Rerandomizable RCCA Encryption

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Thanks to Qualcomm for travel support

Unlinkable Blind Copying

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- Ideally, want scheme to be "non-malleable except for copying"

Strong tradeoff between features and non-malleability:

- Copying is allowed, in very robust sense
- Everything else is forbidden, in the strongest sense

Formalizing the Problem

Rerandomizability

For any $C \leftarrow \mathrm{Enc}_{PK}(m)$, $\mathrm{Rerand}(C)$ has same distribution as $\mathrm{Enc}_{PK}(m)$.

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Scheme is CCA secure, except it may be possible to "maul" an encryption of m into another that decrypts to same m.

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Question [Canetti, Krawczyk, Nielsen]

Are there rerandomizable, RCCA-secure encryption schemes?

Related work

Canetti, Krawczyk, Nielsen [*CRYPTO*'03]

- Defined RCCA, posed problem.

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- Applications of rerandomizability for *anonymous* schemes
- Rerandomizable, CPA-secure scheme

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- Applications of rerandomizability for *anonymous* schemes
- Rerandomizable, CPA-secure scheme

Gröth [*TCC*'04] gives two rerandomizable schemes:

- Achieves weaker variant of RCCA security
- Achieves full RCCA in *generic group* model

Our Results

First rerandomizable RCCA scheme in standard model.

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Full version (and this talk) has improved construction

- http://eprint.iacr.org/2007/119

Recipe

Key Idea #1

Use two "strands" of Cramer-Shoup encryption

- First strand is usual encryption
- Second strand helps with rerandomization

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Key Idea #3

"Twist first strand" (technical)

- Make the 2 strands of different type
- Avoid bad ways of combining the 2 strands together

Double-Strand Idea

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Golle et al. [*RSA*'04] use a double-strand idea with ElGamal.

- In their case, second strand carries public key data needed to rerandomize ElGamal.
- In our case, second strand carries message data needed for rerandomize Cramer-Shoup.

Double-Strand ElGamal

Double-strand idea illustrated with ElGamal [Golle et al]:

- Normal ElGamal encryption of m (public key is g^a):

$$g^x, m\,(g^a)^x \qquad \text{for random } x$$

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- Rerandomize x additively

$$\begin{aligned} & g^x(g^y)^s, \quad m\,g^{ax}(g^{ay})^s, && \text{for rand } s \\ = \quad & g^{x+ys}, \quad m\,g^{a(x+ys)}, \end{aligned}$$

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- Rerandomize x additively, y multiplicatively

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Result is distributed as fresh double-strand encryption

Double-Strands for Cramer-Shoup

We apply double-strand idea to Cramer-Shoup:

- Normal Cramer-Shoup: (μ is hash of first 3 components)

$$g_1^x, g_2^x, m\,B^x, (CD^\mu)^x \qquad \text{for random } x$$

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Possible Attack!

Can "mix-and-match" strands from 2 independent ciphertexts, if they carry the same message (μ).

Tie the Knot

Key Idea #2

"Tie strands together" with shared randomness u; give an additional encryption of u

$$(g_1^x)^u, (g_2^x)^u, mB^x, (CD^\mu)^x, (g_1^y)^u, (g_2^y)^u, B^y, (CD^\mu)^y, \mathsf{Enc}(u)$$

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Encryption of u must be:

- malleable, to allow rerandomization of u
- rerandomizable, for randomness within Enc

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Double-strand "Cramer-Shoup lite" has these properties.

Twist the First Strand

$$g_1^{xu}, g_2^{xu}, mB^x, (CD^\mu)^x, g_1^{yu}, g_2^{yu}, B^y, (CD^\mu)^y, \mathrm{Enc}(u)$$

Possible Attack

Can rerandomize first strand multiplicatively, if plaintext is known.

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Analysis uses linear-algebraic interpretation:

- First strand's randomness x is perturbed by fixed vector
- More components $(g_1, \ldots, g_4)$ needed for linear independence.

