# CS556: Sample Exam 2 Questions

1) What is a vanishing point? Can there be more than one vanishing point in the image?
2) Homogeneous coordinates of point $P$ in the image are $P = [40 \;\; 164 \;\; 2]^T$. Which row and column contains $P$ if homogeneous coordinates are measured from the top left corner of the image?
3) The intrinsic parameters of two cameras are given by

$$K_1 = \begin{bmatrix} 0.6 & 0.3 & -1.4 \\ 0 & 0.4 & 0.9 \\ 0 & 0 & 1 \end{bmatrix}, \qquad K_2 = \begin{bmatrix} 0.4 & 0.3 & 0.9 \\ 0 & 0.6 & 1.4 \\ 0 & 0 & 1 \end{bmatrix}$$

   Which of the two cameras would you prefer to use? Explain your choice.
4) State the camera calibration problem.
5) Given a stereo pair of images, any two points $x$ and $x'$, from the two images respectively, can be related by a homography $x' = H_\pi x$. These points can also be related via the fundamental matrix $x'^T F x = 0$. Derive an expression that relates $F$ and $H_\pi$. What is the rank of $H_\pi$ and $F$?
6) Given a stereo pair of images, can we relate all points $x$ in image 1 with the corresponding points $x'$ in image 2 as $x' = Ax$, where $A$ is a $3 \times 3$ matrix?
7) Suppose we are given a stereo pair of images and corresponding point pairs $(x_i, x_{i'})$, $i = 1, 2, \ldots, n$, where $x_i$ are 2D points in image 1 and $x_{i'}$ are 2D points in image 2, and each pair $(x_i, x_{i'})$ represents the 3D point $X_i$ in the scene. Can we express the point correspondences as $x_i = Ax_{i'}$, $i = 1, 2, \ldots, n$, where $A$ is a $3 \times 3$ matrix? If yes, then what can we say about points $X_i$ $i = 1, 2, \ldots, n$. Is your answer the same when $n$ includes all pixels in the stereo image pair? If yes, then what can we say about the 3D scene.
8) Suppose we are given a stereo pair of images and corresponding point pairs $(x_i, x_{i'})$, $i = 1, 2, \ldots, n$, where $x_i$ are 2D points in image 1 and $x_{i'}$ are 2D points in image 2, and each pair $(x_i, x_{i'})$ represents the 3D point $X_i$ in the scene. We can use RANSAC for identifying the homography matrix $H$ and $m < n$, for which $x_i = Hx_{i'}$, $i = 1, 2, \ldots, m$. What is the name of $m$ point pairs that were used to estimate $H$? What is the name of $n - m$ point pairs that were not used to estimate $H$? Specify the key steps of RANSAC.
9) The intersection of two epipolar lines $l_1$ and $l_2$ is the epipole $e$ in image 1 of a stereo pair. Suppose we are given the image coordinates of two non-planar pairs of points $(x_1, x'_1)$ and $(x_2, x'_2)$ and their respective homographies $x'_1 = H_{\pi_1} x_1$ and $x'_2 = H_{\pi_2} x_2$. Derive the expressions of epipole $e$ and fundamental matrix $F$ in terms of these points and their homographies.
10) In a stereo pair of images, corresponding 2D points $x$ and $x'$ that are generated from 3D co-planar points $X$ that belong to a plane, $\pi$, can be related by a homography, $x' = H_\pi x$. These points can also be related via the fundamental matrix $x'^T F x = 0$. Given coordinates of two corresponding pairs of points $(x_1, x'_1)$ and $(x_2, x'_2)$, where $x'_1 = H_{\pi_1} x_1$, and $x'_2 = H_{\pi_2} x_2$, and $\pi_1 \neq \pi_2$, and given homographies $H_{\pi_1}$ and $H_{\pi_2}$, compute $F$?
11) What is the motion between two images of a stereo pair whose epipoles are located: (a) in the center of the images, (b) in the infinity;
12) Given the epipoles $e$ and $e'$ of image 1 and image 2 of a stereo pair, and their fundamental matrix $F$, compute $e'^T F =?$ and $Fe =?$

