Zero-Knoweldge Proofs, with applications to Sudoku & Where's Waldo?

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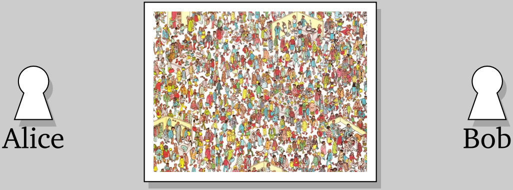
December 10, 2008

Scenario: Where's Waldo?



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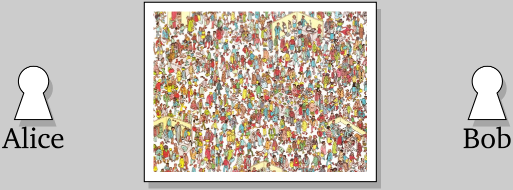
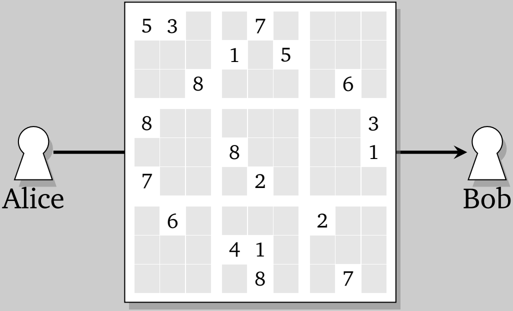


A “Hey Bob, I found Waldo!”

B “That was way too fast, I don't believe you.”

Scenario: Sudoku



A "Hey Bob, check out this brutal Sudoku puzzle!"

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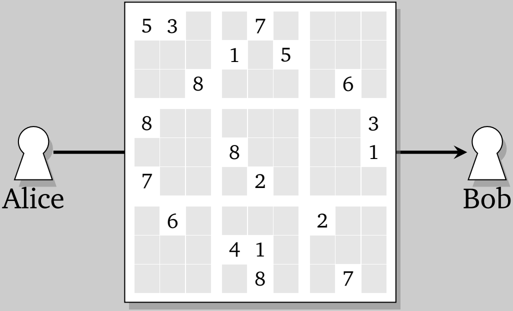

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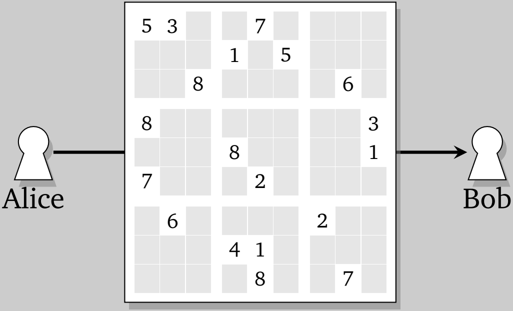
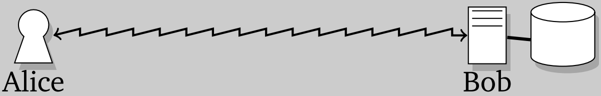


A "Hey Bob, check out this brutal Sudoku puzzle!"

B "Last week you gave me a puzzle with no solution. I wasted 3 hours."

A "This one has a solution, trust me."

Scenario: Authentication



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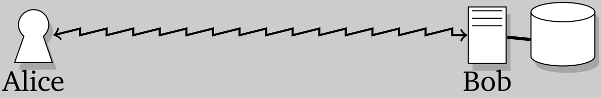

A "Can I have access to the database? It's me, Alice."

B "OK, send me your password so I know it's you."

A Problem of Trust and Information

Alice wants to convince Bob of something

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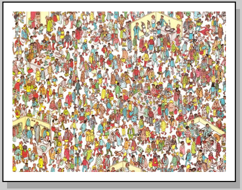
What might a possible solution look like?

