Reconciling Non-malleability & Homomorphic Encryption

Manoj Prabhakaran (U Illinois) · Mike Rosulek (U Montana)

University of Maryland · January 16, 2012

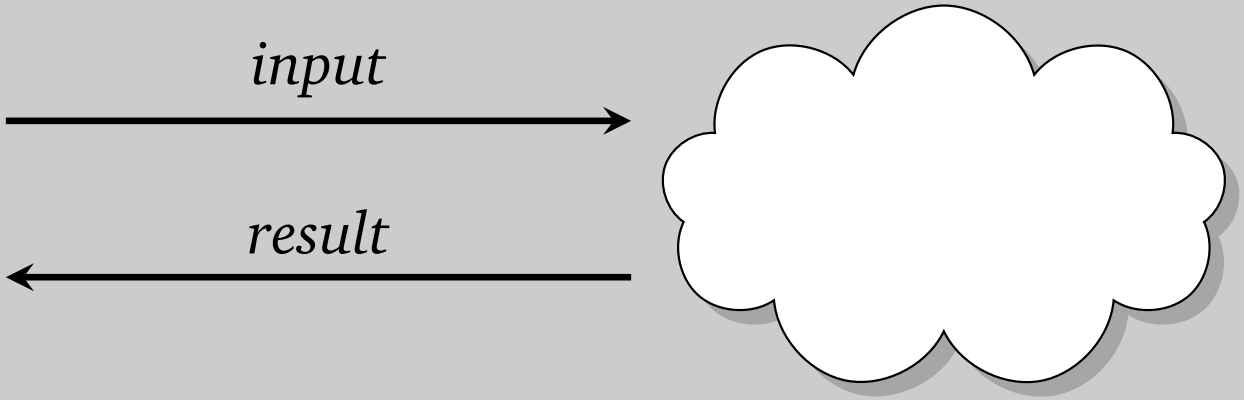
Discussing papers:

[PR07] Manoj Prabhakaran & Mike Rosulek. Rerandomizable RCCA Encryption. In *CRYPTO* 2007.

[PR08] Manoj Prabhakaran & Mike Rosulek. Homomorphic Encryption with CCA Security. In *ICALP* 2008.

[PR08b] Manoj Prabhakaran & Mike Rosulek. Towards Robust Computation on Encrypted Data. In *ASIACRYPT* 2008.

computing on encrypted data



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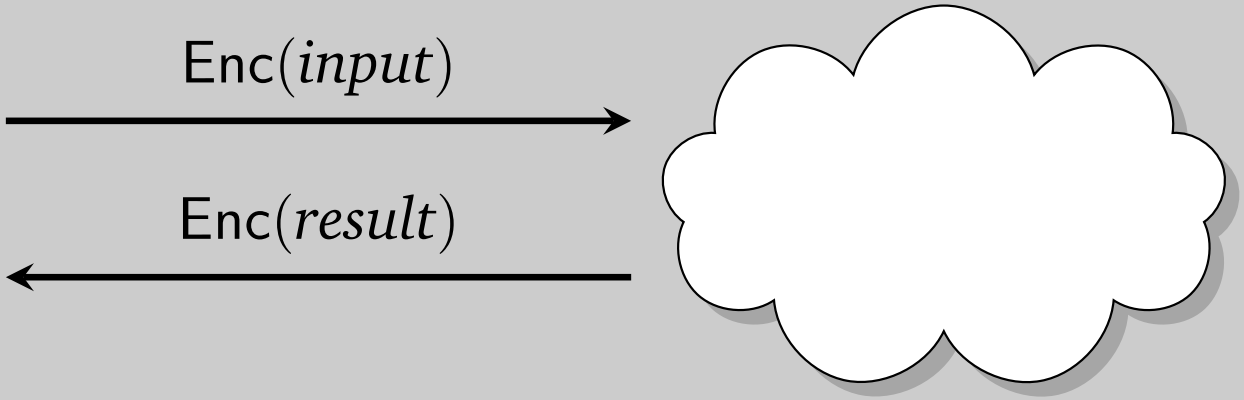

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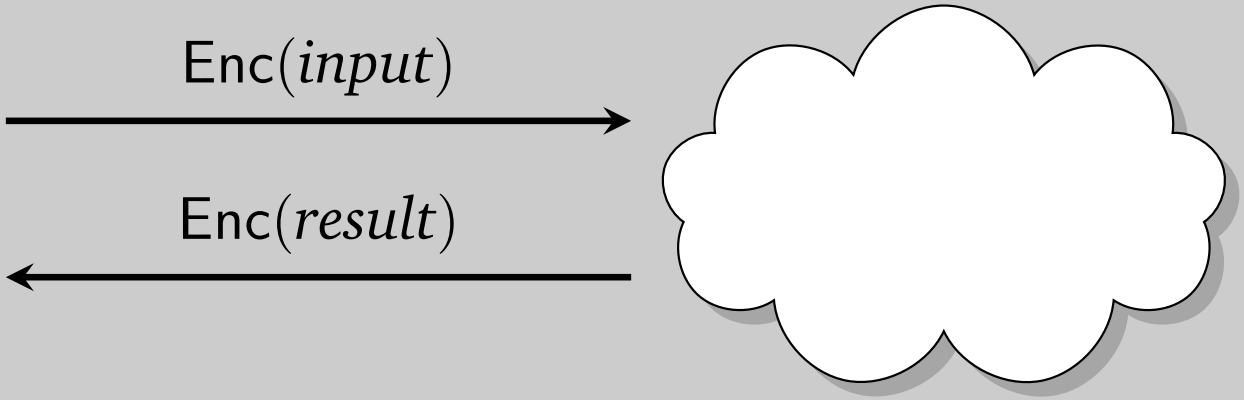

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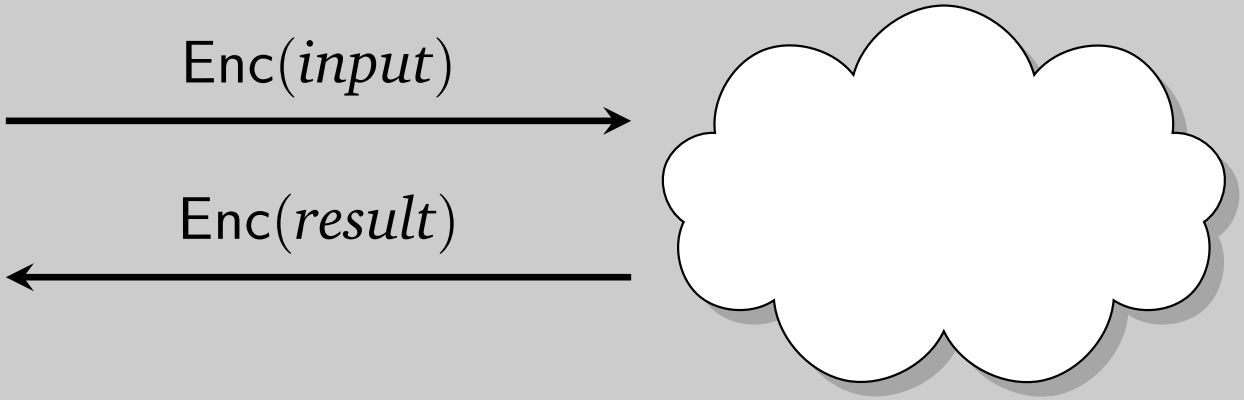

Cloud should be able to manipulate encrypted data.

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Cloud shouldn't be able to perform incorrect computation.

- Non-malleability!
- "Can't generate certain related ciphertexts"

malleability: all-or-nothing

Homomorphic encryption?

- Can't say anything about non-malleability
- Resort to ZK proofs for integrity

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Non-malleable encryption?

