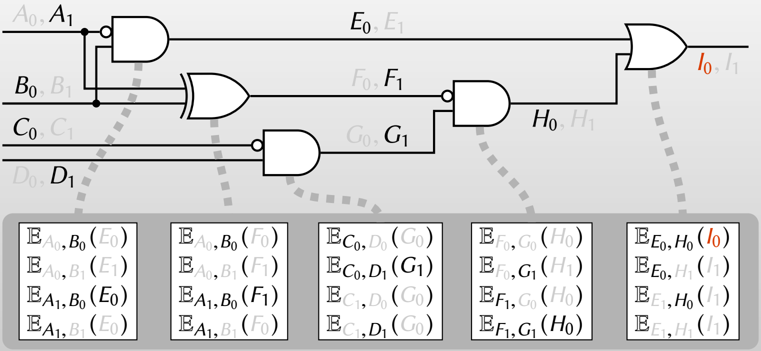


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# Applications: 2PC and more

$x$

$y$

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garbled circuit $f$

$\longrightarrow$

garbled input $x$,
output wire labels

$x$

$y$

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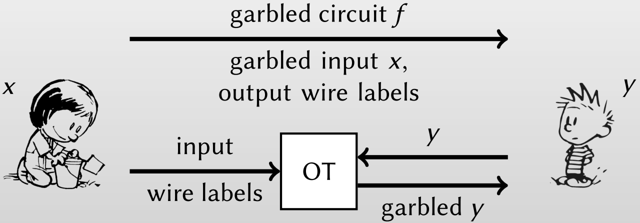

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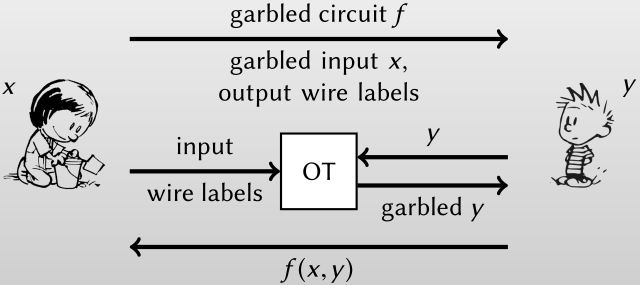

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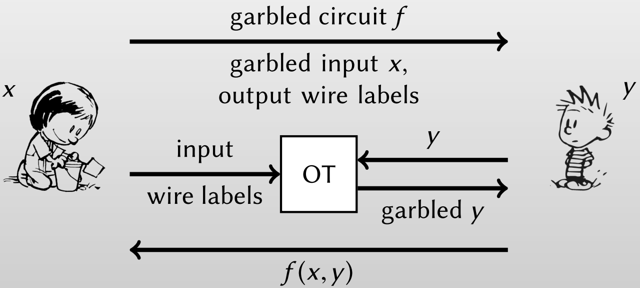


Private function evaluation, zero-knowledge proofs, encryption with key-dependent message security, randomized encodings, secure outsourcing, one-time programs, . . .

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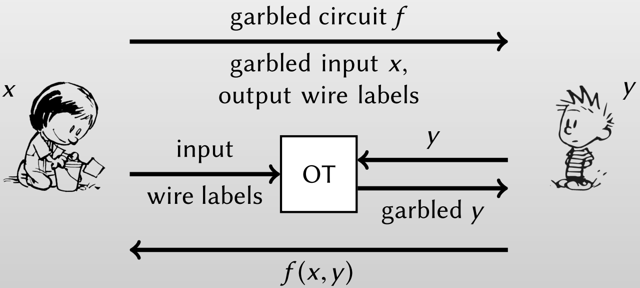


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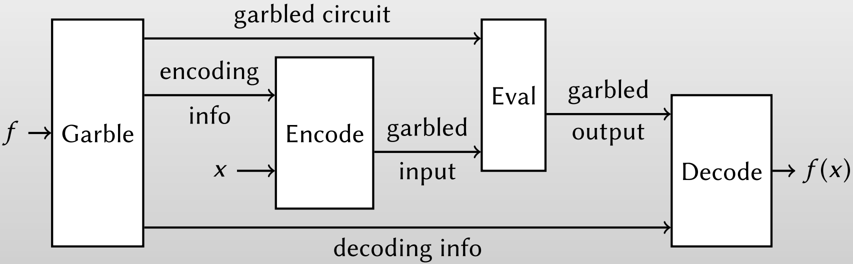
*Garbling is a* ***fundamental primitive*** [BellareHoangRogaway12]

# Syntax [BellareHoangRogaway12]



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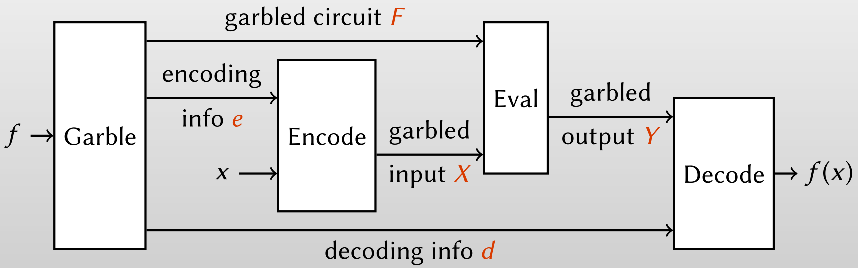

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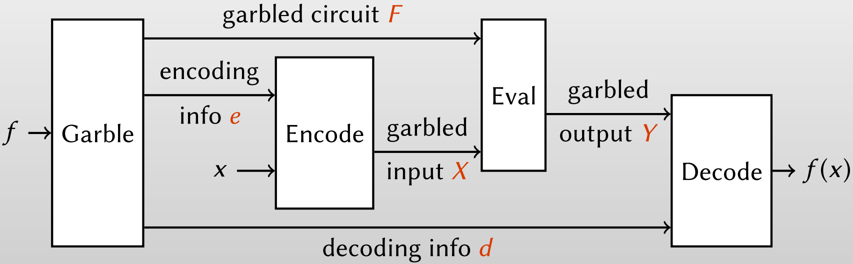


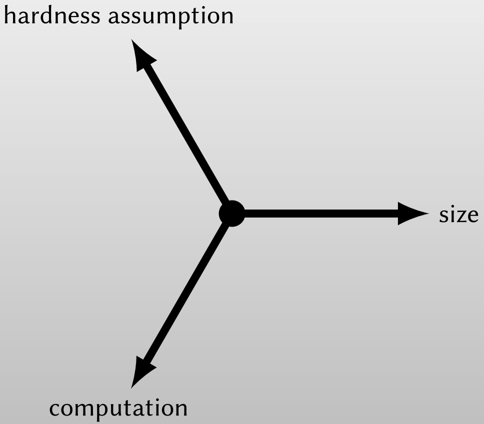

Security properties:

Privacy: $(F, X, d)$ reveals nothing beyond $f(x)$

Obliviousness: $(F, X)$ reveals nothing

Authenticity: given $(F, X)$, hard to find $\widetilde{Y}$ that decodes $\notin \{f(x), \perp\}$

# Parameters to optimize



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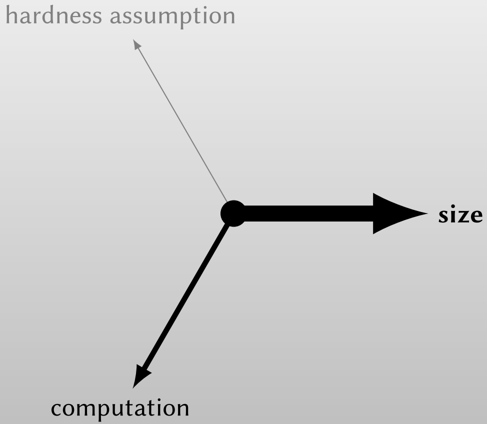
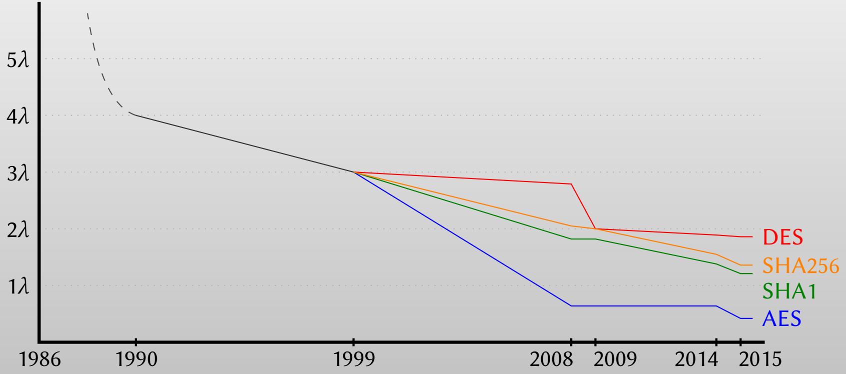


# Average bits per garbled gate



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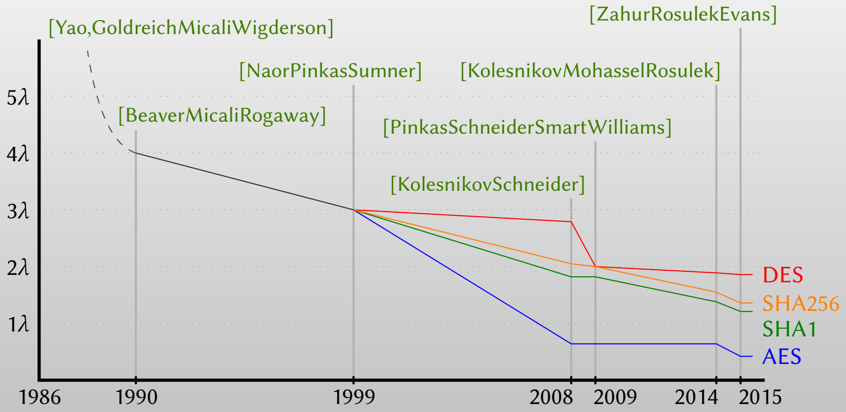

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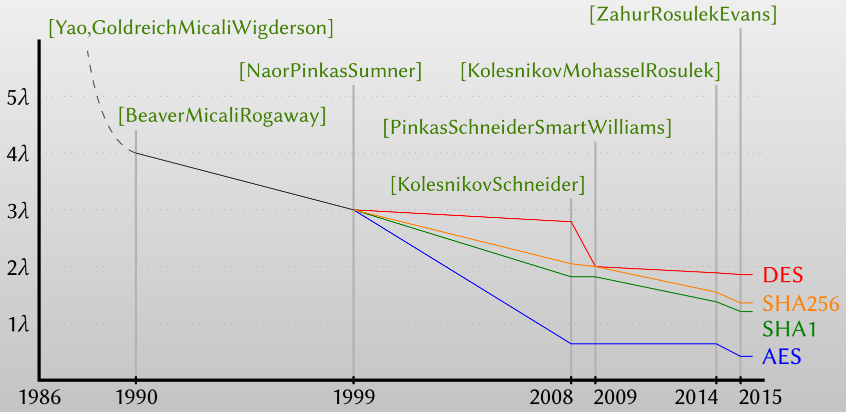
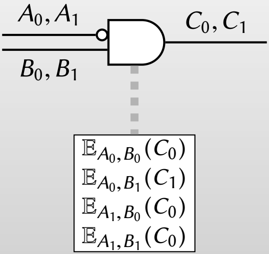



Prediction: by 2026, all garbled circuits will have zero size.

# Murky beginnings [Yao86]




- Position in this list leaks semantic value

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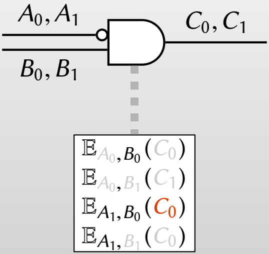


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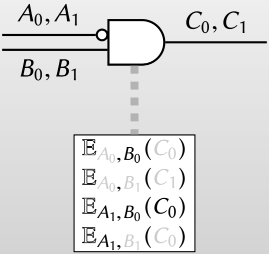


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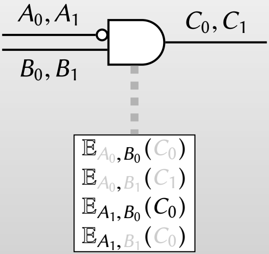


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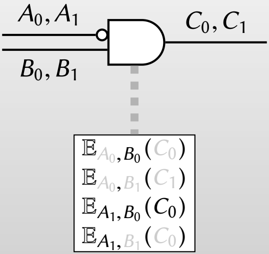


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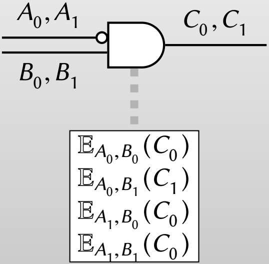
- Need to **detect** [in]correct decryption
- (Apparently) no one knows exactly what Yao had in mind:
  - $\mathbb{E}_{K_0,K_1}(M) = \langle E(K_0, S_0), E(K_1, S_1)\rangle$ where $S_0 \oplus S_1 = M$ [GoldreichMicaliWigderson87]
  - $\mathbb{E}_{K_0,K_1}(M) = E(K_1, E(K_0, M))$ [LindellPinkas09]

# Permute-and-Point [BeaverMicaliRogaway90]



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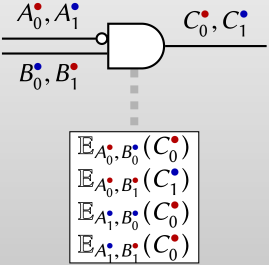