Where's Waldo? Solution

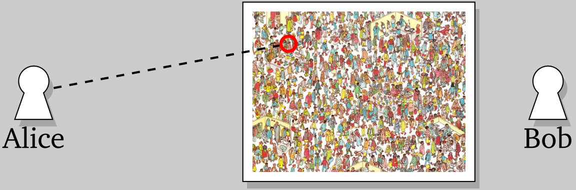
Alice



Bob

Solution:

Where's Waldo? Solution



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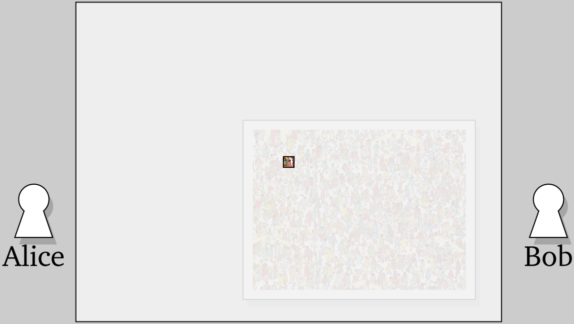

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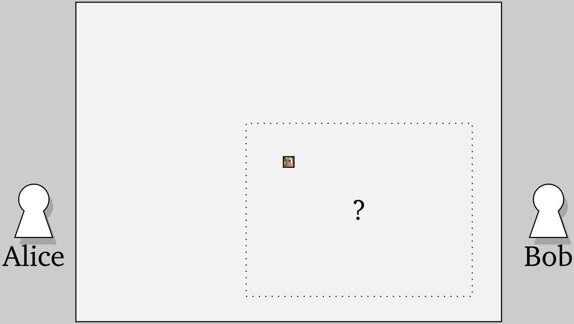

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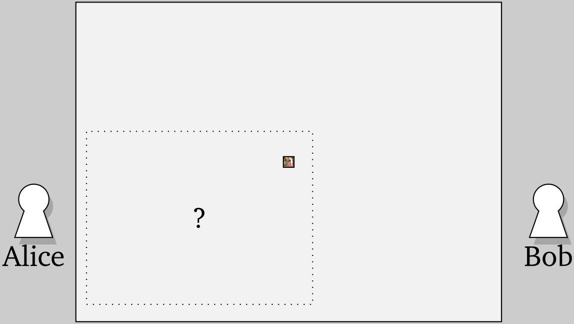

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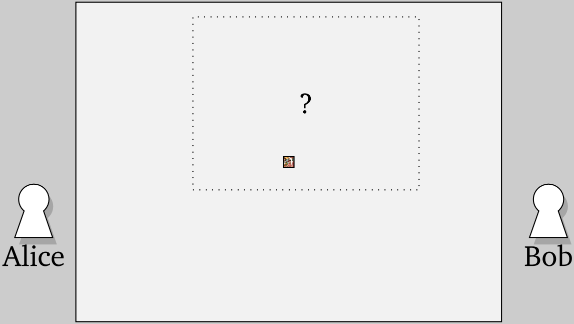

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Philosophy

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A zero-knowledge proof is a way to convince someone of a fact without giving out “any additional information”

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Classical Definition

A proof is a list of logical steps. Something that Alice can write down and send to Bob.

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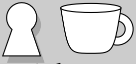

Muriel

Ronald

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- R “Intriguing. Can you prove it?”
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- R “. . .”

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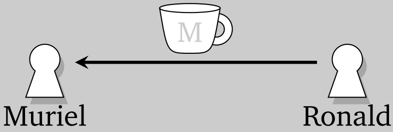

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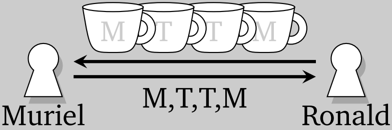

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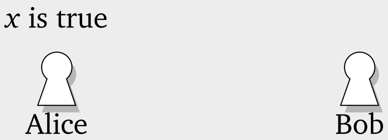
Repeat Repeat n times.

- If no difference in two kinds of teas, she has $(1/2)^n$ chance of guessing *all* correctly.