- Doesn't allow any useful features
- Need additional tools (MPC?) to manipulate encrypted data

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Can we address both concerns in a single encryption scheme?

the problem

Desired security (informal)

"Scheme is *non-malleable, except for explicitly allowed features.*"

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Challenges:

- Convincingly formalize intuitive definition
- Achieve definition with new constructions

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This talk: "allowed features" = homomorphic encryption

1. New general-purpose non-malleability definition
2. New family of constructions
3. Some impossibility results

(unary) homomorphic encryption

Anyone can change $\mathsf{Enc}(m)$ into fresh $\mathsf{Enc}(f(m))$.

- Scheme parameterized by set of allowed f's

Example:

Rerandomizable Replayable-CCA (RCCA) [CKN03,G04,PR07]:

- Only allowed f is identity function
- Non-malleable in any ways that *alter* message

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Example:

Only allowed f's are group operations $\alpha \mapsto \beta\alpha$ (known β):

- Possible to change any message to any other message
- Infeasible to change $\mathsf{Enc}(\alpha)$ into $\mathsf{Enc}(\alpha^k)$
- Infeasible to change $\mathsf{Enc}(\alpha), \mathsf{Enc}(\beta)$ into $\mathsf{Enc}(\alpha\beta)$

contrast with FHE:

Fully homomorphic encryption [G09]:

- Focus: maximum expressivity
- No consideration of non-malleability
- Binary operations: $\mathsf{Enc}(m_1), \mathsf{Enc}(m_2) \rightsquigarrow \mathsf{Enc}(m_1 + m_2)$

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This talk:

Focus: sharp tradeoff between malleability / non-malleability:

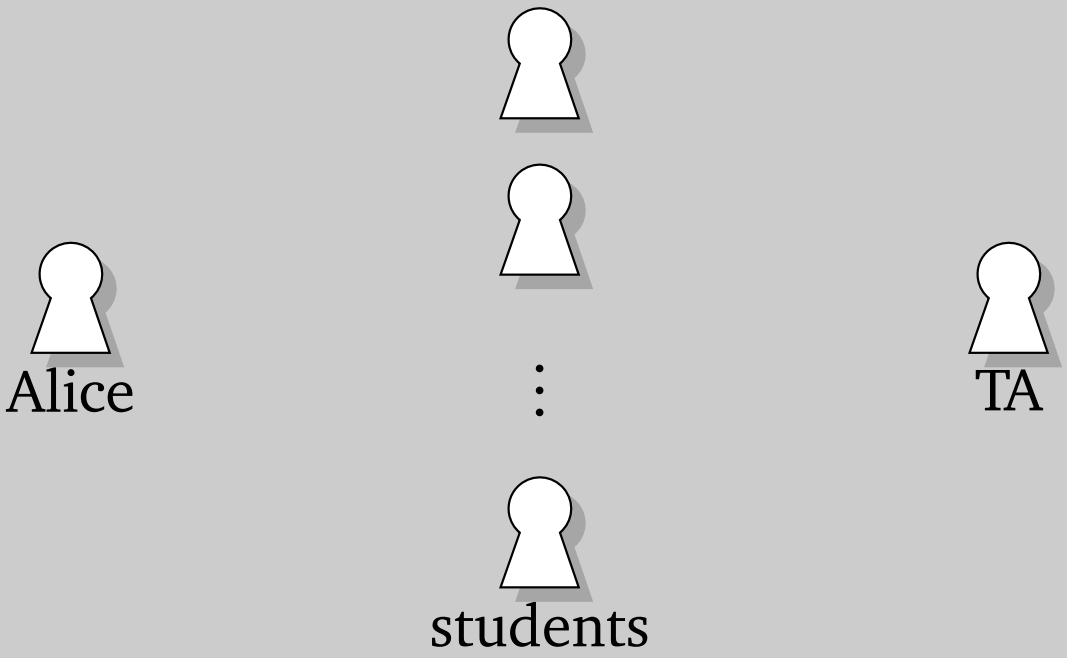
allowed f: available as highly expressive full feature

everything else: computationally infeasible

Challenging even when expressivity is low:

- e.g.: $\mathcal{F}$ contains only one operation

teaching evaluations [PR08b]



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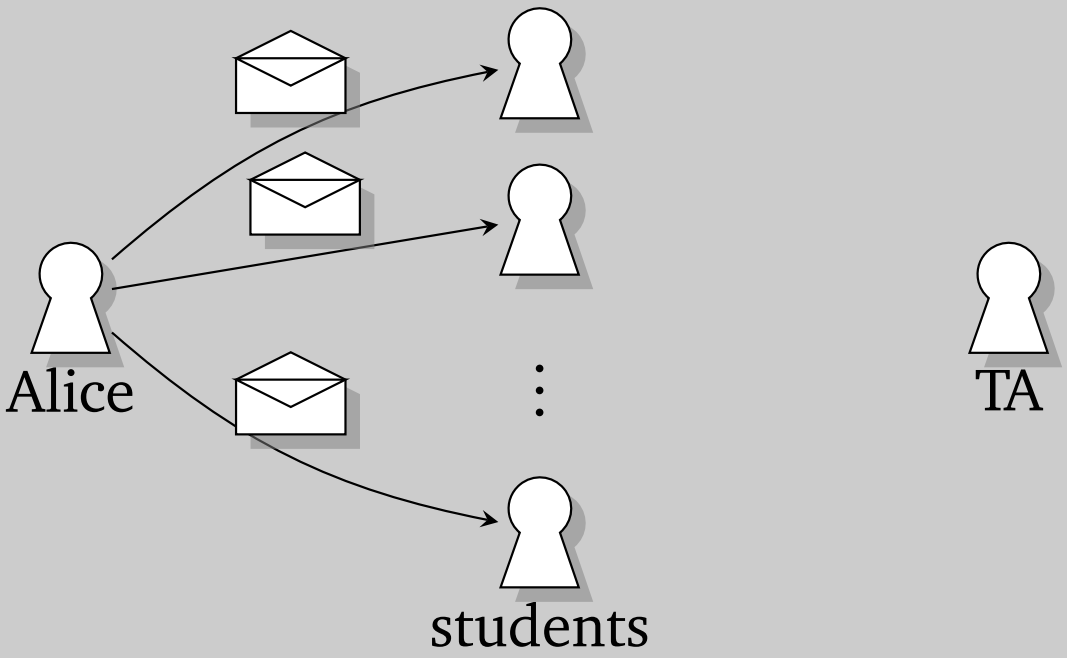

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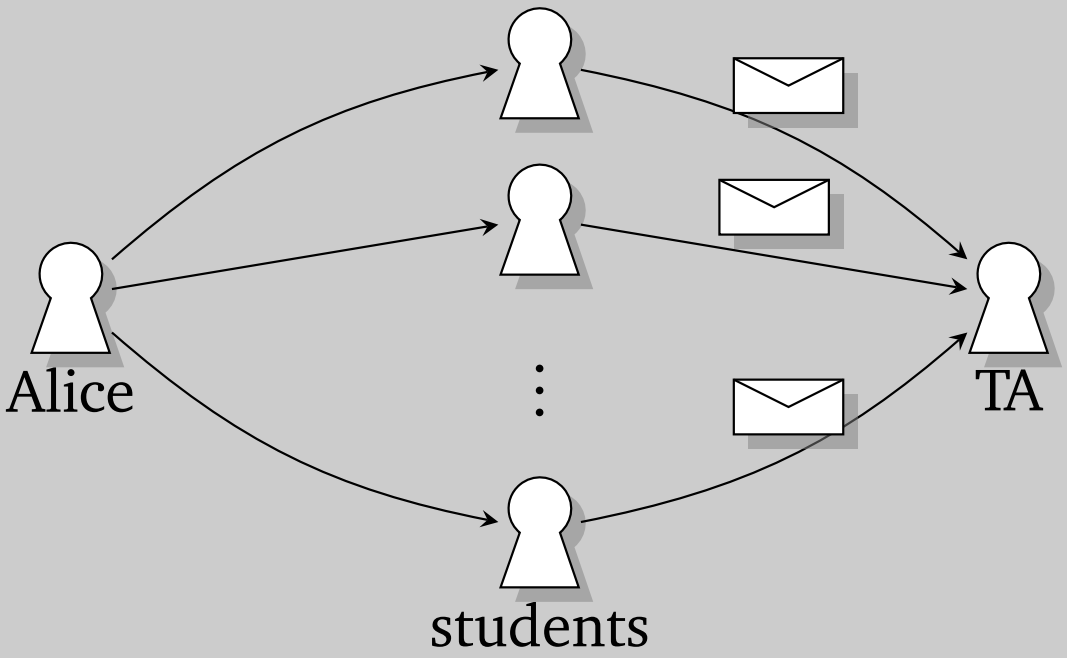

