Linicrypt: A Model for Practical Cryptography

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A Model for "Practical" Cryptography

Block cipher modes

$\underline{\text{CBC.Enc}_k(m_1, \ldots, m_n):}$

$c_0 \leftarrow \{0,1\}^\lambda$
for $i = 1$ to n:
$\quad c_i := E_k(m_i \oplus c_{i-1})$
return $(c_0, \ldots, c_n)$

$\underline{\text{OCB.Enc}_k(m_1, \ldots, m_n):}$

$L := E_k(0^\lambda)$
$R := E_k(N \oplus L)$
for $i = 1$ to n:
$\quad c_i := E_k(m_i \oplus \gamma_i L \oplus R) \oplus \gamma_i L \oplus R$
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- Call block cipher

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- Sample a random block
- XOR blocks
- Call block cipher
- $GF(2^\lambda)$ linear operation

A Model for "Practical" Cryptography

Hash-based signatures

Lamport.KeyGen():

  for $i = 1$ to n and $b \in \{0,1\}$:

    $sk_i^b \leftarrow \{0,1\}^\lambda$

    $vk_i^b = H(sk_i^b)$

Lamport.Sign(sk, m):

  return $(sk_1^{m_1}, \ldots, sk_n^{m_n})$

Winternitz.KeyGen():

  $sk \leftarrow \{0,1\}^\lambda$

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Modern hash-based signatures (e.g., SPHINCS) built from these components.

A Model for "Practical" Cryptography

Garbled circuits

GarbledANDGate(A, B, C, Δ):

output:

$$\begin{aligned} G_1 &= H(A, B) \oplus C \\ G_2 &= H(A, B \oplus \Delta) \oplus C \\ G_3 &= H(A \oplus \Delta, B) \oplus C \\ G_4 &= H(A \oplus \Delta, B \oplus \Delta) \oplus C \oplus \Delta \end{aligned}$$

Eval(A^*, B^*):

return $H(A^*, B^*) \oplus G_i$

(significantly simplified)

A Model for “Practical” Cryptography

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(significantly simplified)

- XOR blocks
- Call hash function function

High-level overview

Linicrypt = class of algorithms that can:

- uniformly **sample** finite field elements,
- **query** a random oracle (input/output = field elements),
- apply **linear** combinations to field elements.

Captures a broad range of practical cryptographic techniques.

Linicrypt Highlights

Efficient Decidability:

Can decide, in polynomial time, whether two Linicrypt programs have indistinguishable outputs (in RO model).

$$\boxed{P_1} \stackrel{?}{\cong} \boxed{P_2}$$

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Fine-Grained:
Possibility of lower-bounds on **concrete constants** (e.g., possible with 3 field elements, impossible with 2). Strong lower bounds in RO model.

Outline

1. **Linicrypt model & technical tools**
2. Synthesizing Linicrypt programs
3. Applications to Garbled Circuits
4. Future work, open problems

Model

Allowed Linicrypt operations:

$v_1 := \mathsf{input}[1]$ // input a field element

$v_2 \leftarrow \mathbb{F}$ // sample the field

$v_3 := H(v_2)$ // query RO with field elem

$v_4 := v_1 + 2\,v_3$ // fixed linear combination

return v_2, v_4 // return any number of vars

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Similar models:

- **Minicrypt** [Impagliazzo95]: no linearity restriction
- **Generic Group Model** [Shoup97]: only linear operations "in the exponent", but can use *secret* coefficients
- **Arithmetic Model** [ApplebaumAvronBrzuska15]: no random oracle, constructions *oblivious* to choice of field

Linicrypt technical tools

Theorem: can decide, in polynomial time, whether two *input-less* Linicrypt programs have **indistinguishable output** distributions.

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Tools:

- **Algebraic representation** - output distribution of Linicrypt programs as collection of matrices/vectors.
- **Normal form** - find a canonical representation.
- **Basis changes** - reorder the variables.

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- **Normal form** - find a canonical representation.
- **Basis changes** - reorder the variables.

Algebraic representation: base variables

P:

$v_1 \leftarrow \mathbb{F}$
$v_2 \leftarrow \mathbb{F}$
$v_3 := v_1 + v_2$
$v_4 := H(v_3)$
$v_5 := v_4 + v_1$
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return (v_4, v_5)

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Def: A **base** variable is the result of a *sampling* step or *call to H*.

Every variable in **P** is a **linear function** of the base variables.