Epistemology: What is Knowledge?

Bad situation



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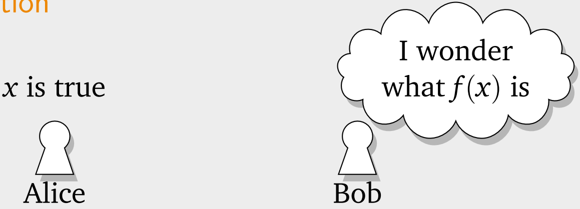

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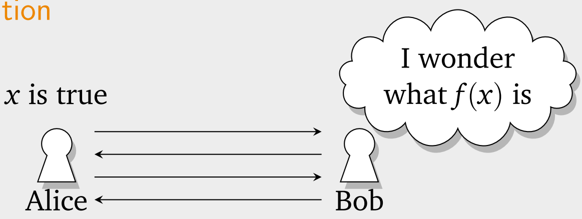

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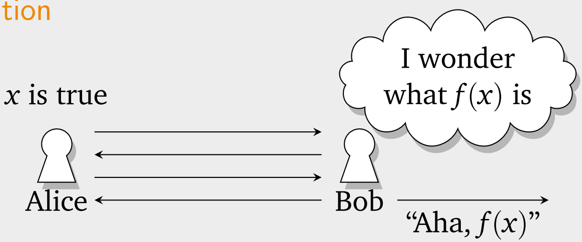

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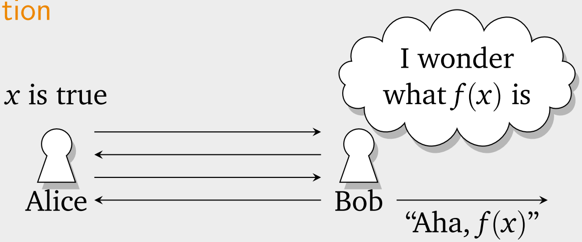

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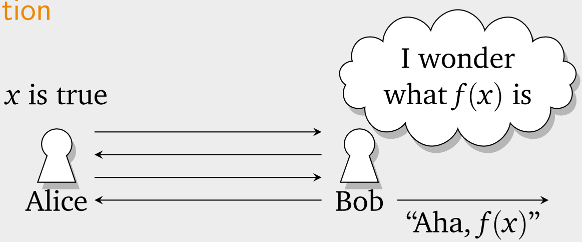

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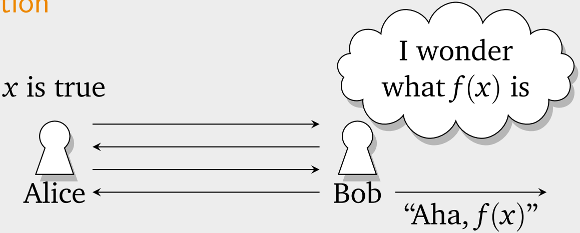

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- Interaction transcript gives him computational power

Want to say:

Everything Bob can compute *after* seeing the transcript, he could have computed *before* seeing the transcript.

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Clever Definition

Interaction is zero-knowledge if Bob could generate transcripts without interacting with Alice:

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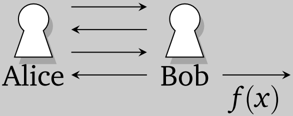

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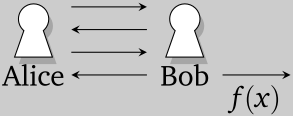

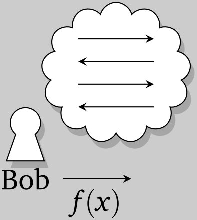

Whatever Bob could compute *after* seeing the transcript ...

... there is a way to compute without interaction!

Apparent Paradox

Paradox?

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B "But I saw her drink it while I timed her!"

Apparent Paradox

B “Alice can tell whether tea is poured into milk or vice-versa!”