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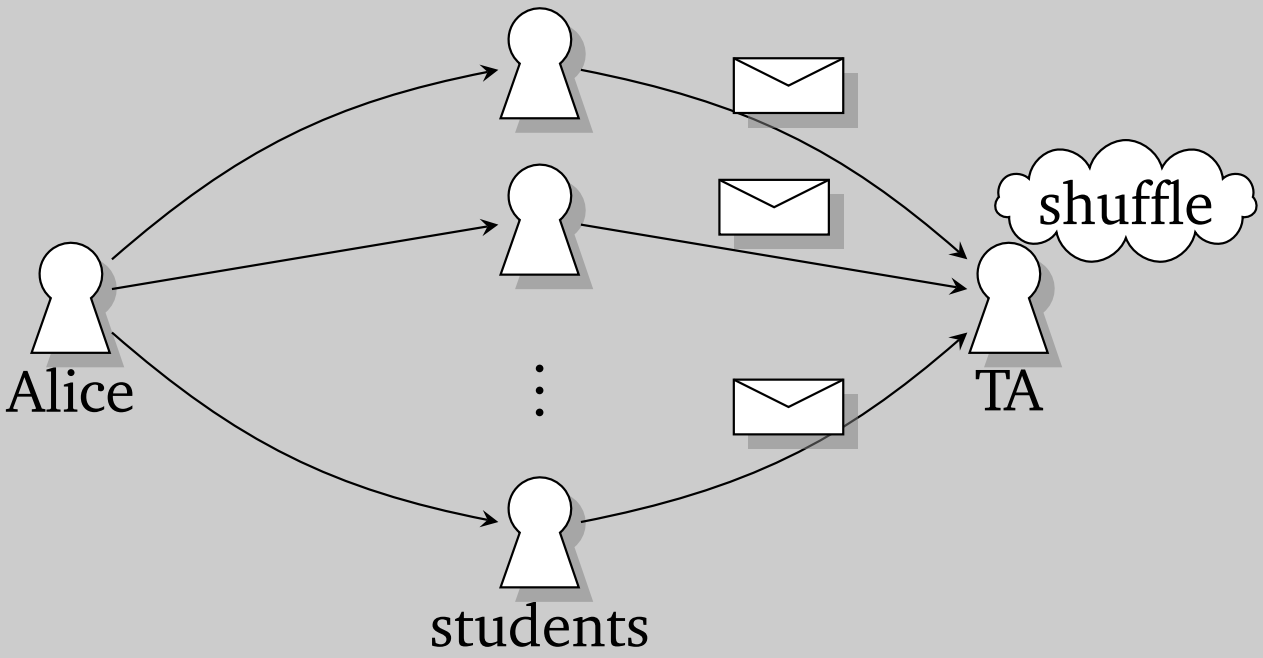

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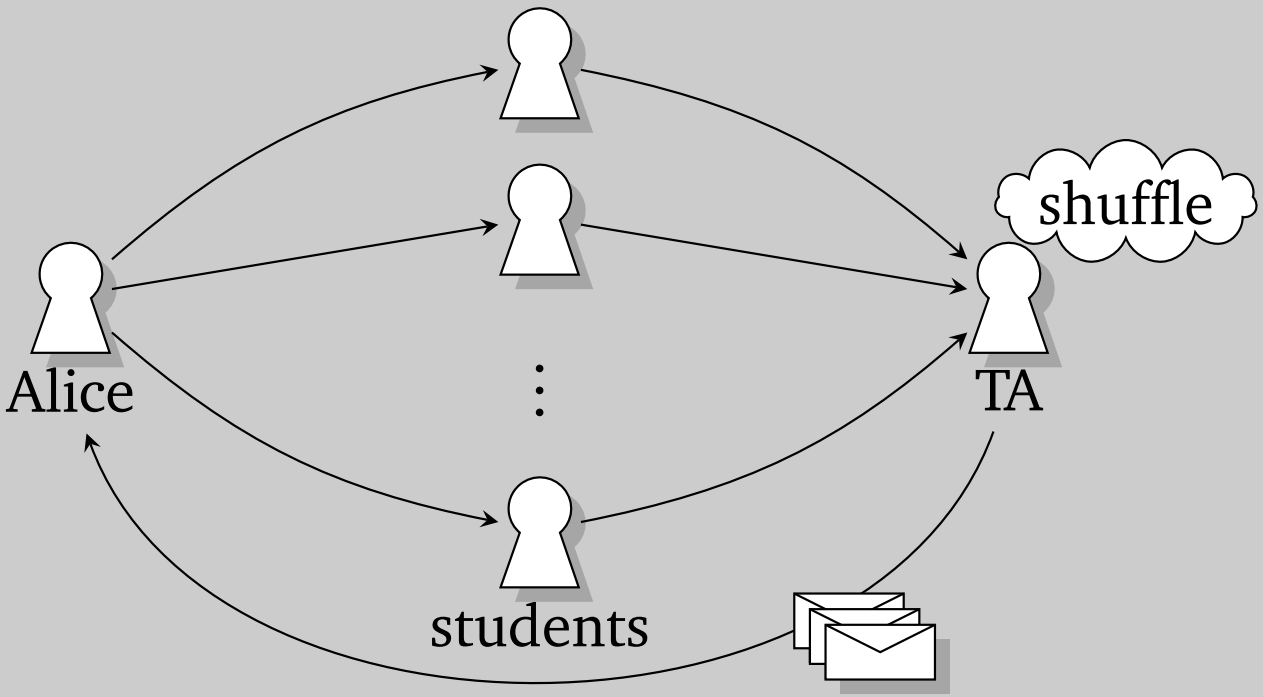

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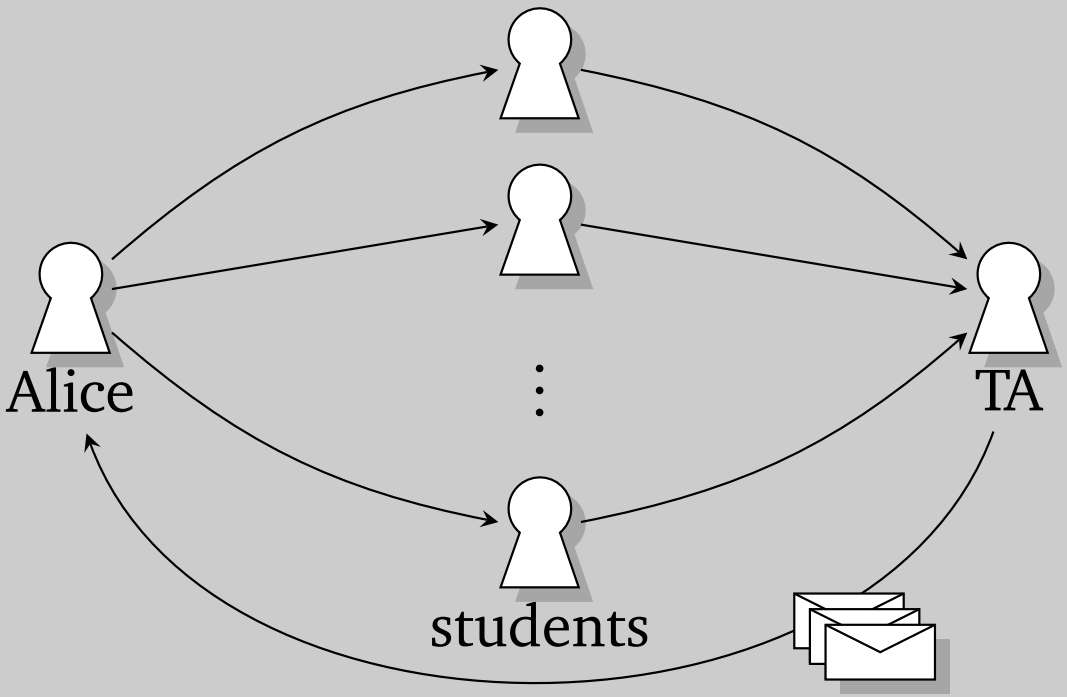

