1) What are "T-junctions"? Why are they useful in computer vision?
2) What is an image edge?
3) Blurring (or smoothing) is one of the common steps in extracting interest points from an image. Image blurring can be done, for example, by convolving the Gaussian filter with the image. In the process, we destroy (i.e., blur) interest points (e.g., corners) that we want to extract. If so, then, why do we do this contradictory operation?
4) The Hessian corner detector computes matrix
   $$M(x, y) = \begin{bmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{bmatrix}$$
   where $I_{xx}$ is the image’s second derivative along the x-axis at location $(x, y)$.
   4.1) Specify the criterion of selecting Hessian corners in terms on the eigenvalues of $M(x, y)$.
   4.2) Is pixel at location $(x, y)$ a corner when the largest eigenvalue of $M(x, y)$ is much larger than the smallest eigenvalue of $M(x, y)$? Why?
   4.3) Are all eigenvalues of $M(x, y)$ positive? Does the criterion of selecting Hessian corners that you specified above work for a negative eigenvalue?
5) Let $I(x, y)$ and $w(x, y)$ denote an image and a smoothing filter. Explain why the following is true:
   $$w(x, y) \ast \frac{\partial I(x, y)}{\partial x} = \frac{\partial w(x, y)}{\partial x} \ast I(x, y).$$
6) Is the Harris corner detector scale invariant? Use formulas to explain your reasoning.
7) Specify the Difference-of-Gaussians (DoG, also called SIFT) detector?
8) Given a discrete, 2D function $f(x, y)$, defined as
   $$f(x, y) = \begin{cases} 1 & x \in \{0, 1, 2\}, y \in \{0, 1, 2\} \\ 0 & \text{otherwise} \end{cases}$$
   compute the matrix $M$ that characterizes an ellipse, $p^T M p = 1$, which best fits $f(x, y)$.
9) Derive a transform $T$ that maps all points $p$ of an ellipse, given by $p^T M p = 1$, to fall onto points $q$ of a unit circle, $q = T p$.
10) Let $I$ denote an identity matrix of size $N \times N$. Also, let $A = [a_{ij}]$ define a square matrix, $i = 1, ..., N$, $j = 1, ..., N$, where each element $a_{ij} = 5$. Compute the expression $\text{Trace}[A^T I] = ?$
11) Suppose you are given the following square matrix of matching costs between two sets of points in two images, where rows correspond to points in image 1, and columns correspond to points in image 2:
   $$A = \begin{bmatrix} 1 & 0 & 1 & 2 & 3 & 3 & 1 \\ 0 & 2 & 3 & 4 & 3 & 5 & 6 \\ 4 & 5 & 1 & 0 & 7 & 2 & 1 \\ 1 & 3 & 4 & 2 & 0 & 3 & 1 \\ 5 & 2 & 3 & 5 & 1 & 3 & 0 \\ 1 & 2 & 0 & 5 & 1 & 2 & 3 \\ 1 & 1 & 3 & 5 & 3 & 0 & 2 \end{bmatrix}$$
   Find the optimal matching between the two sets of points using the Hungarian algorithm.
12) Suppose we are given two sets of SIFT points to be matched: \( V = \{ v_1, v_2, \ldots, v_N \} \) and \( V' = \{ v'_1, v'_2, \ldots, v'_M \} \). Let \( \Psi \) denote the cost matrix between every pair of points from \( V \) and \( V' \), i.e., \( \psi_{vv'} \) measures a cost of matching the SIFT of \( v \in V \) with the SIFT of \( v' \in V' \).

12.1) Using linearization, formulate one-to-one, discrete matching problem as finding those pairs \( f : \{(v, v')\} \) which minimize the total cost of matching pairs \( \sum_{(v, v') \in f} \psi_{vv'} \), and every \( v \in V \) finds its match \( v' \in V' \), and, conversely, every \( v' \in V' \) finds its match \( v \in V \).

12.2) Specify a relaxation that will transform your initial discrete formulation to a continuous one.

12.3) Specify an algorithm which solves your matching formulation in the continuous domain.

13) Suppose we are given two sets of SIFT points to be matched: \( V = \{ v_1, v_2, \ldots, v_N \} \) and \( V' = \{ v'_1, v'_2, \ldots, v'_M \} \). Let, for all \( v \in V \) and \( v' \in V' \), \( \psi_{vv'} \) denote similarity between the SIFT of \( v \) and the SIFT of \( v' \).

13.1) Using linearization, formulate many-to-many, discrete matching problem as finding those pairs \( f : \{(v, v')\} \) which maximize the total similarity of matching pairs \( \sum_{(v, v') \in f} \psi_{vv'} \).

13.2) Specify a relaxation that will transform your initial discrete formulation to a continuous one.

13.3) Specify an algorithm which solves your matching formulation in the continuous domain.

14) What is a vanishing point? Can there be more than one vanishing point in the image?

15) Homogeneous coordinates of point \( P \) in the image are \( P = [40 \ 164 \ 2]^T \). Which row and column contains \( P \) if homogeneous coordinates are measured from the top left corner of the image?

16) Point \( P \) in the image is mapped to point \( P' \) by first translating \( P \) to \( P_1 \) 15 rows in the same direction of \( y \) axis, and 102 columns in the opposite direction of \( x \) axis, then, by rotating \( P_1 \) to \( P_2 \) around the top left image corner by 30° counter clockwise, and finally by scaling the coordinates of \( P_2 \) to \( P' \) by 1.3 along \( x \) axis, and 1.6 along \( y \) axis. Find the affine transformation matrix that captures these transformation steps from \( P \) to \( P' \).

17) Point \( P \) in the 3D scene is mapped to point \( P' \) by first translating \( P \) to \( P_1 \) using the translation vector \( t = [-45 \ 56 \ 78]^T \), and, then, by rotating \( P_1 \) to \( P' \) around the \( y \) axis of the world coordinate center by 45° clockwise. Find the affine transformation matrix that captures these transformation steps from \( P \) to \( P' \).

18) We are given the following set of 1-dimensional descriptors (e.g., intensity) of image features \( D = \{ 0, 3, 1, 45, 23, 56, 107, 207, 110, 255, 2, 101, 98, 34, 40, 111 \} \)

18.1) Compute the codewords of a dictionary with size \( K = 3 \) using the K-means algorithm. Write down all steps of your processing using the K-means.

18.2) Compute the bag of words representation of \( D \) in terms of the dictionary that you computed in the previous task.