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Bob already knew the correct responses to challenges

- Convinced by *how* the transcript was generated (in response to his challenges)

Formal Definition

Definition [GMR 1985]

A zero-knowledge proof is an interactive protocol satisfying:

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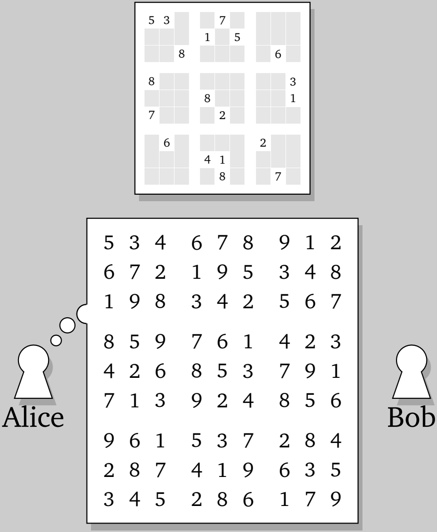
A zero-knowledge proof is an interactive protocol satisfying:

- The prover can always convince the verifier of any true statement
- The verifier can't be convinced of a false statement (even by a cheating prover), except with very low probability
- There is an efficient procedure to output "same-looking" protocol transcripts

Sudoku Zero-Knowledge Proof

Zero-knowledge protocol:



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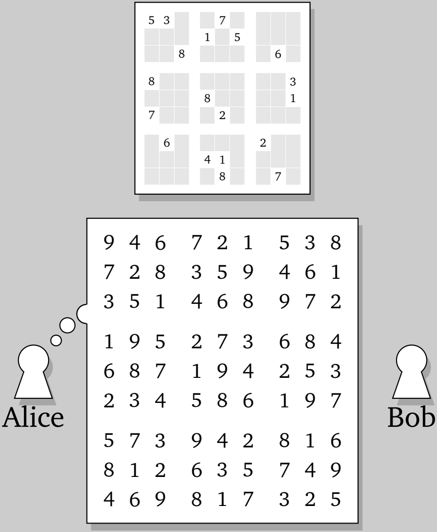

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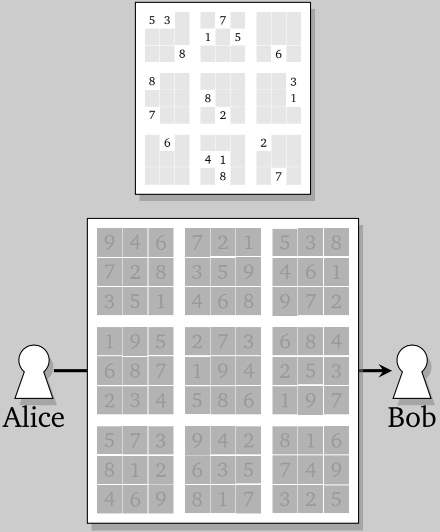

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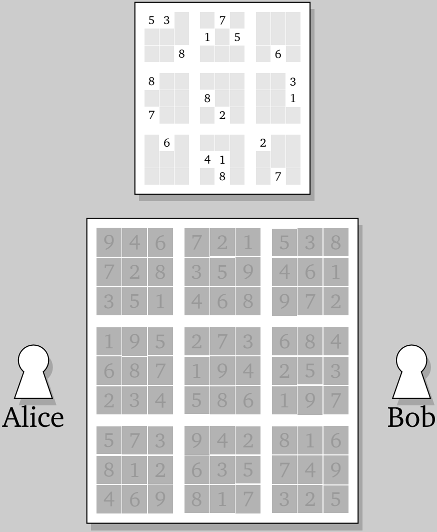