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Functionality: TA must be able to anonymize (shuffle)

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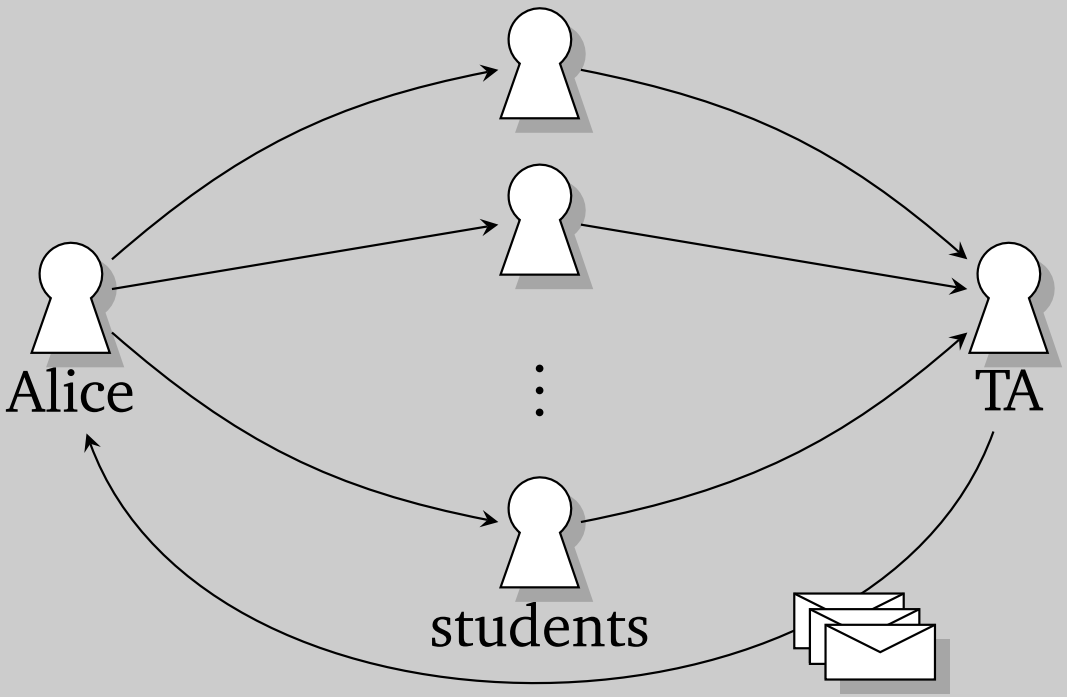

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Verifiable shuffle [G02,GL07a,GL07b]

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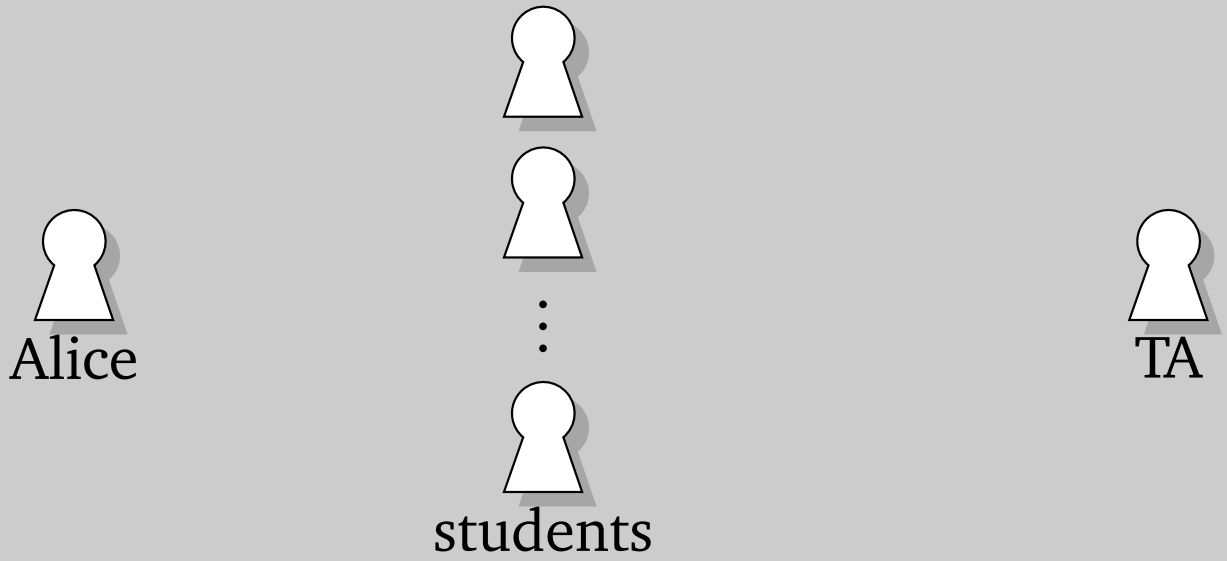

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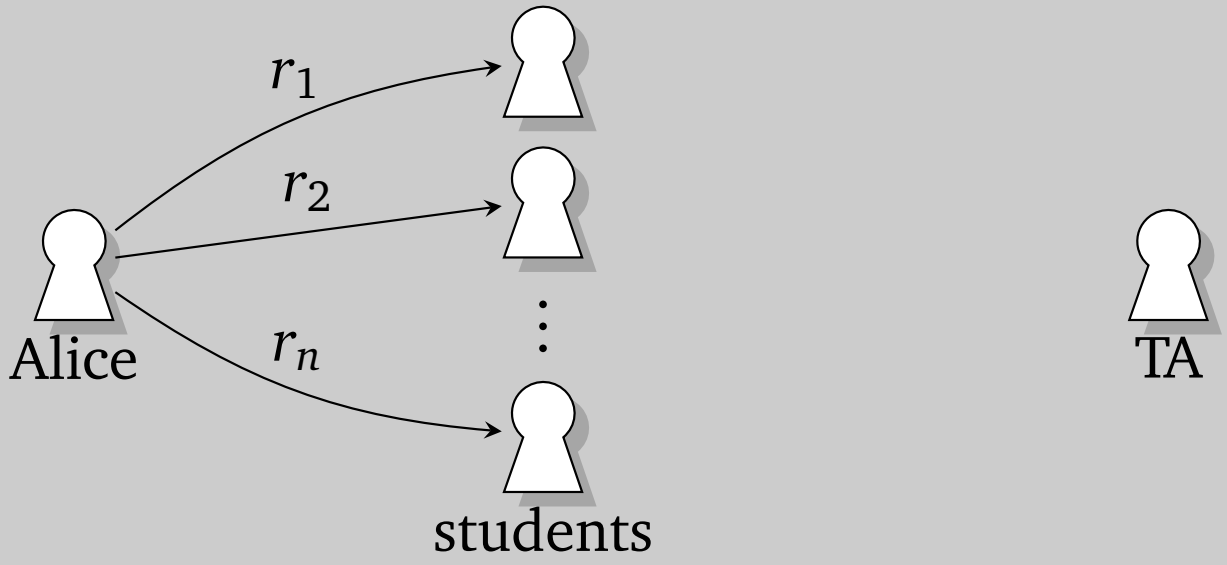