- Randomly assign $({\color{red}\bullet}, {\color{blue}\bullet})$ or $({\color{blue}\bullet}, {\color{red}\bullet})$ to each pair of wire labels
- Include color in the wire label (e.g., as last bit)

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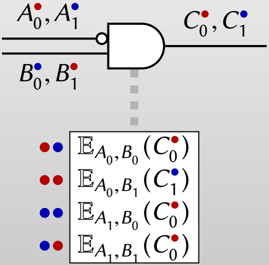


- Randomly assign (●, ●) or (●, ●) to each pair of wire labels
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- Order the 4 ciphertexts canonically, by color of keys

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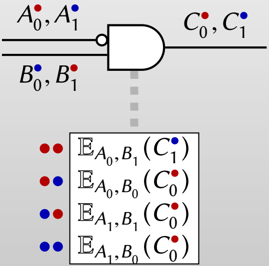


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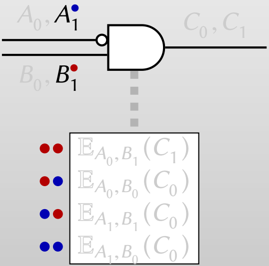


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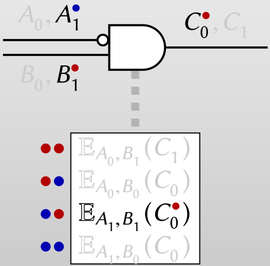


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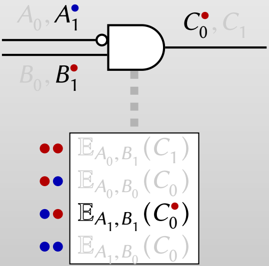


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Can use **one-time-secure** symmetric encryption!

# Computational cost of garbling

| $\mathbb{E}_{A,B}(C)$: | cost to garble AES |
|---|---|
| $\text{PRF}(A, \text{gateID}) \oplus \text{PRF}(B, \text{gateID}) \oplus C$ | ~6s [extrapolated] |
| [NaorPinkasSumner99] | time from Fairplay [MNPS04]: PRF = SHA256 |

# Computational cost of garbling

*2 hash $\gg$ 1 hash*

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| $\mathrm{H}(A \| B \| \mathrm{gateID}) \oplus C$ | 0.15s |
| [LindellPinkasSmart08] | time from [sS12]; H = SHA256 |

# Computational cost of garbling

*2 hash $\gg$ 1 hash $\gg$ 1 block cipher*

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| $\mathrm{AES256}(A\|B, \mathrm{gateID}) \oplus C$<br>[shelatShen12] | 0.12s |

# Computational cost of garbling

*2 hash ≫ 1 hash ≫ 1 block cipher ≫ 1 block cipher w/o key schedule*

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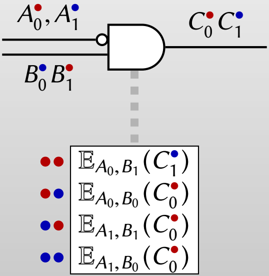
| $\text{AES256}(A \| B, \text{gateID}) \oplus C$ <br> [shelatShen12] | 0.12s |
| $\text{AES}(\text{const}, K) \oplus K \oplus C$ <br> *where* $K = 2A \oplus 4B \oplus \textit{gateID}$ <br> [BellareHoangKeelveedhiRogaway13] | 0.0003s |

# Scoreboard

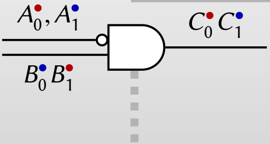
| | size ($\times\lambda$) | garble cost | eval cost | assumption |
|---|---|---|---|---|
| Classical | large? | 8 | 5 | PKE |
| P&P | 4 | 4/8 | 1/2 | hash/PRF |

# Garbled Row Reduction [NaorPinkasSumner99]



# Garbled Row Reduction [NaorPinkasSumner99]

$$C_0 \leftarrow \{0,1\}^n$$
$$C_1 \leftarrow \{0,1\}^n$$




| |
|---|
| $\mathbb{E}_{A_0,B_1}(C_1)$ |
| $\mathbb{E}_{A_0,B_0}(C_0)$ |
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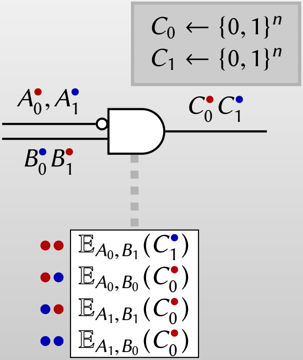
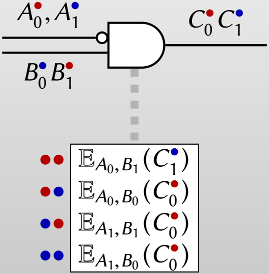



- What wire label will be payload of 1st (●●) ciphertext?

# Garbled Row Reduction [NaorPinkasSumner99]

$C_0 \leftarrow \{0,1\}^n$

$C_1 = \mathbb{E}^{-1}_{A_0,B_1}(0^n)$

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- Choose that label so that 1st ciphertext is $0^n$

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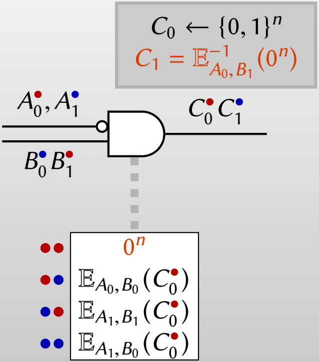


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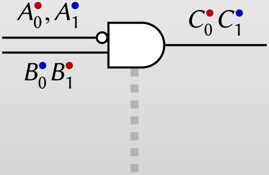


$$\mathbb{E}_{A_0,B_0}(C_0)$$
$$\mathbb{E}_{A_1,B_1}(C_0)$$
$$\mathbb{E}_{A_1,B_0}(C_0)$$

- What wire label will be payload of 1st (••) ciphertext?
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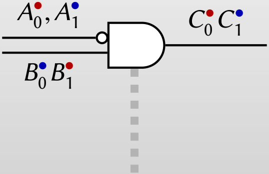


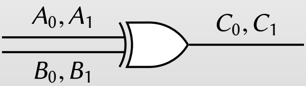

| | |
|---|---|
| ●● | $\mathbb{E}_{A_0,B_0}(C_0)$ |
| ●● | $\mathbb{E}_{A_1,B_1}(C_0)$ |
| ●● | $\mathbb{E}_{A_1,B_0}(C_0)$ |

- What wire label will be payload of 1st (●●) ciphertext?
- Choose that label so that 1st ciphertext is $0^n$
- No need to include 1st ciphertext in garbled gate
- Evaluate as before, but imagine ciphertext $0^n$ if you got ●●.