Algebraic representation: program as matrix

Since every var in **P** is a linear function of the base variables, write them as a **matrix**.

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Algebraic representation: program as matrix

Since **the output of P** is a linear function of the base variables, write it as a matrix.

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$\mathcal{M}$ does not account for the way **variables can be correlated via H.**

Algebraic representation: oracle constraints

An **oracle constraint** captures: *"query H at <u>this</u> linear combination of base variables, receive <u>that</u> combination of base variables."*

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Lemma: $\mathcal{M}$ & $\mathcal{C}$ completely characterize P!

Linicrypt technical tools

Theorem: can decide, in polynomial time, whether two *input-less* Linicrypt programs have **indistinguishable output** distributions.

Tools:

- **Algebraic representation** - output distribution of Linicrypt programs as matrices.
- **Normal form** - remove "extra" oracle queries to generate a canonical representation.
- **Basis changes** - reorder the variables.

Normalize: unreachable oracle queries

What **linear constraints of base variables** does Adv know?

P:
$v_1 \leftarrow \mathbb{F}$
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In this example, first oracle constraint is **unreachable**.

Normalize: unreachable oracle queries

Claim: Pr[Adv calls H on unreachable query] $=$ negl.

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- Query 1 is **unreachable**
- Remove that oracle constraint $\Rightarrow$ negligible effect on distinguisher

Normalize: useless oracle queries

Other kinds of oracle constraints can be removed, too!

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- v_6 never even used in the program, clearly can be removed
- **Definition:** oracle constraint is **useless** if RHS is **linearly independent** of everything else
- Useless oracle constraints can be removed, no effect on Adv

Subtleties about useless constraints

$$\begin{array}{l}\underline{\mathsf{P}}\\ v_1 \leftarrow \mathbb{F}\\ v_2 \leftarrow \mathbb{F}\\ v_3 := H(v_1)\\ v_4 := H(v_3)\\ v_5 = v_2 + v_4\\ \text{return } v_1, v_5\end{array}$$

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- v_4 *syntactically* influences the output, but still **useless**

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- Removing constraint #2 causes constraint #1 to be useless!

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Normalization

normalize(P):

iteratively delete **unreachable** and **useless** oracle constraints from P

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Lemma: **normalize**$(P) \cong P$.

Linicrypt technical tools

Theorem: can decide, in polynomial time, whether two *input-less* Linicrypt programs have **indistinguishable output** distributions.

Tools:

- **Algebraic representation** - output distribution of Linicrypt programs as matrices.
- **Normal form** - remove "extra" oracle queries.
- **Basis changes** - reorder the variables.

Basis changes

- We defined <u>reachable</u> & <u>useful</u> in terms of **linear independence**:
  - Constraint is <u>reachable</u> if LHS is in span of reachable vectors.
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Proposition: Linicrypt program $\mathbf{P}$ is **indistinguishable** from $\mathbf{B}\times\mathbf{P}$, where $\mathbf{B}$ is invertible matrix.

Linicrypt technical tools

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Bringing it all together

Main Theorem:

$P_1 \cong P_2$ if and only if
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- Choose basis change for **normalize**(P_1) so that it coincides with **normalize**(P_2) "as much as possible"
- If they completely coincide, then $P_1 \cong P_2$

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Basic outline:

- Choose basis change for **normalize**(P_1) so that it coincides with **normalize**(P_2) "as much as possible"
- If they completely coincide, then $P_1 \cong P_2$
- Otherwise, there is an oracle constraint that is ...
  - ... present in P_1 but not in P_2 (by symmetry)
  - ... **reachable** $\Rightarrow$ distinguisher can query it
  - ... **useful** $\Rightarrow$ result involved in some linear relation

  To distinguish, make this query, check that result satisfies the linear relation.

Outline

1. Linicrypt model & technical tools
2. **Synthesizing Linicrypt programs**
3. Applications to Garbled Circuits
4. Future work, open problems

Existential Formulas

Claim: Linicrypt properties can be expressed in an **existential formula.**

Existential Formulas

> **Claim:** Linicrypt properties can be expressed in an **existential formula.**

1. Are two Linicrypt programs indistinguishable?
2. Does the **composition** of two Linicrypt programs have a certain property

Trick: basis change witness

Problem: process for identifying **unreachable** oracle constraints is **highly iterative**.