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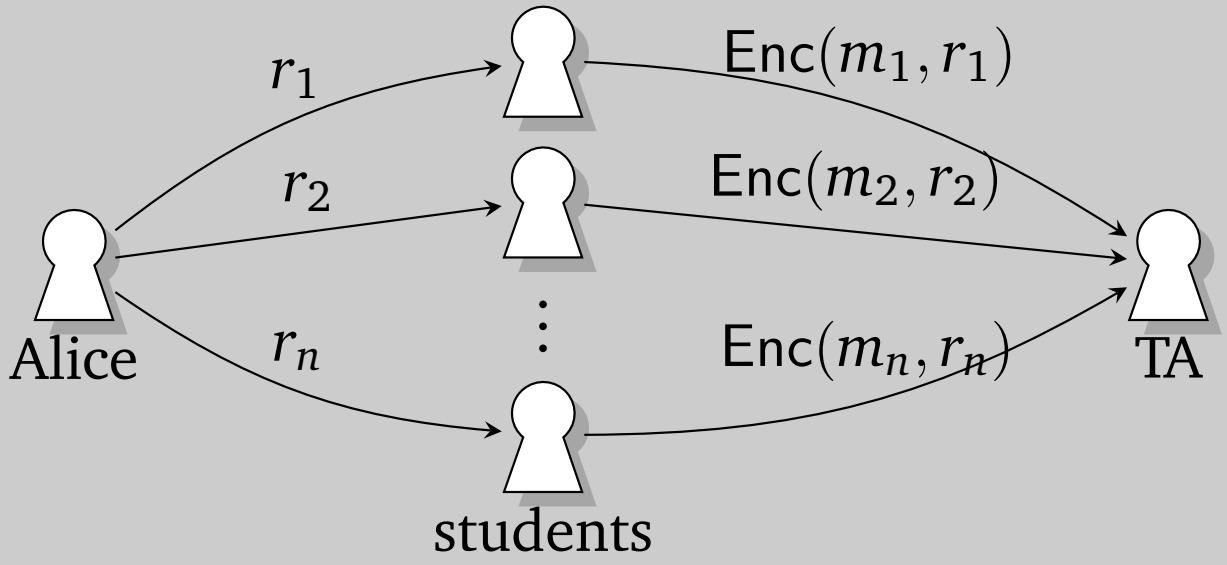

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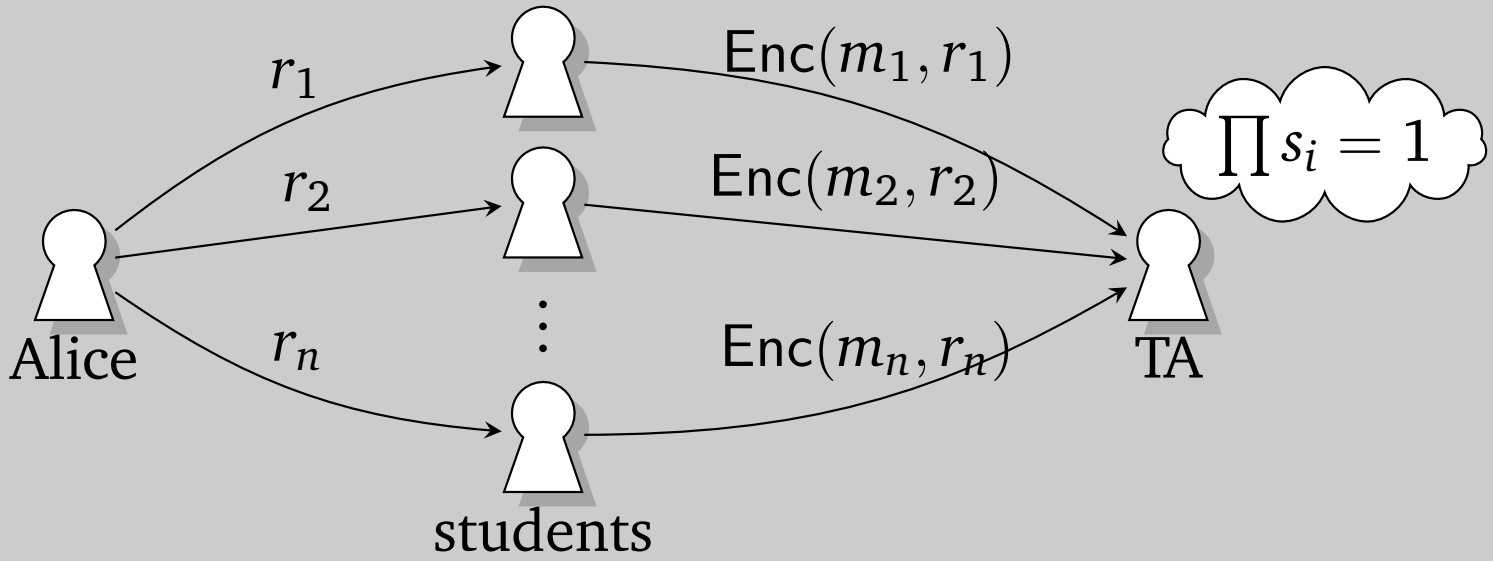

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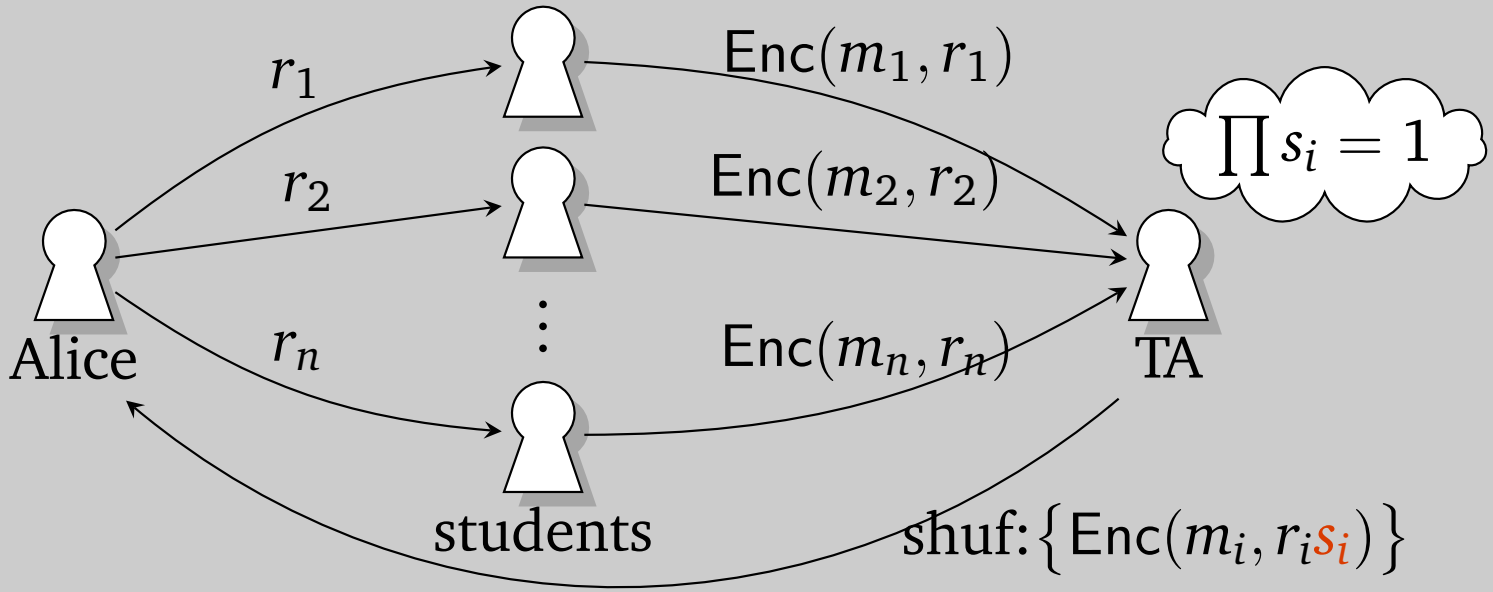

