CS556: Sample Exam 1 Questions

1) What are "T-junctions"? Why are they useful in computer vision?
2) What is an image edge? What is an image region?
3) The Hessian corner detector computes matrix $M(x, y) = [I_{xx} \quad I_{xy}; \; I_{xy} \quad I_{yy}]$, where I_{xx} is the image's second derivative along the x-axis at location (x, y).
   3.1) Specify the criterion of selecting Hessian corners in terms on the eigenvalues of $M(x, y)$.
   3.2) Is the pixel at location (x, y) a corner when the largest eigenvalue of $M(x, y)$ is much larger than the smallest eigenvalue of $M(x, y)$? Why?
   3.3) Are all eigenvalues of $M(x, y)$ positive? Does the criterion of selecting Hessian corners that you specified above work for a negative eigenvalue?
4) 4.1) Specify the matrix form of the objective function $E(x, y)$ for detecting pixel variations around a given pixel coordinate (x, y), which we use for detecting Harris corners in image $I(x, y)$.
   4.2) Specify the objective function $\rho(\sigma)$ for selecting Harris corners, where σ denotes the scale of the smoothing filter used. Specify an algorithm that will handle undefined values of $\rho(\sigma)$. Specify the scale-invariant algorithm that selects optimal Harris corners.
5) Given an image, $I(x, y)$, an interest point detector typically computes matrix $M(x, y)$ which captures the changes of pixel intensities in a neighborhood of each pixel location, $[x, x + u]$ and $[y, y + v]$, in the image. A pixel-intensity change at location (x, y) can be computed by filtering $I(x, y)$ with a filter, $h(x, y) * I(x, y)$. Let $\lambda_1(x, y)$ and $\lambda_2(x, y)$ denote the two eigenvalues of $M(x, y)$.
   5.1) Specify the exact expressions of a 3×3 filter $h(x, y)$ and matrix $M(x, y)$ for the Harris detector. Explain your notation.
   5.2) Specify the exact expressions of a 3×3 filter $h(x, y)$ and matrix $M(x, y)$ for the Hessian detector. Explain your notation.
   5.3) What kind of image feature is present at image location (x, y) when:
      5.3.1) $0 < \lambda_1(x, y) \ll \lambda_2(x, y)$ of the Harris corner detector?
      5.3.2) $\lambda_1(x, y) \approx \lambda_2(x, y) \gg 0$ of the Hessian point detector?
      5.3.3) $\lambda_1(x, y) = \lambda_2(x, y) = 0$ of the Hessian point detector?
      5.3.4) $\lambda_1(x, y) = \lambda_2(x, y) = 0$ of the Harris point detector?
      5.3.5) $\lambda_1(x, y) < 0$ or $\lambda_2(x, y) < 0$ of the Harris point detector?
6) Let $I(x, y)$ and $w(x, y)$ denote an image and a smoothing filter. Explain why the following is true: $w(x, y) * \frac{\partial I(x,y)}{\partial x} = \frac{\partial w(x,y)}{\partial x} * I(x, y)$.
7) Let I and $\mathbf{1}$ denote the $N \times N$ identity matrix and the matrix of all ones, respectively. Also, let $A = [a_{ij}]$ define a square matrix, $i = 1, ..., N$, $j = 1, ..., N$, where each element $a_{ij} = 5$. Finally, let B denote an $N \times N$ diagonal matrix, whose every diagonal element is equal to 5, and all other elements are equal to zero. Compute the expressions: Trace$[A^T I]$ =?, Trace$[A^T \mathbf{1}]$ =?, Trace$[B^T I]$ =?, Trace$[B^T \mathbf{1}]$ =?.
8) Suppose you are given the following square matrix of matching costs between two sets of points in two images, where rows correspond to points in image 1, and columns correspond to points in image 2:

$$A = \begin{bmatrix} 1 & 0 & 1 & 2 & 3 & 3 & 1 \\ 0 & 2 & 3 & 4 & 3 & 5 & 6 \\ 4 & 5 & 1 & 0 & 7 & 2 & 1 \\ 1 & 3 & 4 & 2 & 0 & 3 & 1 \\ 5 & 2 & 3 & 5 & 1 & 3 & 0 \\ 1 & 2 & 0 & 5 & 1 & 2 & 3 \\ 1 & 1 & 3 & 5 & 3 & 0 & 2 \end{bmatrix}$$

Find the optimal matching between the two sets of points using the Hungarian algorithm.

9) Suppose we are given two sets of SIFT points to be matched: $V = \{v_1, v_2, \ldots, v_N\}$ and $V' = \{v'_1, v'_2, \ldots, v'_M\}$. Let Ψ denote the cost matrix between every pair of points from V and V', i.e., $\psi_{vv'}$ measures a cost of matching the SIFT of $v \in V$ with the SIFT of $v' \in V'$.
   - 9.1) Using linearization, formulate one-to-one, discrete matching problem as finding those pairs $f : \{(v, v')\}$ which minimize the total cost of matching pairs $\sum_{(v,v')\in f} \psi_{vv'}$, and every $v \in V$ finds its match $v' \in V'$, and, conversely, every $v' \in V'$ finds its match $v \in V$.
   - 9.2) Specify a relaxation that will transform your initial discrete formulation to a continuous one.
   - 9.3) Specify an algorithm which solves your matching formulation in the continuous domain.

10) Suppose we are given two sets of descriptors extracted from two images to be matched: $V = \{v_1, v_2, \ldots, v_N\}$ and $V' = \{v'_1, v'_2, \ldots, v'_N\}$, $N = 4$. The cost matrix of matching every pair of descriptors is given below.

$$A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 2 & 0 \\ 0 & 2 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

   - 10.1) Compute trace$(A^T I) =?$, where I is the $N \times N$ identity matrix.
   - 10.2) Find the $N \times N$ matrix X which minimizes the following problem, and explain the meaning of your solution in the context of matching two sets of points V and V'.

$$\min_X \ \text{trace}(A^T X)$$

subject to

$$\forall v_i \in V, \quad \forall v'_i \in V', \quad x_{vv'} \in [0, 1]$$

   - 10.3) Find the $N \times N$ matrix X which minimizes the following problem, and explain the meaning of your solution in the context of matching two sets of points V and V'.

$$\min_X \ \text{trace}(A^T X)$$

subject to

$$\forall v \in V, \quad \forall v' \in V', \quad x_{vv'} \in [0, 1]$$
$$\forall v \in V, \quad \textstyle\sum_{v'\in V'} x_{vv'} = 1, \quad \text{and} \quad \forall v' \in V', \ \textstyle\sum_{v\in V} x_{vv'} = 1$$

11) What is a vanishing point? Can there be more than one vanishing point in the image?
12) Homogeneous coordinates of point P in the image are $P = [40 \ \ 164 \ \ 2]^T$. Which row and column contains P if homogeneous coordinates are measured from the top left corner of the image?
13) The intrinsic parameters of two cameras are given by

$$K_1 = \begin{bmatrix} 0.6 & 0.3 & -1.4 \\ 0 & 0.4 & 0.9 \\ 0 & 0 & 1 \end{bmatrix}, \qquad K_2 = \begin{bmatrix} 0.4 & 0.3 & 0.9 \\ 0 & 0.6 & 1.4 \\ 0 & 0 & 1 \end{bmatrix}$$

Which of the two cameras would you prefer to use? Explain your choice.

14) State the camera calibration problem.