# Scoreboard

| | size ($\times\lambda$) | garble cost | eval cost | assumption |
|---|---|---|---|---|
| Classical | large? | 8 | 5 | PKE |
| P&P | 4 | 4/8 | 1/2 | hash/PRF |
| **GRR3** | **3** | **4/8** | **1/2** | **hash/PRF** |

# Free XOR [KolesnikovSchneider08]



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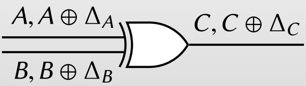


- Wire's **offset** ≡ XOR of its two labels

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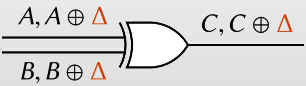


- Wire's **offset** ≡ XOR of its two labels
- Choose all wires to have same (secret) offset $\Delta$

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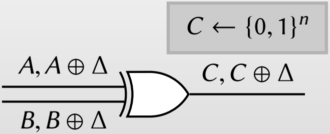


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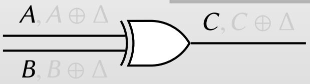


$$\underbrace{A}_{\text{FALSE}} \oplus \underbrace{B}_{\text{FALSE}} = \underbrace{A \oplus B}_{\text{FALSE}}$$

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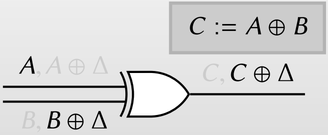


$$\underbrace{A\quad}_{\text{FALSE}} \oplus \underbrace{B \oplus \Delta}_{\text{TRUE}} = \underbrace{A \oplus B \oplus \Delta}_{\text{TRUE}}$$

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- Evaluate by XORing input wire labels (no crypto)

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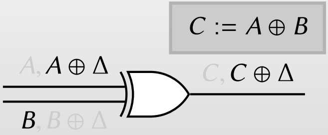


$$\underbrace{A \oplus \Delta}_{\text{TRUE}} \oplus \underbrace{B \quad}_{\text{FALSE}} = \underbrace{A \oplus B \oplus \Delta}_{\text{TRUE}}$$

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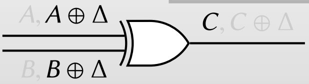


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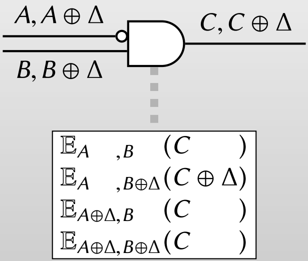
# Freedom at a cost. . .




- Still need to garble AND gates

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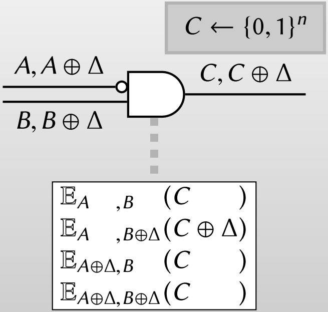


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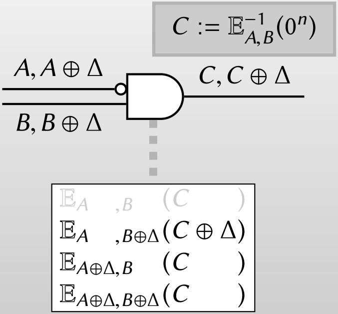


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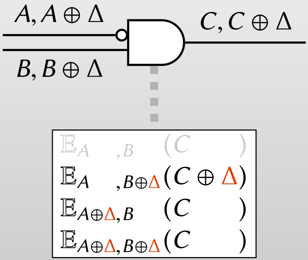


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- Secret $\Delta$ used in key and payload of ciphertexts!

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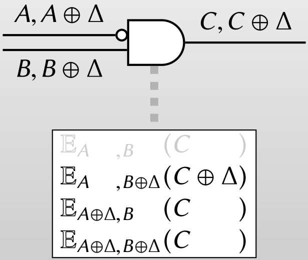


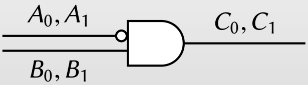

- Still need to garble AND gates
- Compatible with garbled row-reduction
- Secret $\Delta$ used in key and payload of ciphertexts!
- Requires related-key + circularity assumption [ChoiKatzKumaresanZhou12]

# Scoreboard

| | size ($\times\lambda$) | | garble cost | | eval cost | | assumption |
|---|---|---|---|---|---|---|---|
| | XOR | AND | XOR | AND | XOR | AND | |
| Classical | large? | | 8 | | 5 | | PKE |
| P&P | 4 | 4 | 4/8 | 4/8 | 1/2 | 1/2 | PRF/hash |
| GRR3 | 3 | 3 | 4/8 | 4/8 | 1/2 | 1/2 | PRF/hash |
| **Free XOR** | **0** | **3** | **0** | **4** | **0** | **1** | **circ. hash** |

# Row reduction ++ [PinkasSchneiderSmartWilliams09]

Garbled gates with only **2** ciphertexts!



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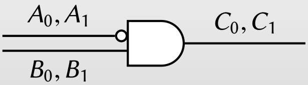

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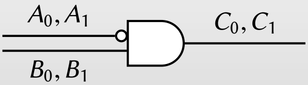

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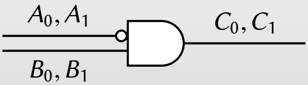


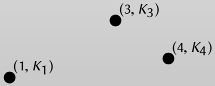


$$(1, K_1), (3, K_3), (4, K_4)$$

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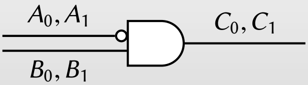


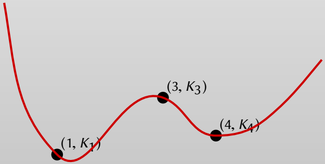


$P$ = uniq deg-2 poly thru

$(1, K_1), (3, K_3), (4, K_4)$

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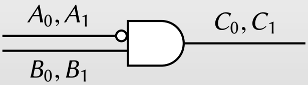


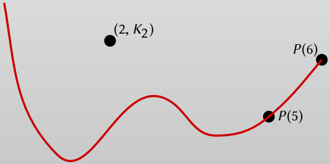


$P$ = uniq deg-2 poly thru

$(1, K_1), (3, K_3), (4, K_4)$

$(2, K_2), (5, P(5)), (6, P(6))$

# Row reduction ++ [PinkasSchneiderSmartWilliams09]

Garbled gates with only **2** ciphertexts!