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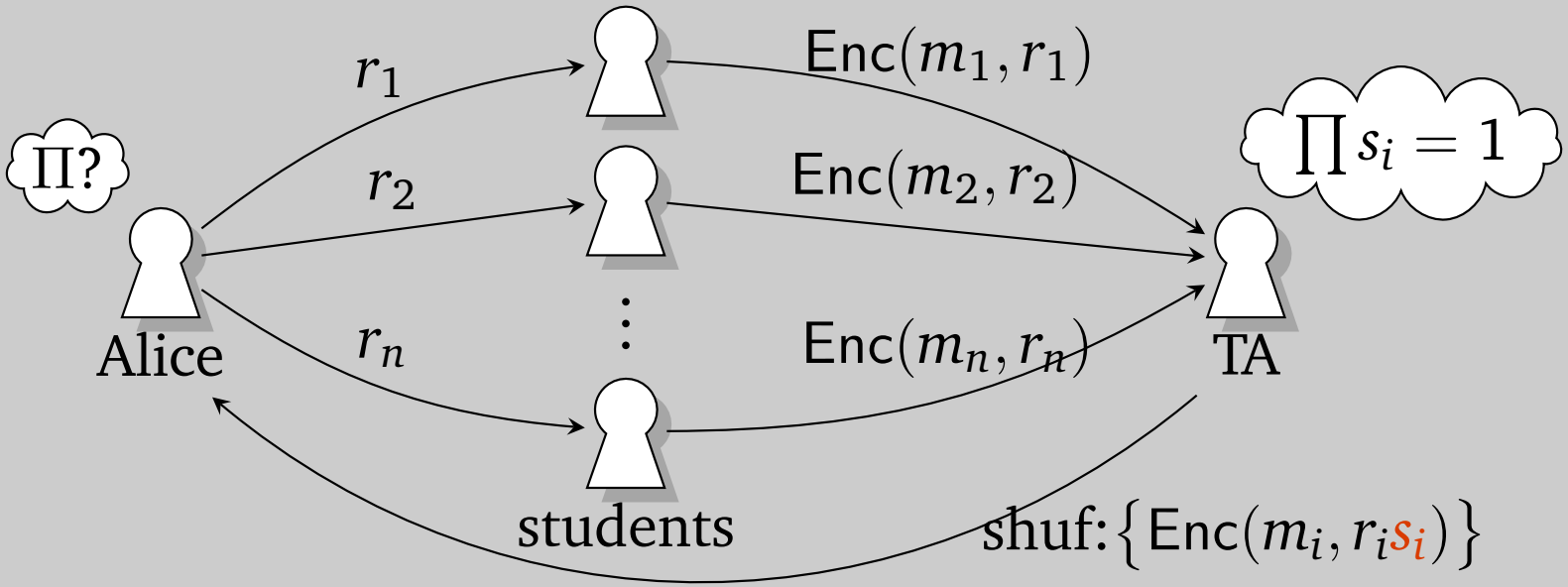

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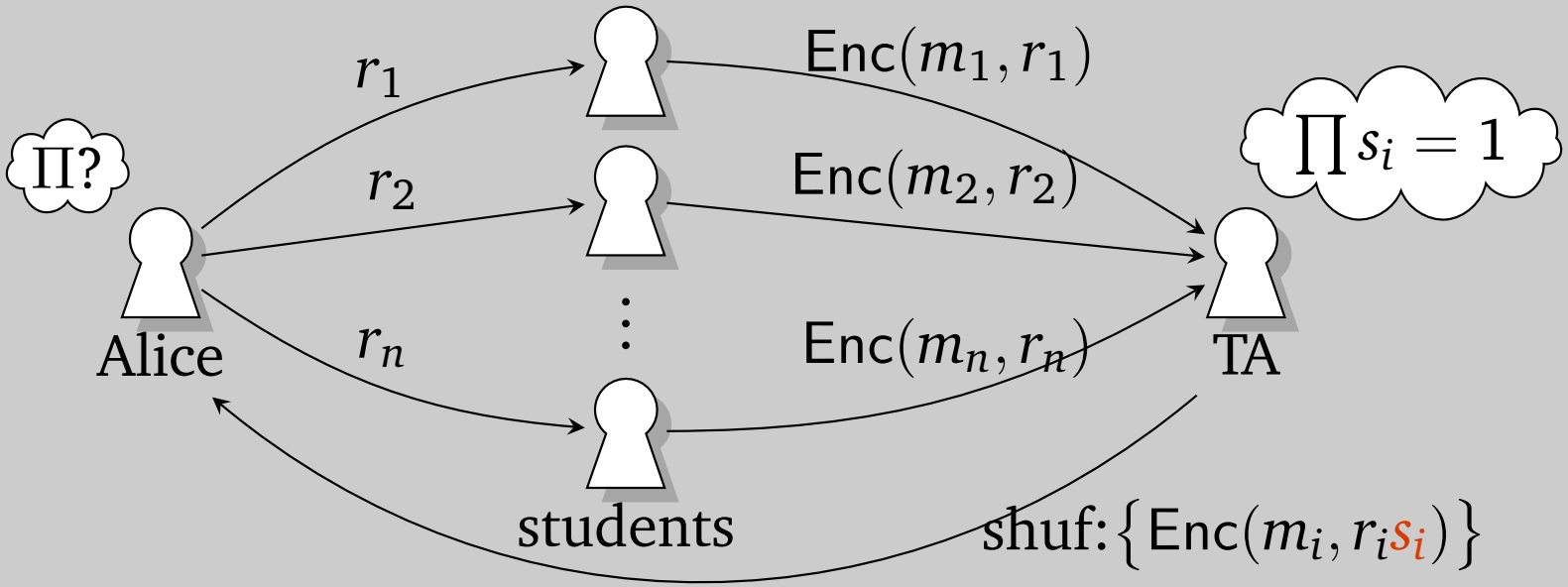

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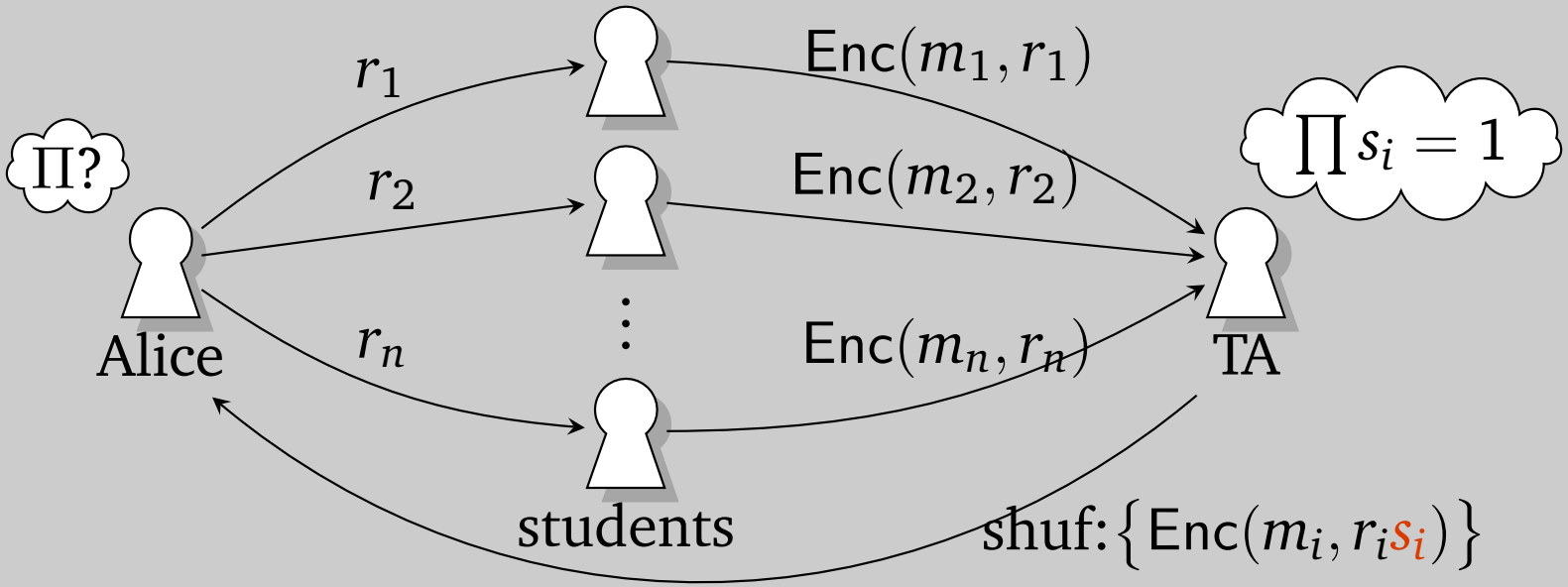