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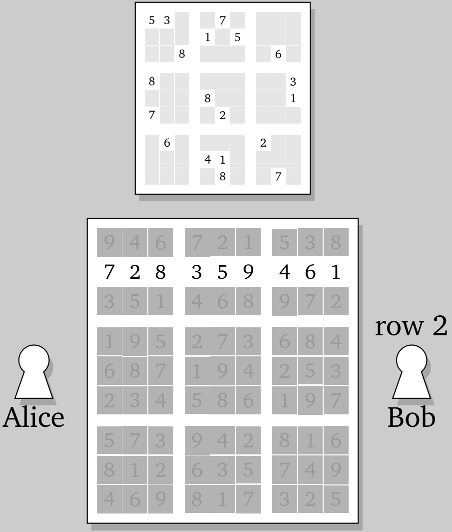

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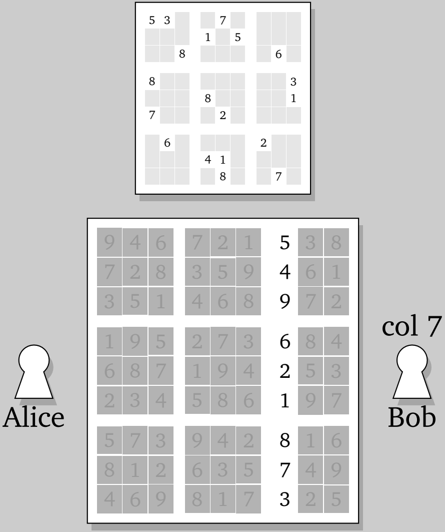

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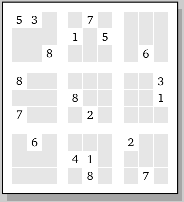

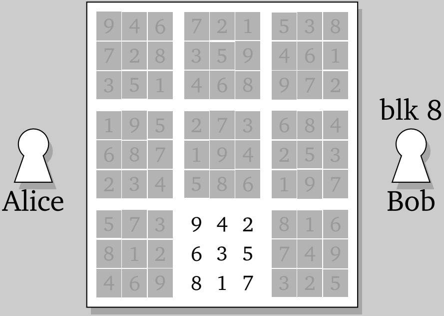

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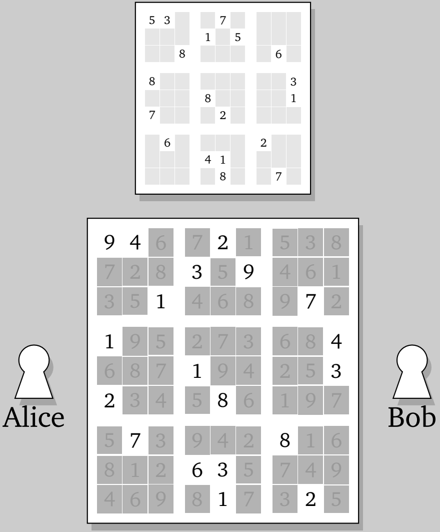

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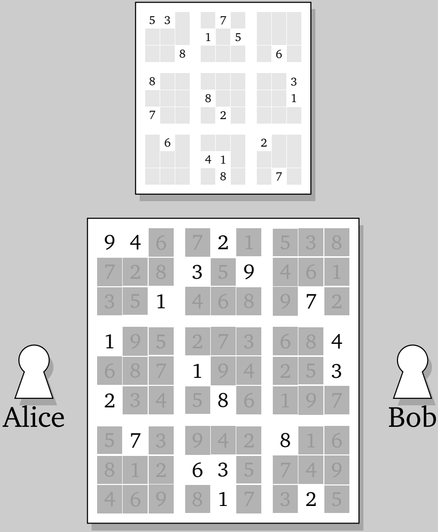
4. Repeat n times



Sudoku Zero-Knowledge Proof, Analysis

Observation

If Alice can answer all challenges successfully, her scratch card satisfies:

- Every row, column, block is permutation of $\{1, \ldots, 9\}$
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What if Alice is cheating (there really is no solution)?

$\Rightarrow$ No scratch card can correctly answer all challenges.

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Suppose Alice tries to prove an incorrect statement. Let c be a challenge that is bad for Alice's scratch card.