13) Suppose we are given: (i) the projection camera matrices $M$ and $M'$ for image 1 and image 2 of a stereo pair, (ii) the coordinates of two corresponding points $x = MX$ and $x' = M'X$, and (iii) the point in the 3D scene, $C$, that projects onto the camera center of image 1 (i.e. $MC = 0$). Derive the equations of the epipolar lines $l$ and $l'$, and fundamental matrix $F$, in terms of $M, M', x, x', C$. What is the expression for $F$ when the camera centers of image 1 and image 2 are the same (i.e., the stereo images are related only through a rotation at the same camera center)?

14) Given the projection camera matrices $M$ and $M'$ for image 1 and image 2 of a stereo pair, the fundamental matrix $F$, and N pairs of corresponding points $(x, x')$ derive the least squares error-minimization algorithm for 3D scene reconstruction.

15) Given a stereo pair of images, let $e$ and $e'$ denote the epipoles of image 1 and image 2, and $\boldsymbol{l}$ and $\boldsymbol{l}'$ denote epipolar lines in image 1 and image 2, respectively. Also, let $F$ denote the fundamental matrix. Prove that:
    15.1) $e'^T F = 0$
    15.2) $l' = F\boldsymbol{k} \times \boldsymbol{l}$, where $\boldsymbol{k}$ is any line that does not pass though the epipole $e$.

16) A pinhole camera projects 3D point $P$, located at coordinate $[1, 0, 0]^T$ in the world coordinate system, onto the image at point $p$ using the projection matrix $M$, as $p = MP$, where

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

    16.1) Compute the row and column of $p$ in the image?
    16.2) Compute the following camera parameters:
        16.2.1) Focus
        16.2.2) Skew
        16.2.3) Offset along x axis
        16.2.4) Offset along y axis
        16.2.5) Scaling

17) From the specification of $M$, and the standard definition of the 3D rotation matrix,

$$R = \begin{bmatrix} cos\theta \cos\psi & -\cos\phi \sin\psi + \sin\phi \sin\theta \cos\psi & \sin\phi \sin\psi + \cos\phi \sin\theta \cos\psi \\ \cos\theta \sin\psi & \cos\theta \cos\psi + \sin\phi \sin\theta \sin\psi & -\sin\phi \cos\psi + \cos\phi \sin\theta \sin\psi \\ -\sin\theta & \sin\phi \cos\theta & \cos\phi \cos\theta \end{bmatrix},$$

the transform that maps the world coordinate system of $P$ to the camera coordinate system is characterized by the following rotation angles $\theta = 90°$, $\phi = 0$, and $\psi = 0$. Compute the Euclidean distance between $P$ and the camera center at the moment of capturing the image.

18) Image 1 and image 2 are a stereo pair, characterized by the fundamental matrix $F = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ -1 & 0 & -1 \end{bmatrix}$.
In image 1, lines $k = [1, -1, 2]$ and $l = [1, 0, -1]$ intersect in point $x$.
    18.1) Analytically prove whether point $x'$ at coordinates $[0, 1]$ in image 2 correspond to point $x$ in image 1 (i.e., can they represent the same 3D point in the scene).
    18.2) Compute all locations in image 2 where one can expect to find corresponding points of $x$.

19) The projection of 3D points $\mathbf{P}_i = [X_i \quad Y_i \quad Z_i \quad 1]^T$, whose coordinates are defined in the camera coordinate system, onto 2D points $\mathbf{p}_i = [x_i \quad y_i \quad 1]^T$ can be defined as $\mathbf{p}_i = K\mathbf{P}_i$, where $K$ is the matrix of intrinsic camera parameters. For estimating $K$, we can establish $N$ correspondences between pairs of points $(\mathbf{P}_i, \mathbf{p}_i)$, $i = 1, 2, ..., N$, and derive the following system of equations:

$$W_{N\times 9} \cdot [k_{11} \quad k_{12} \quad k_{13} \quad k_{21} \quad k_{22} \quad k_{23} \quad k_{31} \quad k_{32} \quad k_{33}]^T = \mathbf{0}_{N\times 1},$$

where $W_{N\times 9}$ is a matrix of size $N \times 9$, specified in terms of coordinates of $(\mathbf{P}_i, \mathbf{p}_i)$, $i = 1, 2, ..., N$, and each $k_{ij}$ represents the element in $i$th row and $j$th column of $K$.