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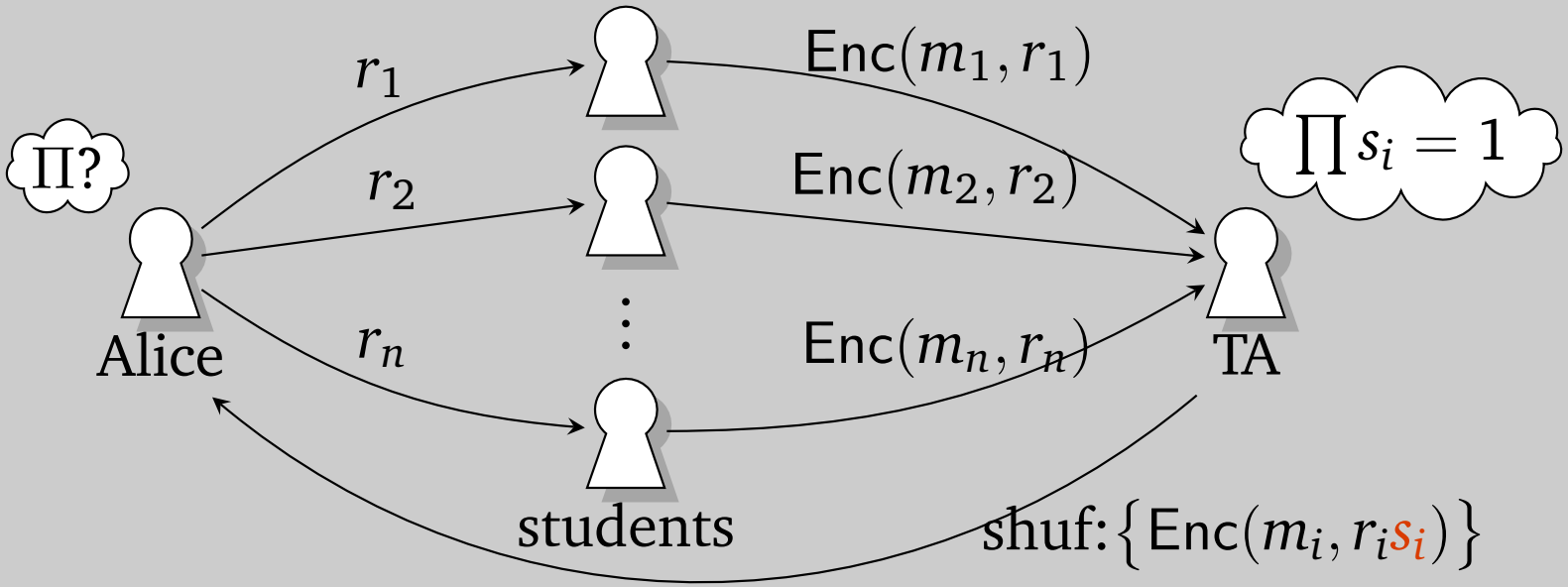

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- Can't get dependence on an r_i without its m_i intact

approach to definitions

$\mathcal{F}$-unlinkability

Given unknown $\mathsf{Enc}(m)$, anyone can generate *fresh* $\mathsf{Enc}(f(m))$, for any $f \in \mathcal{F}$.

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Goal: Simultaneously achieve $\mathcal{F}$-unlinkability and $\mathcal{F}$-HCCA.

generalizing IND-CCA game

1. Generate keypair, give PK.
2. Provide Dec oracle.
3. Adversary chooses m_0, m_1.
4. Give $C \leftarrow \mathsf{Enc}(m_0)$.

5. Provide Dec oracle, except:
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- Anything that could be legitimately derived from C

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Derivatives of C

Ciphertexts that could have been *legitimately* derived from C (i.e., via scheme's allowed features).

Examples:

CCA: C' is derivative $\Leftrightarrow C' = C$

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Examples:

CCA: C' is derivative $\Leftrightarrow C' = C$

gCCA: C' is derivative $\Leftrightarrow R(C', C) = 1$ [S01,ADR02]

RCCA: C' is derivative $\Leftrightarrow \mathsf{Dec}(C') = \mathsf{Dec}(C)$ [CKN03]

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why this won't work

Example: $\mathcal{F} = \{$ group operations, $\alpha \mapsto \beta\alpha$ $\}$

These distributions are *identical*:

- $\mathsf{Enc}(\beta)$ derived by legitimately multiplying $\mathsf{Enc}(\alpha)$ by β/α
- $\mathsf{Enc}(\beta)$ obtained independently by encrypting β

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Problem:

- Dec oracle cannot identify derived ciphertexts
- Alternately, every ciphertext flagged as (possibly) derived from C
  - Game collapses to IND-CCA1.

rigging the game

Key idea: C need not be actual encryption of some m_1:

1. Generate keypair, give PK.
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"Rigged" Ciphertexts

Challenge "ciphertext" can have embedded tracking information.
Extraction procedure determines how C' derived from C.

interpreting the security game

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Suppose adversary can generate C' whose value depends on C

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Response is $f(m^*)$ for some f.

$\Longrightarrow$ Dependence is $\mathsf{Dec}(C') = f(\mathsf{Dec}(C))$, for some $f \in \mathsf{range}(\mathsf{RigExtract})$.

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Suppose RigExtract never outputs f^*:

- Scheme cannot be malleable via f^* operation.

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Homomorphic-CCA (HCCA) Security [PR08]

Scheme is non-malleable except for $\mathcal{F}$ if $\exists$ RigEnc, RigExtract:

1. two worlds indistinguishable in the HCCA game
2. range(RigExtract) $\subseteq \mathcal{F}$

Fine print:

- Oracles for RigEnc and RigExtract should be provided, too.
- RigEnc, RigExtract needed only for security analysis.

interpreting HCCA

Theorem [PR08]

CCA, gCCA, RCCA are all special cases of HCCA

In each case:

- The only allowed transformation is identity function
  - "non-malleable in ways that change the plaintext"
- RigEnc simply uses Enc honestly

interpreting HCCA

Theorem [PR08]

HCCA and unlinkability imply UC-secure protocol for "natural" ideal functionality

Analogous to [C01,CKN03], use UC model to define encryption security.