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Our scheme satisfies:

- RCCA security (Canetti, Krawczyk, Nielsen [*CRYPTO*'03])
- Perfect rerandomizability
- Several correctness properties

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Proof uses:

- DDH assumption in 2 groups (for CS and CS-lite)
  - Requires 3 large primes of special form (Cunningham chain)
- Linear algebra interpretation of our scheme

On Security Definitions

Some philosophical thoughts:

- Want scheme that is non-malleable except for unlinkable copying feature

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On Security Definitions

Some philosophical thoughts:

- Want scheme that is non-malleable except for unlinkable copying feature
- Can also imagine different tradeoff (e.g, non-malleable other than some homomorphic operation)
- What are the "right" security, correctness definitions?
- Details motivated by natural UC formulation...

Universal Composition (UC)

Universal Composition (UC) framework for security definitions

- Proposed by Canetti [*FOCS*'01]
- Simulation-based security with arbitrary interactive environment
- "Natural" formulations of security via ideal functionalities

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- Proposed by Canetti [*FOCS*'01]
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- "Natural" formulations of security via ideal functionalities
- No secure protocols for most tasks (unless model is significantly weakened)!

Our Results

We define a powerful new UC functionality.

- Users send private messages to each other
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Theorem

Any rerandomizable, RCCA-secure scheme (with our correctness properties) is a secure realization of this functionality.

- Justifies our security definitions, correctness properties
- Positive UC result in standard model.
- Can easily extend to add features

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First rerandomizable, RCCA-secure encryption scheme under standard assumption

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Natural characterization in UC framework

- Justifies choice of security definitions
- Sophisticated positive result for standard UC model

Open Problems

Anonymous, rerandomizable RCCA scheme?

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- Most interesting applications of rerandomizability also require anonymity

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Other tradeoffs between features and non-malleability in encryption schemes

- Features can be performed in an unlinkable way
- Scheme is non-malleable otherwise

Thanks for your attention!

fin.

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- ... and as the malleable operation of CS-lite:

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For 2 operations to coincide, must have:

- CS-lite group is subgroup of $\mathbb{Z}_p^*$.
- DDH in both groups (that of CS and CS-lite)

Is it an unreasonable relationship between 2 groups?

Cunningham Chains

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- $\mathbb{QR}^*_{4q+3}$ and $\mathbb{QR}^*_{2q+1}$ have desired relationship
- DDH believed to hold ($4q+3$ and $2q+1$ are *safe primes*).
- Cunningham chains known to exist for $q \sim 2^{20,000}$.

Security Proof Outline

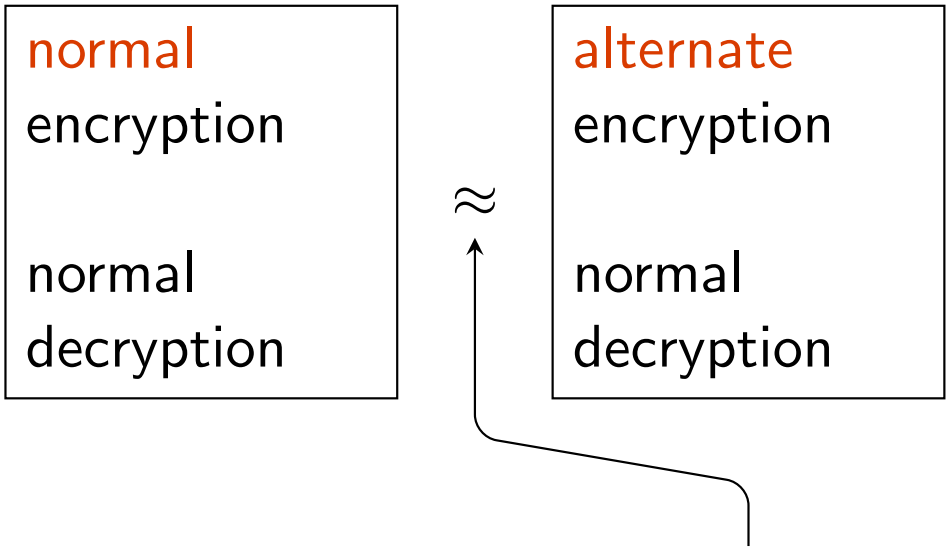
What is adversary's advantage in RCCA experiment?

normal encryption ?

normal decryption ?