- Evaluator can know exactly one of:

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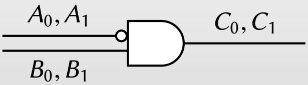


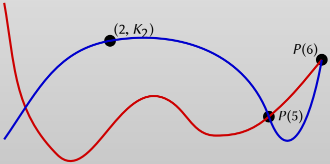


$P$ = uniq deg-2 poly thru $(1, K_1), (3, K_3), (4, K_4)$

$Q$ = uniq deg-2 poly thru $(2, K_2), (5, P(5)), (6, P(6))$

# Row reduction ++ [PinkasSchneiderSmartWilliams09]

$C_0 = P(0); C_1 = Q(0)$

Garbled gates with only **2** ciphertexts!

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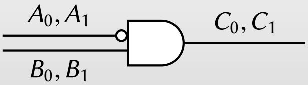


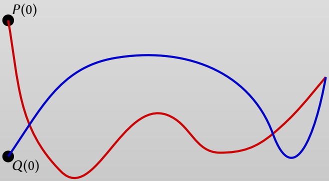


$P$ = uniq deg-2 poly thru $(1, K_1), (3, K_3), (4, K_4)$

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# Row reduction ++ [PinkasSchneiderSmartWilliams09]

$C_0 = P(0);\ C_1 = Q(0)$

Garbled gates with only **2** ciphertexts!

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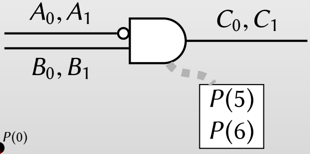


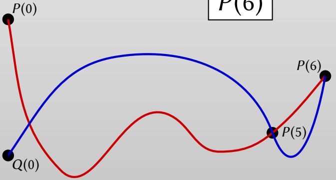


$P =$ uniq deg-2 poly thru

$(1, K_1), (3, K_3), (4, K_4)$

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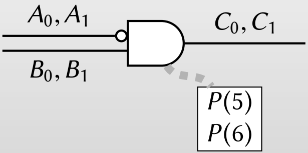


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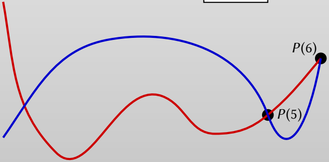
- Evaluate by interpolating poly thru $K_i$, $P(5)$ and $P(6)$




$P$ = uniq deg-2 poly thru $(1, K_1), (3, K_3), (4, K_4)$

$Q$ = uniq deg-2 poly thru $(2, K_2), (5, P(5)), (6, P(6))$

# Row reduction ++ [PinkasSchneiderSmartWilliams09]

Garbled gates with only **2** ciphertexts!

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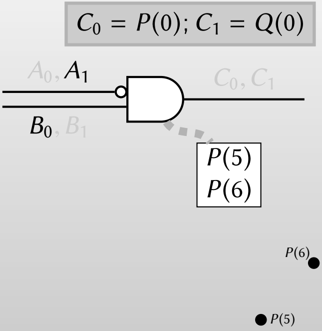


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# Row reduction ++ [PinkasSchneiderSmartWilliams09]

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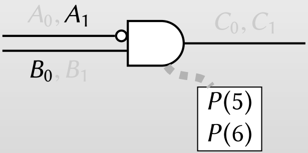


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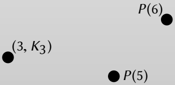
- Evaluate by interpolating poly thru $K_i$, $P(5)$ and $P(6)$

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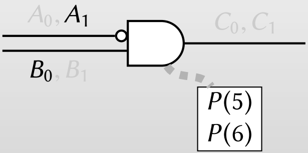
# Row reduction ++ [PinkasSchneiderSmartWilliams09]

$C_0 = P(0); C_1 = Q(0)$




Garbled gates with only **2** ciphertexts!

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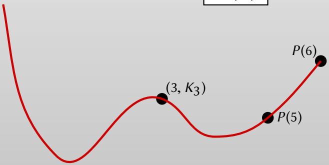


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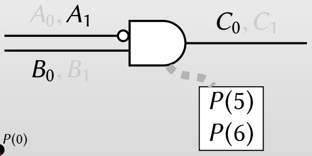


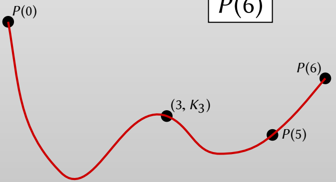


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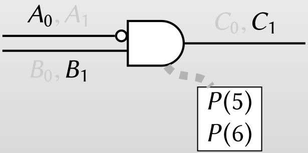


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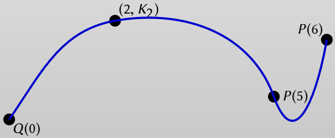


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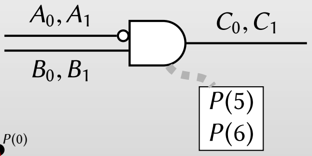


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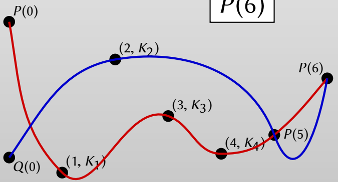
- Evaluate by interpolating poly thru $K_i$, $P(5)$ and $P(6)$
- **Incompatible** with Free-XOR: can't ensure $C_0 \oplus C_1 = \Delta$

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# Scoreboard

| | size ($\times\lambda$) | | garble cost | | eval cost | | assumption |
|---|---|---|---|---|---|---|---|
| | XOR | AND | XOR | AND | XOR | AND | |
| Classical | large? | | 8 | | 5 | | PKE |
| P&P | 4 | 4 | 4/8 | 4/8 | 1/2 | 1/2 | hash/PRF |
| GRR3 | 3 | 3 | 4/8 | 4/8 | 1/2 | 1/2 | PRF/hash |
| Free XOR | 0 | 3 | 0 | 4 | 0 | 1 | circ. hash |
| **GRR2** | **2** | **2** | 4/8 | 4/8 | 1/2 | 1/2 | PRF/hash |

# FleXOR [KolesnikovMohasselRosulek14]

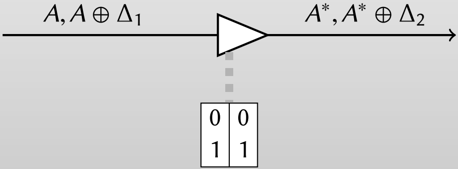
$$A, A \oplus \Delta_1$$
$$\longrightarrow$$

# FleXOR [KolesnikovMohasselRosulek14]

$A, A \oplus \Delta_1 \longrightarrow A^*, A^* \oplus \Delta_2$

- Translate to a new wire offset

# FleXOR [KolesnikovMohasselRosulek14]




- Translate to a new wire offset (unary $a \mapsto a$ gate)

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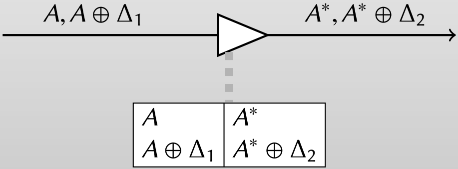


