

Rerandomizable RCCA Encryption

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Unlinkable Blind Copying

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Strong tradeoff between features and non-malleability:

- ▶ Copying is allowed, in very robust sense
- ▶ **Everything else** is forbidden, in the strongest sense

Formalizing the Problem

Rerandomizability

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Scheme is CCA secure, except it may be possible to “maul” an encryption of m into another that decrypts to same m .

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Question [Canetti, Krawczyk, Nielsen]

Are there rerandomizable, RCCA-secure encryption schemes?

Related work

Canetti, Krawczyk, Nielsen [*CRYPTO*'03]

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- ▶ Applications of rerandomizability for *anonymous* schemes
- ▶ Rerandomizable, CPA-secure scheme

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Gröth [*TCC*'04] gives two rerandomizable schemes:

- ▶ Achieves weaker variant of RCCA security
- ▶ Achieves full RCCA in *generic group* model

Our Results

First rerandomizable RCCA scheme in standard model.

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- ▶ Positive result for sophisticated functionality in standard UC model

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Full version (and this talk) has improved construction

- ▶ <http://eprint.iacr.org/2007/119>

Recipe

Key Idea #1

Use two “strands” of Cramer-Shoup encryption

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- ▶ Second strand helps with rerandomization

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Key Idea #3

“Twist first strand” (technical)

- ▶ Make the 2 strands of different type
- ▶ Avoid bad ways of combining the 2 strands together

Double-Strand Idea

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Golle et al. [RSA'04] use a **double-strand** idea with ElGamal.

- ▶ In their case, second strand carries public key data needed to rerandomize ElGamal.
- ▶ In our case, second strand carries message data needed for rerandomize Cramer-Shoup.

Double-Strand ElGamal

Double-strand idea illustrated with ElGamal [Golle et al]:

- ▶ Normal ElGamal encryption of m (public key is g^a):

$$g^x, m (g^a)^x \quad \text{for random } x$$

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- ▶ Rerandomize x additively

$$\begin{aligned}
 & g^x (g^y)^s, \quad m g^{ax} (g^{ay})^s, && \text{for rand } s \\
 = & g^{x+ys}, \quad m g^{a(x+ys)},
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- ▶ Rerandomize x additively, y multiplicatively

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 & g^x (g^y)^s, \quad m g^{ax} (g^{ay})^s, \quad (g^y)^t, \quad (g^{ay})^t \quad \text{for rand } s, t \\
 = & g^{x+ys}, \quad m g^{a(x+ys)}, \quad g^{yt}, \quad g^{a(yt)}
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 \end{aligned}$$

Result is distributed as fresh double-strand encryption

Double-Strands for Cramer-Shoup

We apply double-strand idea to Cramer-Shoup:

- ▶ Normal Cramer-Shoup: (μ is hash of first 3 components)

$$g_1^x, g_2^x, m B^x, (CD^\mu)^x \quad \text{for random } x$$

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Possible Attack!

Can “mix-and-match” strands from 2 independent ciphertexts, if they carry the same message (μ).

Tie the Knot

Key Idea #2

“Tie strands together” with shared randomness u ; give an additional encryption of u

$$(g_1^x)^u, (g_2^x)^u, mB^x, (CD^\mu)^x, (g_1^y)^u, (g_2^y)^u, B^y, (CD^\mu)^y, \text{Enc}(u)$$

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Double-strand “Cramer-Shoup lite” has these properties.

Twist the First Strand

$$g_1^{xu}, g_2^{xu}, mB^x, (CD^\mu)^x, g_1^{yu}, g_2^{yu}, B^y, (CD^\mu)^y, \text{Enc}(u)$$

Possible Attack

Can rerandomize first strand multiplicatively, if plaintext is known.

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Analysis uses linear-algebraic interpretation:

- ▶ First strand's randomness x is perturbed by fixed vector
- ▶ More components (g_1, \dots, g_4) needed for linear independence.

Security of our Scheme

Our scheme satisfies:

- ▶ RCCA security (Canetti, Krawczyk, Nielsen [*CRYPTO*'03])
- ▶ Perfect rerandomizability
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Proof uses:

- ▶ DDH assumption in 2 groups (for CS and CS-lite)
 - ▶ Requires 3 large primes of special form (Cunningham chain)
- ▶ Linear algebra interpretation of our scheme

On Security Definitions

Some philosophical thoughts:

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On Security Definitions

Some philosophical thoughts:

- ▶ Want scheme that is non-malleable except for unlinkable copying feature
- ▶ Can also imagine different tradeoff (e.g, non-malleable other than some homomorphic operation)
- ▶ What are the “right” security, correctness definitions?
- ▶ Details motivated by natural UC formulation...