Message privacy:	Message handles reveal nothing; only recipient can obtain underlying message
Homomorphism:	Anyone can generate a "derived message" by giving f and existing handle
Unlinkability:	Same internal behavior for both kinds of posts
Non-malleability:	No one can use unauthorized f

our construction [PR07,PR08]

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Simplest example: $\mathcal{F} = \{\text{identity function}\}$:

$\mathcal{F}$-unlinkability = rerandomizability

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Question [CKN03]

Are there rerandomizable, RCCA-secure encryption schemes?

our construction [PR07]

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- Rerandomize $\qquad y \mapsto yt$ (multiplicative)

$$(g^y)^t, \quad (A^y)^t \qquad \text{for rand } t$$

$$= \qquad g^{yt}, \quad A^{yt}$$

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- Rerandomize $x \mapsto x + ys$ (additive), $y \mapsto yt$ (multiplicative)

$$\begin{aligned} & g^x(g^y)^s, \quad mA^x(A^y)^s, \quad (g^y)^t, \quad (A^y)^t \qquad \text{for rand } t, s \\ = \; & g^{x+ys}, \quad mA^{x+ys}, \quad g^{yt}, \quad A^{yt} \end{aligned}$$

double-strand Cramer-Shoup

Normal Cramer-Shoup: (μ is hash of first 3 components)

$$g_1^x, g_2^x, m\,B^x, (CD^\mu)^x \qquad \text{for random } x$$

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Possible Attack!

"Mix-and-match" strands from 2 independent ciphertexts

- Result is valid $\Leftrightarrow$ same plaintext (μ)

tie the knot

Key Idea #2

"Tie strands together" with shared randomness u; give an additional encryption of u

$$(g_1^x)^u, (g_2^x)^u, mB^x, (CD^\mu)^x, (g_1^y)^u, (g_2^y)^u, B^y, (CD^\mu)^y, \mathsf{Enc}(u)$$

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What's that $\mathsf{Enc}(u)$ doing?

- Receiver must remove u to decrypt
- $\mathsf{Enc}(u)$ malleable, to allow rerandomization of u ($u \mapsto ru$)
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"Cramer-Shoup lite" has these properties [fine print].

“twist” the first strand

$$g_1^{xu}, g_2^{xu}, mB^x, (CD^\mu)^x, g_1^{yu}, g_2^{yu}, B^y, (CD^\mu)^y, \mathsf{Enc}(u)$$

Possible Attack

Can test candidate plaintext by “rerandomizing” first strand multiplicatively:

$$g_1^{xu}, g_2^{xu}, mB^x, (CD^\mu)^x \quad \rightsquigarrow \quad (g_1^{xu})^2, \quad (g_2^{xu})^2, \quad \frac{(mB^x)^2}{m'}, \quad ((CD^\mu)^x)^2$$

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Decryption succeeds iff $m = m'$.

“twist” the first strand (cont.)

Key Idea #3

Make first strand fundamentally different, so it can only be rerandomized in the prescribed way

“twist” the first strand (cont.)

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Make first strand fundamentally different, so it can only be rerandomized in the prescribed way

Analysis uses linear-algebraic interpretation:

- First strand’s randomness x additively perturbed by fixed vector
- More components $(g_1, \ldots, g_4)$ needed for linear independence.

$$\begin{array}{cccccccc} g_1^{xu}, & g_2^{xu}, & g_3^{xu}, & g_4^{(x+1)u}, & mB^x, & (CD^\mu)^x, \\ g_1^{yu}, & g_2^{yu}, & g_3^{yu}, & g_4^{yu}, & B^y, & (CD^\mu)^y, \\ & & & & & \mathsf{Enc}(u) \end{array}$$

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- Multiplication in *exponent* of Cramer-Shoup group ($\mathbb{Z}_p^*$):

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For 2 operations to coincide, must have:

- CS-lite group is subgroup of $\mathbb{Z}_p^*$.
- DDH in both groups (that of CS and CS-lite)

Is it an unreasonable relationship between 2 groups?

cunningham chains

Definition

A Cunningham chain is 3 primes of the form $q, 2q+1, 4q+3$.

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A Cunningham chain is 3 primes of the form $q, 2q+1, 4q+3$.

- $\mathbb{QR}^*_{4q+3}$ and $\mathbb{QR}^*_{2q+1}$ have desired subgroup relationship
- DDH believed to hold ($4q+3$ and $2q+1$ are safe primes).
- Cunningham chains known to exist for $q \sim 2^{20,000}$.

security

Theorem [PR07]

Our construction is unlinkable & RCCA-secure under DDH assumption in 2 groups of related size.

generalization

Generalized construction [PR08] allows operations of the form:

$$\mathsf{Enc}(x_1, \ldots, x_n) \rightsquigarrow \mathsf{Enc}(x_1 h_1, \ldots, x_n h_n) \text{ for any } \vec{h} \in \mathbb{H}$$

... where parameter $\mathbb{H}$ can be any subgroup of $\mathbb{G}^n$

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Example instantiations:

- $\mathbb{H} = \{\mathbf{1}\}$: Cannot change plaintext, only rerandomize
- $\mathbb{H} = \mathbb{G}^n$: Can freely "multiply" coordinate-wise
- $\mathbb{H} = \{\mathbf{1}\} \times \mathbb{G}$: Only first component non-malleable
- $\mathbb{H} = \{(h, \ldots, h) \mid h \in \mathbb{G}\}$: "Scalar multiplication" of vector

beyond unary operations

What about binary operations?

$$\mathsf{Enc}(x) \otimes \mathsf{Enc}(y) \rightsquigarrow \mathsf{Enc}(x+y)$$

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Impossible to achieve unlinkable-HCCA-security, for *binary group operation*.

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Impossible to achieve unlinkable-HCCA-security, for *binary group operation*.

Proof sketch:

1. Give n "rigged" ciphertexts to adversary: $C_1, \ldots, C_n$
2. Adversary picks $S \subseteq [n]$, requests decryption of $C^* = \bigotimes_{i \in S} C_i$
3. Simulator must extract "derivation history" for C^* (i.e., S)
4. For n sufficiently large:

$$\#\text{ possible histories} = 2^n \gg \#\text{ possible ciphertexts}$$

relaxed requirements

We can achieve relaxed requirements [PR08b]:

- Ciphertexts have "length" parameter
- Ciphertexts leak length parameter but *nothing else*
- Non-malleable except for unlinkable operation:

$$\mathsf{Enc}(\alpha;\ell), \mathsf{Enc}(\beta;\ell') \rightsquigarrow \mathsf{Enc}(\alpha * \beta;\ell+\ell')$$

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Uses unary HCCA as a building block.

- Similar to [SYY99] notion of "cryptocomputing" with ciphertexts of increasing length.

taming malleability

"Non-malleable except for prescribed [strong] operations:"

taming malleability

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Controlled-non-malleable NIZK: Chase et al. [CKLM12]:

NIZK non-malleable except unlinkable operations (for $f \in \mathcal{F}$):

$$\text{“}\exists w : R(x, w)\text{”} \rightsquigarrow \text{“}\exists w' : R(f(x), w')\text{”}$$

Modular construction of unlinkable-HCCA-secure encryption (CPA + NIZK paradigm)

taming malleability (cont.)

Computing on Authenticated Data: Ahn et al. [ABCHSW12]:

Signature non-malleable except unlinkable operations (for $f \in \mathcal{F}$):

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Definitions support binary operations.

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Targeted-malleable encryption: Boneh et al. [BSW12]:

- Relaxation of HCCA security
- Supports binary operations, ciphertext size grows

open problems

More of the same:

- Improved constructions; Hybrid encryption with HCCA-secure KEM?
- Extend “HCCA-style” ideas to other primitives

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Applications:

- Have a hammer; nails sought
- Relaxations of HCCA security for natural applications
- Secret-key? Special features of the scheme *key-activated*?

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Technical: avoid pairings via "nested Cunningham groups" ?

- Pairings: have g^a, g^b, get $e(g^a, g^b) = e(g, g)^{ab}$.
- Nested DDH: have g^a and malleable $\mathsf{Enc}(b)$, get $(g^a)^b$.

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Anything else?

Thanks for your attention!

fin.