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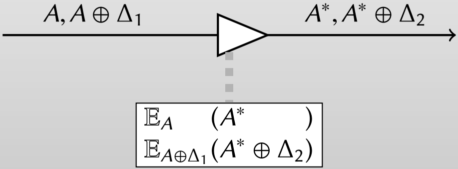


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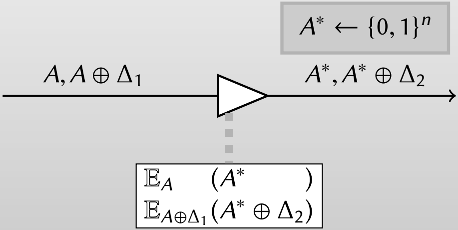


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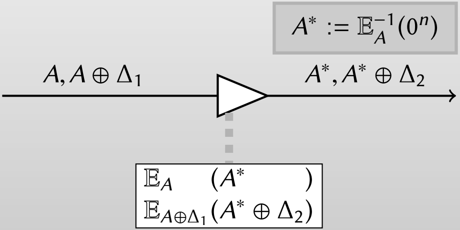


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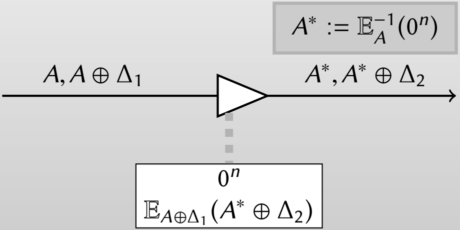


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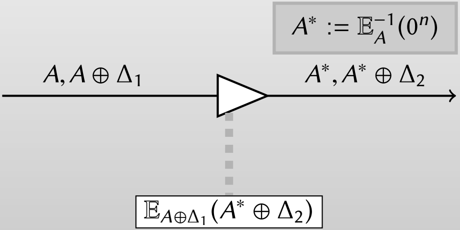


- Translate to a new wire offset (unary $a \mapsto a$ gate) using 1 ciphertext

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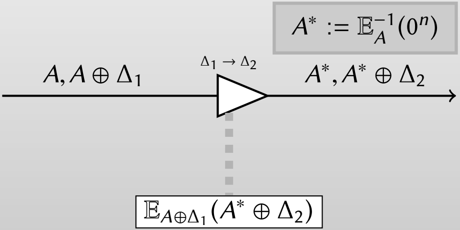


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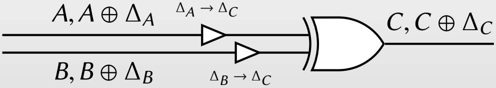


- Adjust inputs to target offset $\Delta_C$ (1 ciphertext each)

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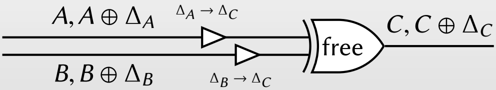


- Adjust inputs to target offset $\Delta_C$ (1 ciphertext each), then XOR is free

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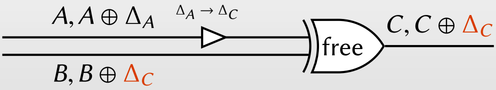


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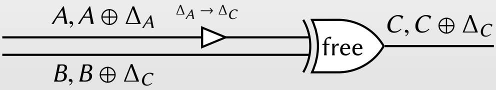


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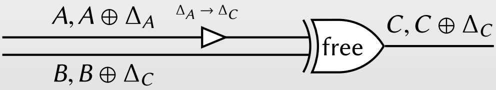


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**Combinatorial optimization problem:** Choose an offset for each wire, minimizing total cost of XOR gates

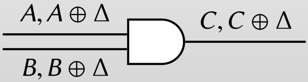
- Subj. to compatibility with 2-ciphertext row-reduction of AND gates
- (or) Subj. to removing circularity property of free-XOR

# Scoreboard

| | size ($\times\lambda$) | | garble cost | | eval cost | | assumption |
|---|---|---|---|---|---|---|---|
| | XOR | AND | XOR | AND | XOR | AND | |
| Classical | large? | | 8 | | 5 | | PKE |
| P&P | 4 | 4 | 4/8 | 4/8 | 1/2 | 1/2 | hash/PRF |
| GRR3 | 3 | 3 | 4/8 | 4/8 | 1/2 | 1/2 | PRF/hash |
| Free XOR | 0 | 3 | 0 | 4 | 0 | 1 | circ. hash |
| GRR2 | 2 | 2 | 4/8 | 4/8 | 1/2 | 1/2 | PRF/hash |
| **FleXOR** | $\{0, 1, 2\}$ | **2** | $\{0, 1, 2\}$ | **4** | $\{0, 1, 2\}$ | **1** | **circ. hash** |

# Half Gates [ZahurRosulekEvans15]

*What if garbler knows in advance the truth value on one input wire?*



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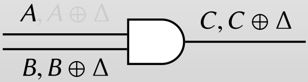

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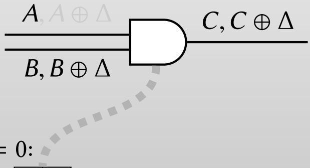


if $a = 0$:

| | |
|---|---|
| 0 | 0 |
| 1 | 0 |

unary gate $b \mapsto 0$

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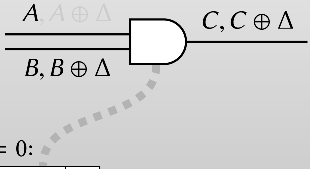


if $a = 0$:

| | |
|---|---|
| $B$ | $C$ |
| $B \oplus \Delta$ | $C$ |

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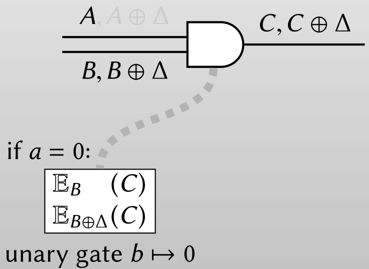

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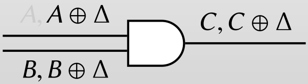

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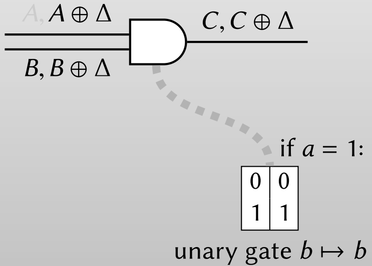

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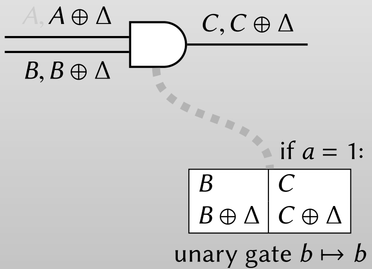