- Bob picks random challenge (28 choices)
- With probability 1/28, Bob chooses c and Alice is caught!
- With probability $\leq 27/28$, Alice's cheating undetected

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When $n = 2500$, Alice caught with 99% probability.

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If Alice follows protocol (there is a solution), then each round transcript is:

Sudoku Zero-Knowledge Proof, Analysis

9	4	6	7	2	1	5	3	8
7	2	8	3	5	9	4	6	1
3	5	1	4	6	8	9	7	2
1	9	5	2	7	3	6	8	4
6	8	7	1	9	4	2	5	3
2	3	4	5	8	6	1	9	7
5	7	3	9	4	2	8	1	6
8	1	2	6	3	5	7	4	9
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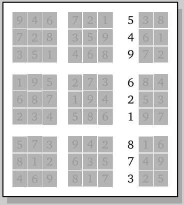
Sudoku Zero-Knowledge Proof, Analysis



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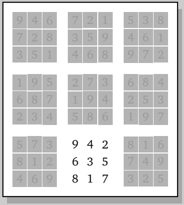

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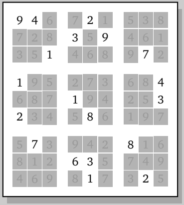

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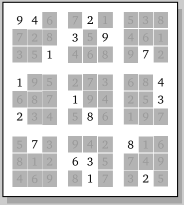

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Each of these Bob can generated himself (without the solution)!

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We have a zero-knowledge proof protocol for Sudoku, so what?

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Theorem

Every NP statement can be proven in zero-knowledge.

- More efficient protocols for many classes of statements

What are They Good For?

Lots of things!

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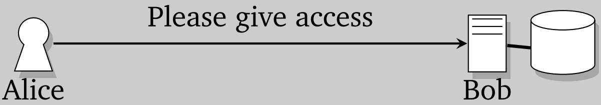
Disclaimer: ZK proofs **very bad** for teaching courses:

- Students convinced that professor knows a lot
- Students gained no additional knowledge

ZK Proof of Identity

Public Key DB:

Alice:	`583ffb4b3..`
Bob:	`0fae6535d..`
Charlie:	`0dd1dd8de..`
...	...



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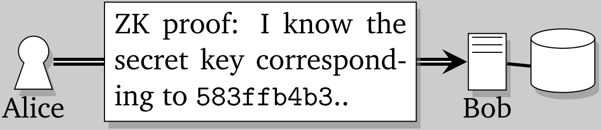

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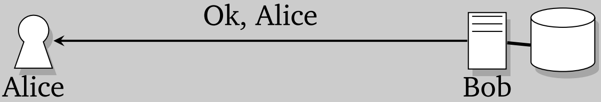

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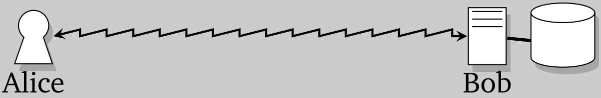

- Alice has her PK published anyway
- No one else knows/can compute corresponding secret key

Security "Compiler"

Problem

- Want protocols that give security guarantee, even against malicious parties who deviate from protocol

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Clever Idea: Security “Compiler” [GMW 1987]

1. Design a protocol that is secure if everyone follows protocol (not too hard)

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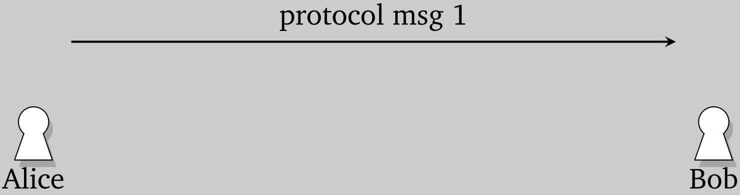
Clever Idea: Security "Compiler" [GMW 1987]

1. Design a protocol that is secure if everyone follows protocol (not too hard)
2. Parties must prove that they follow protocol at each step