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Reachable space is the smallest subspace $\mathcal{V}$ s.t.:

- $\mathcal{V}$ contains all rows of $\mathcal{M}$
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Idea: guess a basis change under which $\mathcal{V} = \mathbb{F}^d \times \{0\}^{n-d}$.

- Guess basis B and its inverse. Check that $B \times B^{-1} = I$
- **A vector is in $\mathcal{V}$ $\Leftrightarrow$ rightmost $n - d$ entries are all zero.**

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Reachable space is the smallest subspace $\mathcal{V}$ s.t.:

- $\mathcal{V}$ contains all rows of $\mathcal{M}$
- For all oracle constraints, if LHS in $\mathcal{V}$ then RHS in $\mathcal{V}$

Idea: guess a basis change under which $\mathcal{V} = \mathbb{F}^d \times \{0\}^{n-d}$.

- Guess basis B and its inverse. Check that $B \times B^{-1} = I$
- **A vector is in $\mathcal{V}$ $\Leftrightarrow$ rightmost $n-d$ entries are all zero.**

Now easy to verify that we've identified the reachable space:

- After basis change, all rows of $\mathcal{M}$ in $\mathcal{V}$?
- After basis change, LHS of constraint in $\mathcal{V}$ $\Rightarrow$ RHS is too?

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Composition:

Use a **basis change** to “line up” base vars.

Composed output is second program’s output.

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$r \leftarrow \mathbb{F}$
return $(r, m + H(r))$

$$\rightarrow \begin{array}{ccc} m & r & H(r) \end{array} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \left\{ [0\ 1\ 0] \mapsto [0\ 0\ 1] \right\}$$

$\underline{\mathrm{Dec}(r, c)\text{:}}$
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$\underline{\mathsf{Dec}(r, m + H(r))}$:
return $(m + H(r)) - H(r)$

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Bonus: **B** is a witness to the **correctness** of this scheme!

$$B = \begin{array}{c c} & \begin{array}{ccc} m & r & H(r) \end{array} \\ \begin{array}{c} r \\ c \\ H(r) \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

Outline

1. Linicrypt model & technical tools
2. Synthesizing Linicrypt programs
3. **Applications to Garbled Circuits**
4. Future work, open problems

Garbled Circuits

Existing garbled-circuit constructions are Linicrypt*:

	cost per gate
Textbook GC [Yao80s]	4 field elements
[NaorPinkasSumner99]	3 field elements
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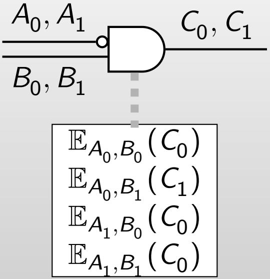
Can we use Linicrypt to **discover new** GC constructions?
prove **lower bounds** about GC constructions?

(focus on constructions to garble small **gates**)

Point/permute [BeaverMicaliRogaway90]



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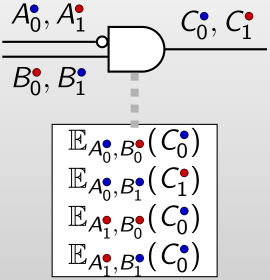

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- Include color in the wire label (e.g., as last bit)

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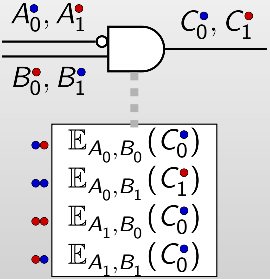

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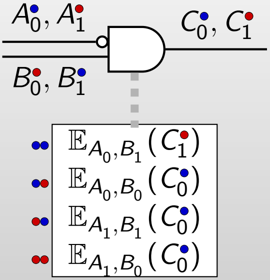

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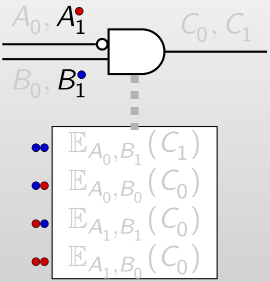

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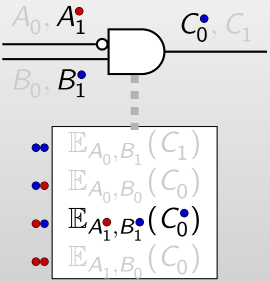

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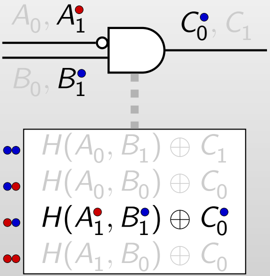

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$\Rightarrow$ smaller garbled circuit, encryption/decryption "linear"

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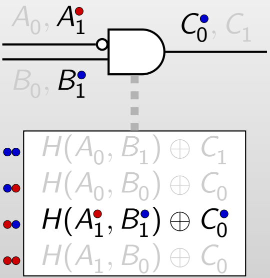

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$\Rightarrow$ smaller garbled circuit, encryption/decryption "linear"

✗ Evaluator's **choice of linear operation** depends on color bit in a non-linear way!