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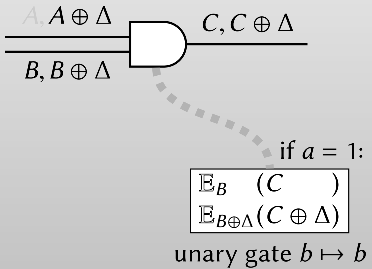

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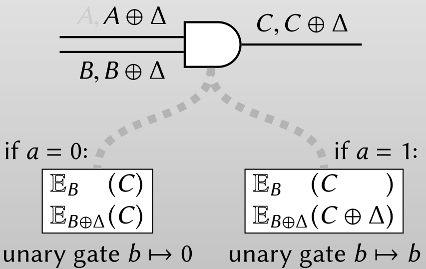

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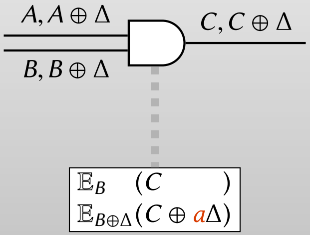

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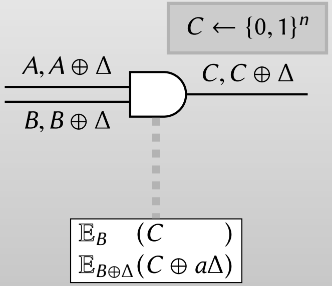

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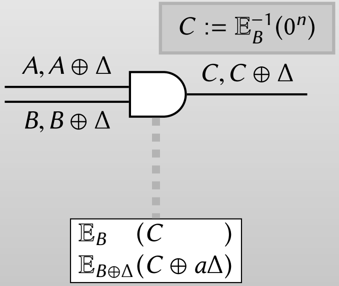

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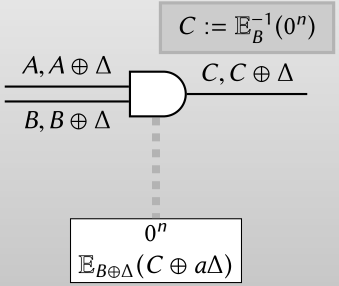

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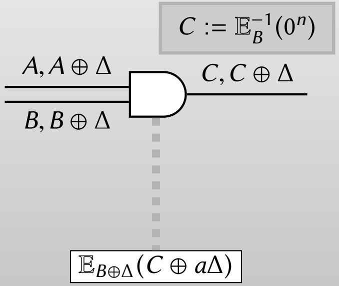


Fine print: permute ciphertexts with permute-and-point.

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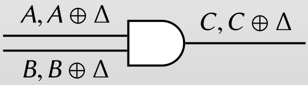

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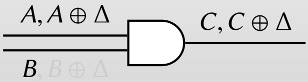

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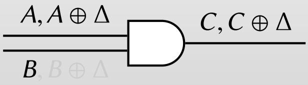


Evaluator has $B$ (knows FALSE):

⇒ should obtain $C$ (FALSE)

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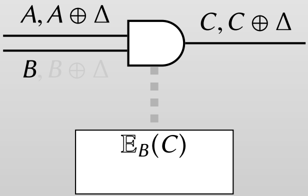


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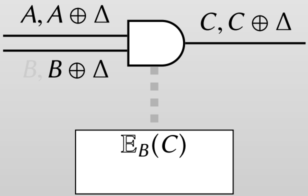


Evaluator has $B$ (knows FALSE):

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Evaluator has $B \oplus \Delta$ (knows TRUE):

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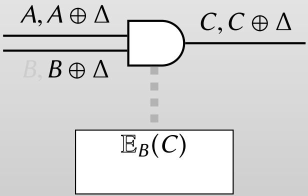


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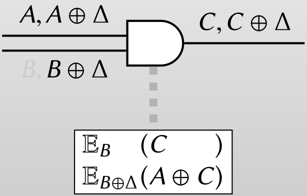


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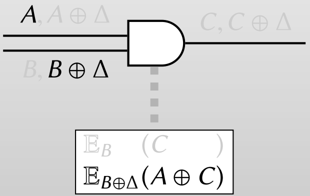


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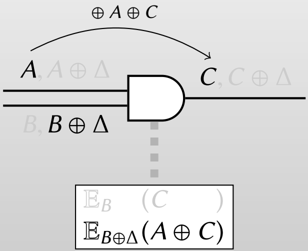


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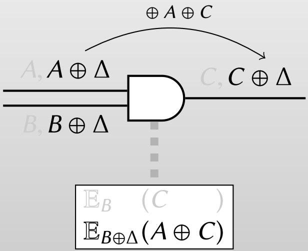


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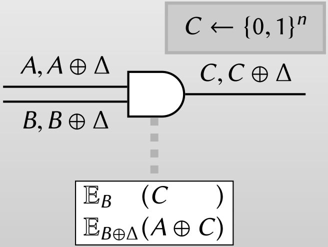


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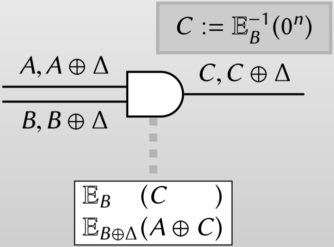


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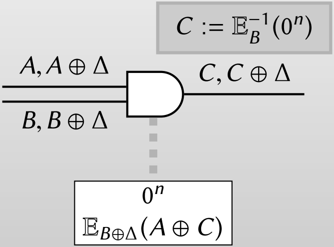


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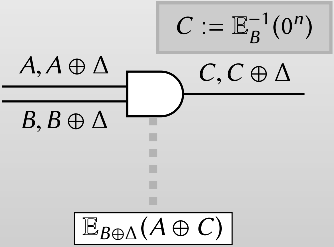


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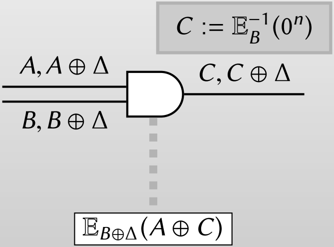


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Fine print: no need for permute-and-point here

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  - $r$ = color bit of FALSE wire label $A$
- Arrange for evaluator to learn $a \oplus r$ in the clear
  - $a \oplus r$ = color bit of wire label evaluator gets ($A$ or $A \oplus \Delta$)
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# Scoreboard

| | size ($\times\lambda$) | | garble cost | | eval cost | | assumption |
|---|---|---|---|---|---|---|---|
| | XOR | AND | XOR | AND | XOR | AND | |
| Classical | large? | | 8 | | 5 | | PKE |
| P&P | 4 | 4 | 4/8 | 4/8 | 1/2 | 1/2 | hash/PRF |
| GRR3 | 3 | 3 | 4/8 | 4/8 | 1/2 | 1/2 | PRF/hash |
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| FleXOR | $\{0, 1, 2\}$ | 2 | $\{0, 1, 2\}$ | 4 | $\{0, 1, 2\}$ | 1 | circ. symm |
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| [XYZ26]? | 0 | $<2$? | ? | ? | ? | ? | ? |

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*Garbling a single* AND *gate requires 2 ciphertexts ($2\lambda$ bits), if garbling scheme is "linear" in this sense.*

Half-gates construction is *size-optimal* among schemes that:

... use "known techniques"

... work gate-by-gate in {XOR, AND, NOT} basis

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4: Use weaker security when situation calls for it.