Security Compiler



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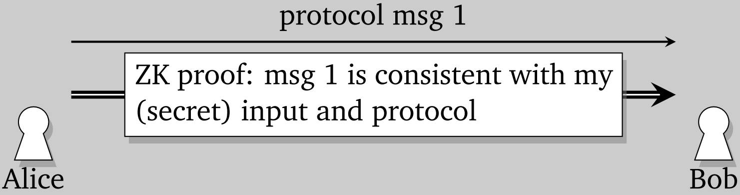

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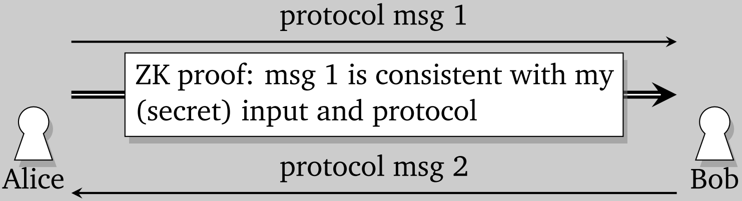

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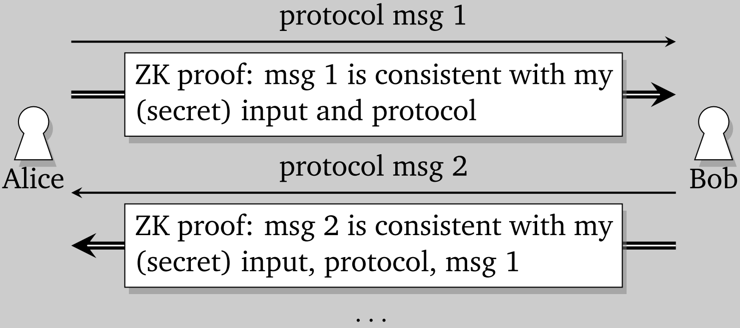

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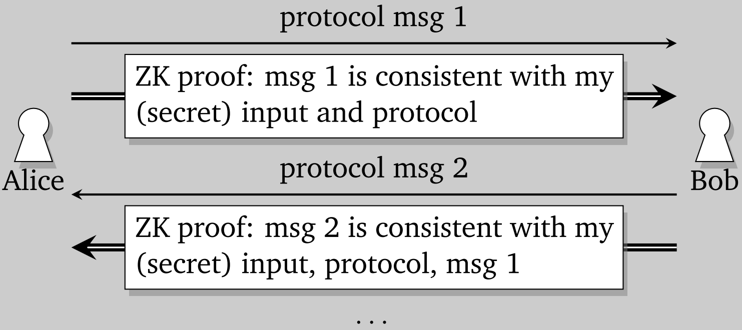

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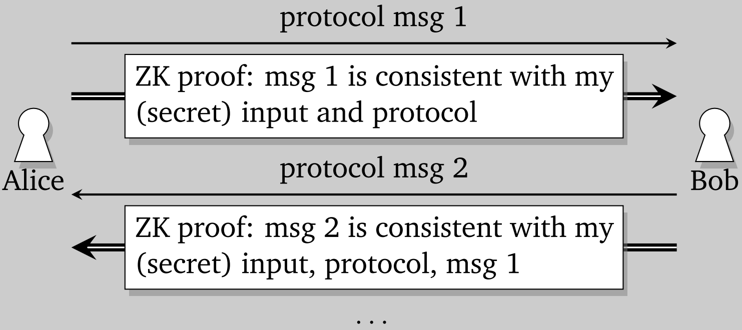