19.1) Specify $W$ in terms of $\mathbf{P}_i$ and $\mathbf{p}_i$, $i = 1, 2, ..., N$? Can 3D points $\{\mathbf{P}_i : i = 1, 2, ..., N\}$ lie on a single plane in the scene? Why?

19.2) Let $\boldsymbol{k} = [k_{11} \;\; k_{12} \;\; k_{13} \;\; k_{21} \;\; k_{22} \;\; k_{23} \;\; k_{31} \;\; k_{32} \;\; k_{33}]^T$. Specify a non-trivial solution $\boldsymbol{k}$ of the above homogeneous system of equations: $W \cdot \boldsymbol{k} = \mathbf{0}$.

20) Let $x_1$ and $x_2$ be 2D projections of point $X$ from the scene onto image 1 and image 2, respectively, where the images form a stereo pair. Let $M_1$ and $M_2$ denote the perspective projection matrices for image 1 and image 2, respectively, and $F$ denote the fundamental matrix of the stereo pair. Suppose you are given $x_1, x_2, M_1, M_2, F$.

20.1) The problem of finding the point $X$ in the scene can be formulated such that the estimated point $\hat{X}$ has the minimum least squared error from the true $X$. This problem formulation can be specified as:

$$\min_X \sum_i \|\mathbf{a}_i - A_i X\|_2^2, \quad i = 1, 2, 3$$

Define $\mathbf{a}_i$ and $A_i$, $i = 1, 2, 3$, in terms of the given $x_1, x_2, M_1, M_2, F$.

20.2) Which algorithm would you use to solve the above problem?

20.3) Specify all the computational steps of that algorithm in terms of the given $x_1, x_2, M_1, M_2, F$.

20.4) Is this algorithm guaranteed to converge?

21) Explain the main steps of the Canny edge detector. The Canny edge detector uses two thresholds of image gradients to identify edges. Why?

22) Suppose we applied the Canny edge detector to an image. Can we use intersections of detected Canny edges as interest points?

23) Which procedure is better for detecting salient edges in the image:

23.1) Compute the image gradients and then smooth these gradients using the Gaussian filter;

23.2) Smooth the image pixel values and then compute the image gradients;

23.3) Filter the image using the gradient of the Gaussian filter.

24) Given a contour, specify its:

24.1) Shape-context descriptor. Is this descriptor rotation-invariant?

24.2) Beam-angle descriptor. Is this descriptor rotation-invariant?

25) Suppose we are give two open sequences of points: $s_1 = \{s_{1,1}, \ldots, s_{1,4}\}$ and $s_2 = \{s_{2,1}, \ldots, s_{2,5}\}$, and their cost matrix, $C$. The rows of C correspond to the points in $s_1$, and the columns of C correspond to the points in $s_2$, and each element of $C$ represents the cost of matching the corresponding pair of points $C(i, j) = d(s_{1,i}, s_{2,j})$. The cost matrix is specified as

$$C = \begin{bmatrix} 1 & 3 & 0 & 2 \\ 2 & 4 & 1 & 1 \\ 0 & 1 & 4 & 4 \\ 1 & 0 & 4 & 2 \end{bmatrix}$$

Compute the output of Dynamic Time Warping algorithm (DTW). To this end, transform $C$ into an auxiliary matrix DTW that will record

$$\text{DTW}(i, j) = C(i, j) + \min\left[\text{DTW}(i-1, j), \text{DTW}(i, j-1), \text{DTW}(i-1, j-1)\right]$$

and then compute the optimal path in matrix DTW.