# ZK via garbled circuits [JawurekKerschbaumOrlandi13]

“$\exists w : R(x, w) = 1$”

$x, w$

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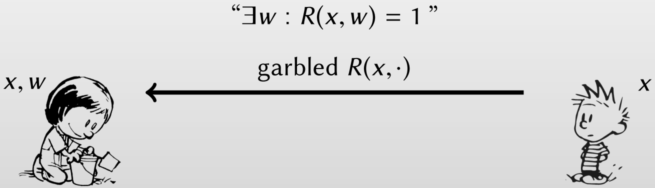

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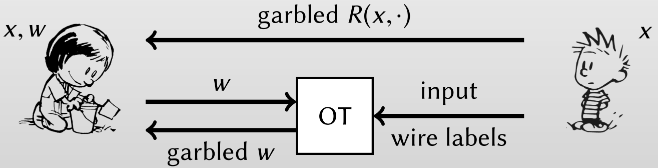

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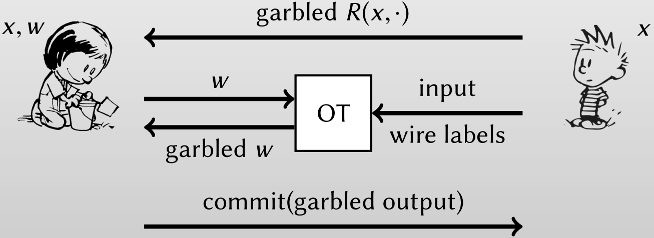

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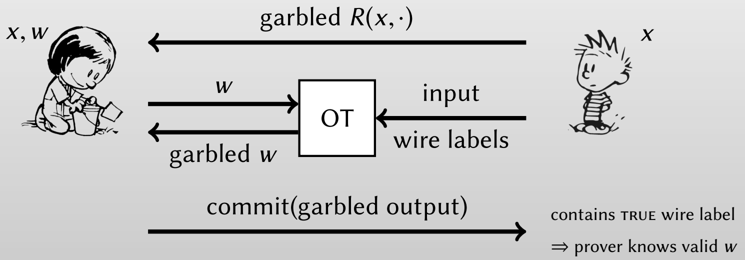

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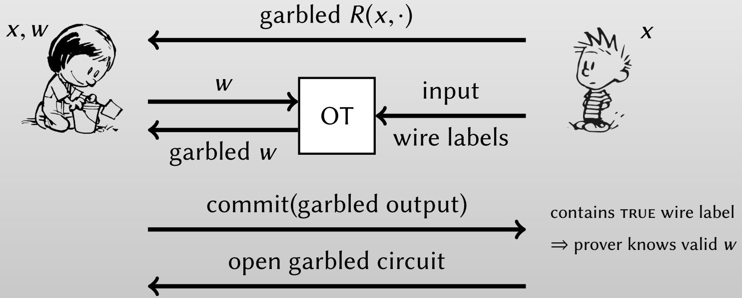

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garbled $R(x, \cdot)$

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OT

input

wire labels

garbled $w$

$x$

commit(garbled output)

contains TRUE wire label $\Rightarrow$ prover knows valid $w$

open garbled circuit

correct GC $\Rightarrow$ garbled output leaks nothing about $w$

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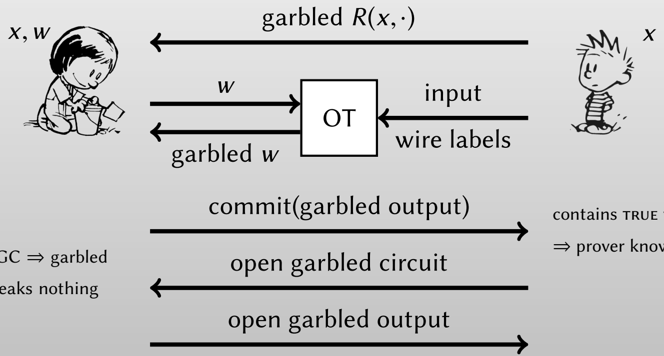


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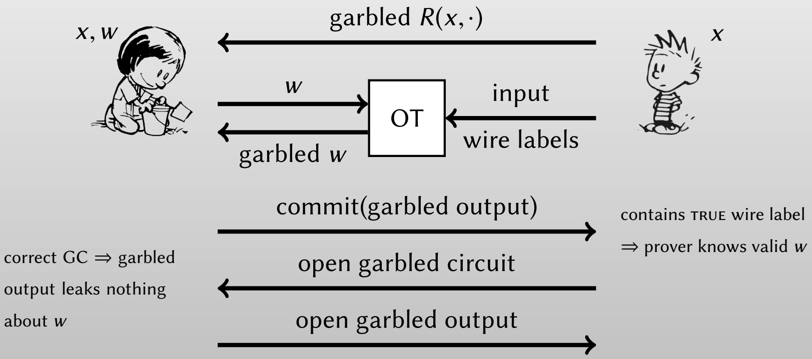


Prover knows entire input to garbled circuit!

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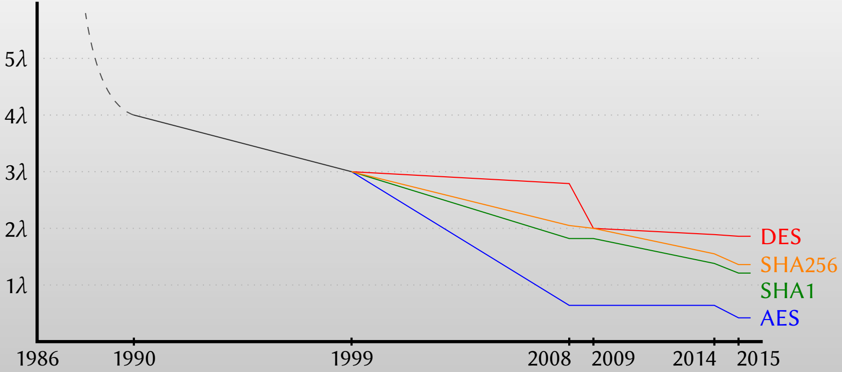
| **PrivFree** * | **0** | **1** | **0** | **2** | **0** | **1** | circ. hash |

# A success story!




- Reduction in size by 10x
- Reduction in computation by 10000x

# the end!