Security Proof Outline

Define an "alternate encryption."



Lemma

Alternate encryption indistinguishable from normal encryption (DDH assumption).

Security Proof Outline

normal encryption normal decryption	$\approx$	alternate encryption ✓ normal decryption

Lemma

Alternate encryption independent of choice of plaintext.

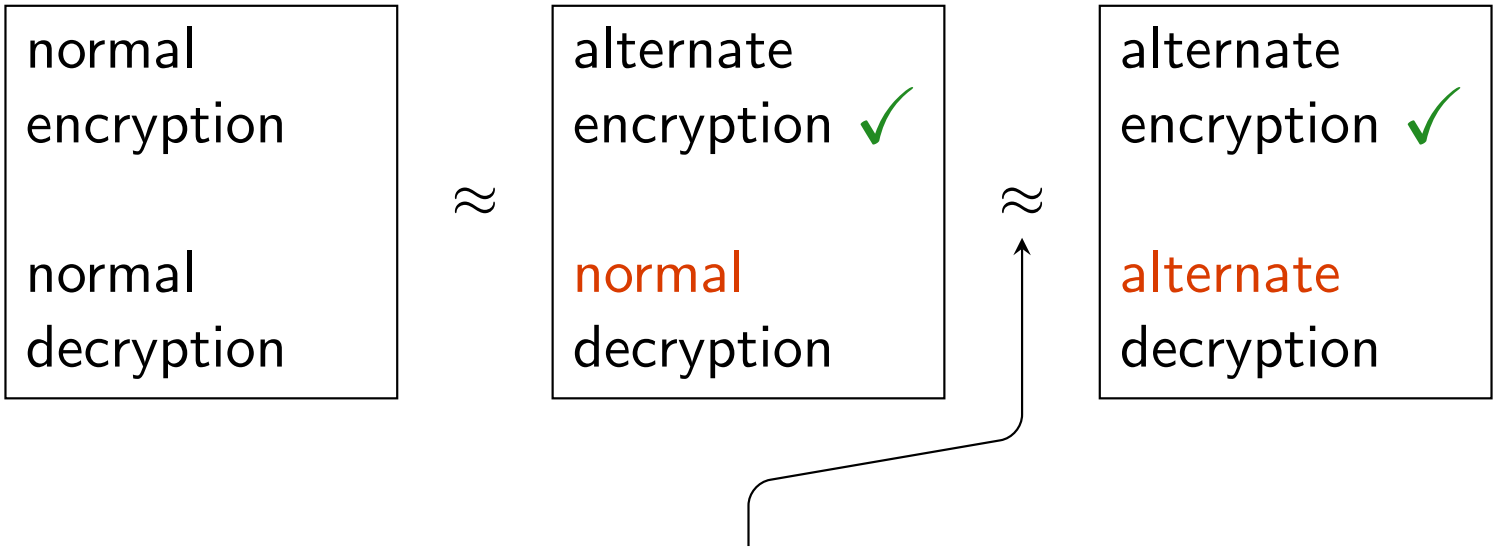
Security Proof Outline

Decryption answers might leak information about private key!

normal encryption normal decryption	$\approx$	alternate encryption ✓ normal decryption ?

Security Proof Outline

Define a (computationally unbounded) "alternate decryption."



Lemma

Alternate decryption indistinguishable from normal decryption (linear algebra analysis).

Security Proof Outline

Lemma

Alternate decryption computed using only public key and challenge ciphertext.

Adversary's entire view independent of choice of plaintext!