Universal Composition (UC)

Universal Composition (UC) framework for security definitions

- ▶ Proposed by Canetti [*FOCS*'01]
- ▶ Simulation-based security with arbitrary interactive environment
- ▶ “Natural” formulations of security via ideal functionalities

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Universal Composition (UC) framework for security definitions

- ▶ Proposed by Canetti [*FOCS*'01]
- ▶ Simulation-based security with arbitrary interactive environment
- ▶ “Natural” formulations of security via ideal functionalities
- ▶ No secure protocols for most tasks (unless model is significantly weakened)!

Our Results

We define a powerful new UC functionality.

- ▶ Users send private messages to each other
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Theorem

Any rerandomizable, RCCA-secure scheme (with our correctness properties) is a secure realization of this functionality.

- ▶ Justifies our security definitions, correctness properties
- ▶ Positive UC result in standard model.
- ▶ Can easily extend to add features

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Natural characterization in UC framework

- ▶ Justifies choice of security definitions
- ▶ Sophisticated positive result for standard UC model

Open Problems

Anonymous, rerandomizable RCCA scheme?

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Other tradeoffs between features and non-malleability in encryption schemes

- ▶ Features can be performed in an unlinkable way
- ▶ Scheme is non-malleable otherwise

Thanks for your attention!

fin.

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- ▶ ... and as the malleable operation of CS-lite:

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For 2 operations to coincide, must have:

- ▶ CS-lite group is subgroup of \mathbb{Z}_p^* .
- ▶ DDH in both groups (that of CS and CS-lite)

Is it an unreasonable relationship between 2 groups?

Cunningham Chains

Definition

A **Cunningham chain** is 3 primes of the form $q, 2q + 1, 4q + 3$.

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A **Cunningham chain** is 3 primes of the form $q, 2q + 1, 4q + 3$.

- ▶ \mathbb{QR}_{4q+3}^* and \mathbb{QR}_{2q+1}^* have desired relationship
- ▶ DDH believed to hold ($4q + 3$ and $2q + 1$ are *safe primes*).
- ▶ Cunningham chains known to exist for $q \sim 2^{20,000}$.

Security Proof Outline

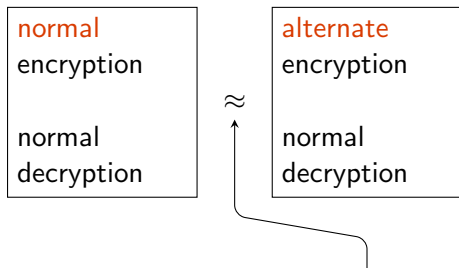
What is adversary's advantage in RCCA experiment?

normal
encryption ?

normal
decryption ?

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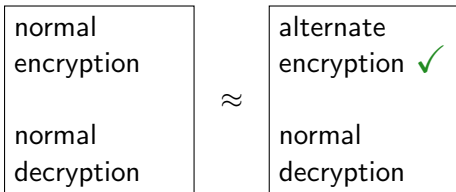
Define an “alternate encryption.”



Lemma

Alternate encryption indistinguishable from normal encryption (DDH assumption).

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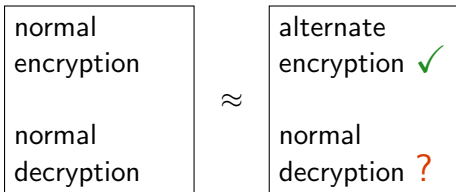


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Alternate encryption independent of choice of plaintext.

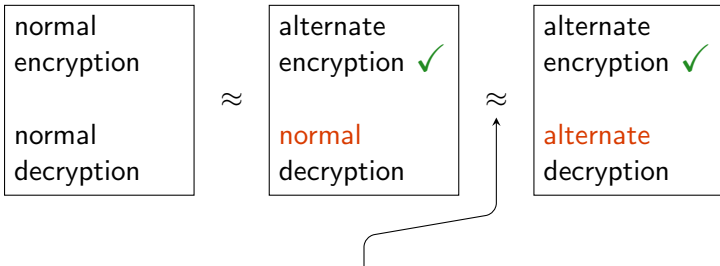
Security Proof Outline

Decryption answers might leak information about private key!



Security Proof Outline

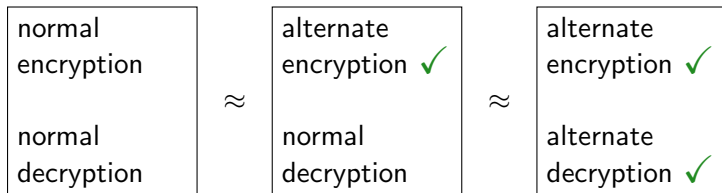
Define a (computationally unbounded) “alternate decryption.”



Lemma

*Alternate decryption indistinguishable from normal decryption
(linear algebra analysis).*

Security Proof Outline



Lemma

Alternate decryption computed using only public key and challenge ciphertext.

Adversary's entire view independent of choice of plaintext!