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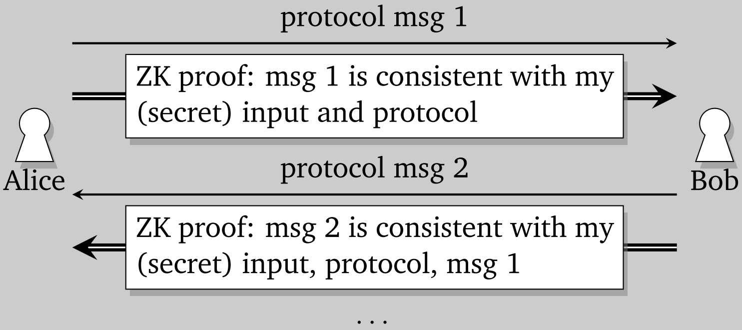


- ZK proofs leak no further information about secret inputs
- If proofs succeed, then parties ran protocol honestly
  - Security is guaranteed
- If proof fails, abort the protocol!

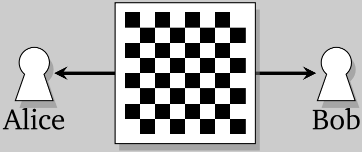
Conning the Chess Grandmasters

Alice

Bob

B "Hey Alice, let's play chess! If you win, I'll give you \$5. Otherwise, you give me \$10. You play black."

Conning the Chess Grandmasters



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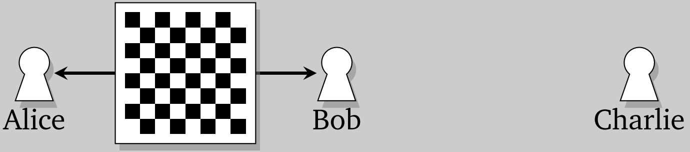

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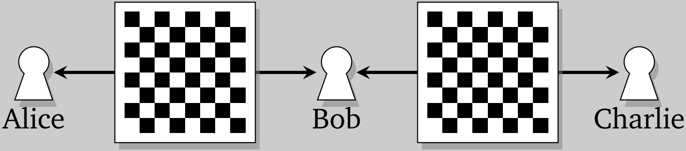

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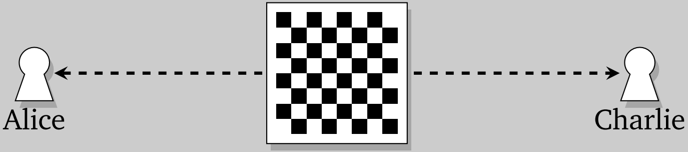

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Bob relays moves to synchronize both chess games.

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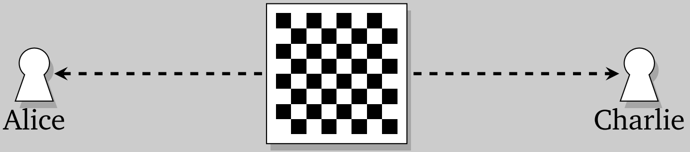

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- "Man-in-the-middle attack"

Chess Grandmasters: Lesson

Key Idea:

"Weird things" can happen when multiple protocol instances run concurrently.

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- Bob will always lose to Alice or Charlie 1-on-1.
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Chess:

- Bob will always lose to Alice or Charlie 1-on-1.
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Zero-Knowledge:

- Bob gets no information from Alice's ZK proof 1-on-1.
- Bob can prove the same statement to Charlie concurrently.

Better Network Security Models

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Is the ZK proof still zero-knowledge if a player participates in:

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Other concerns for ZK definitions:

- "This Sudoku puzzle has a solution", vs.
- "I *know* a solution to this Sudoku puzzle"

Conclusions

Defining zero-knowledge:

- A proof can be randomized, interactive, have small error probability
- Bob "learns nothing" from an interaction if he could have generated transcripts himself

Achieving zero-knowledge:

- Scratch-off card protocol for Sudoku (inefficient)
- Every statement can be expressed in terms of Sudoku

Using zero-knowledge:

- Authenticate without a password
- "Compile" any simple protocol (secure when players are honest) into a robust one

Thanks for your attention!

fin.

I hope this was a “talk about zero-knowledge,”
not a “zero-knowledge talk.”