Modeling Garbled Circuits in Linicrypt

- Garbler's behavior is Linicrypt program, after fixing **association between T/F and** $\bullet/\bullet$ on both wires
- Evaluator's behavior is Linicrypt program, after fixing **color bits of input wire labels**

Modeling Garbled Circuits in Linicrypt

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Our approach: Synthesize collection of Linicrypt programs:

$$\{\mathsf{Garble}^{{\color{red}\bullet\bullet}}, \mathsf{Garble}^{{\color{red}\bullet}{\color{blue}\bullet}}, \mathsf{Garble}^{{\color{blue}\bullet}{\color{red}\bullet}}, \mathsf{Garble}^{{\color{blue}\bullet\bullet}}\}$$

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. . . that comprise a point-permute-style GC construction

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... that comprise a point-permute-style GC construction

Details I won't discuss:

- GC security $\Leftrightarrow$ indistinguishability of input-less Linicrypt program
- How to express correctness of GC scheme via composition
- We specialized to Free-XOR-compatible schemes

Linisynth : `github.com/osu-crypto/linisynth`

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Results

name	τ	size	H_{gb}	H_{ev}	time	sat
free-xor	$\oplus : 2 \to 1$	0	0	0	1s	1
half-gate	$\wedge : 2 \to 1$	2	4	2	5s	1
one-third-gate	$\wedge : 2 \to 1$	1	4	2	74s	0
half-gate-cheaper	$\wedge : 2 \to 1$	2	4	1	6.2h	0
1-out-of-2-mux	$\text{MUX} : 3 \to 1$	2	4	2	29s	1
2-bit-eq	$= : 4 \to 1$	2	4	2	6m	1
2-bit-eq-small	$= : 4 \to 1$	1	4	2	6m	0
2-bit-leq	$\leq : 4 \to 1$	1	2	1	77s	0
2-bit-lt	$< : 4 \to 1$	2	4	2	3.5h	0

half-gate size $= 2$ calls$_{\mathsf{gb}} = 4$

$\wedge : \{0,1\}^2 \to \{0,1\}$ time $= 5\mathsf{s}$ calls$_{\mathsf{ev}} = 2$

$\underline{\mathsf{GateGb}^H(\sigma, A, B, \Delta):}$

$$
\begin{aligned}
&h_1 = H(A) \\
&h_2 = H(A + \Delta) \\
&h_3 = H(A + B) \\
&h_4 = H(A + B + \Delta) \\
&G_0 = [0,2]\Delta + h_3 + h_4 \\
&G_1 = A + B + [0,2]\Delta + h_1 + h_2 + h_3 + h_4 \\
&C_0 = B + [0]\Delta + [0,2]h_1 + [1,3]h_2 + [1,2]h_3 + [0,3]h_4 \\
&\text{return } G_0, G_1, C_0
\end{aligned}
$$

$\underline{\mathsf{GateEv}^H(\chi, A^*, B^*, G_0, G_1):}$

$$
\begin{aligned}
\text{return } &[1,3]A^* + [0,2]B^* + \\
&[0,1]G_0 + [1,3]G_1 + \\
&H(A^*) + H(A^* + B^*)
\end{aligned}
$$

Lower Bounds

Existential formula is satisfiable **if and only if**
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SAT solver returns $\textsc{false} \Rightarrow$ **lower bound** for garbled circuit constructions in Linicrypt

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Future Work

Theoretical questions about the model:

1. Security properties involving adversarial inputs (vs. input-less)
2. Support "fancier" random oracles (ideal cipher)
3. Which Linicrypt programs have security from standard assumptions (vs. random oracle)?
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Application areas (synthesis & lower bounds):

1. Block cipher modes, tweakable block ciphers
2. Authenticated encryption modes, MACs
3. Hash-based signatures
4. More garbled circuits
5. Memory-hard functions??

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Thanks!