CS556: HOMEWORK 2
due 02/27/2017 by 12pm

Please submit your homework reports **in PDF format** to the TEACH website:

`https://secure.engr.oregonstate.edu:8000/teach.php?type=want_auth`

This homework is about stereo geometry. For this assignment, you will use the **five** stereo pairs of images provided on the class website. For **each stereo pair** of images, perform the following tasks:

Assignment

1) (5×2 points)
   1.1) Manually select $N^{(0)} = 10$ points $\mathbf{p}_i = [x_i, y_i, 1]^\top$ in image 1 and their corresponding points $\mathbf{q}_{i'} = [x_{i'}, y_{i'}, 1]^\top$ in image 2, as precisely as possible, and record their coordinates:
$$M^{(0)} = \{(\mathbf{p}_i, \mathbf{q}_{i'}): \quad i = 1, \ldots, N^{(0)}, i' = 1, \ldots, N^{(0)}\}$$
   1.2) Estimate the fundamental matrix $F^{(0)}$ based on the set of point pairs $M^{(0)}$, and **present** $F^{(0)}$ **in your PDF report**.
2) (5×6 points)
   2.1) Detect 100 strongest interest points in each image. You may use your HW1 implementation, or any other interest point detector of your choice. Let these points be located at coordinates $\mathbf{p}_i = [x_i, y_i, 1]^\top$ in image 1, $i = 1, \ldots, 100$, and at coordinates $\mathbf{q}_{i'} = [x_{i'}, y_{i'}, 1]^\top$ in image 2, $i' = 1, \ldots, 100$. Compute feature descriptors of these points, denoted as $\mathbf{f}(\mathbf{p}_i)$ and $\mathbf{f}(\mathbf{q}_{i'})$. You may use your HW1 implementation, or any other feature descriptor of your choice.
   2.2) Compute the $(100 \cdot 100) \times 1$ vector of matching *similarities*, $\boldsymbol{w} = [w_{ii'}]$, as:
$$w_{ii'} = \begin{cases} \exp[-\|\mathbf{f}(\mathbf{p}_i) - \mathbf{f}(\mathbf{q}_{i'})\|_2^2] & , \text{ if } \sqrt{(x_i - x_{i'})^2 + (y_i - y_{i'})^2} < \theta \\ 0 & , \text{ otherwise,} \end{cases} \tag{1}$$
   where threshold $\theta = 20$.
   2.3) Find a set of matching pairs of points in image 1 and image 2,
$$M^{(1)} = \{(\mathbf{p}_i, \mathbf{q}_{i'})\},$$
   using the following matching formulation:
$$\max_{\mathbf{z}} \ \boldsymbol{w}^\top \mathbf{z}, \quad \text{s.t. } \|\mathbf{z}\|_2^2 = 1, \quad \mathbf{z} \in [0,1]^{(100 \cdot 100) \times 1}, \tag{2}$$
   where $\mathbf{z} = [z_{ii'}]$ is a $(100 \cdot 100) \times 1$ a real column vector indicating matches, such that $z_{ii'}$ closer to 1 means that $\mathbf{p}_i$ in image 1 is more likely to match $\mathbf{q}_{i'}$ in image 2, and $z_{ii'}$ closer to 0 means that $\mathbf{p}_i$ and $\mathbf{q}_{i'}$ should not get matched. After finding the solution $\mathbf{z}^*$ from Eq. (2), you need to threshold $\mathbf{z}^*$ to obtain binary values, and thus the matched pairs of points $M^{(1)}$.
   2.4) From your matching solution $M^{(1)}$, automatically select $N = 30$ pairs $(\mathbf{p}_i, \mathbf{q}_{i'})$ for which the fundamental equation (closely) holds:
$$(\mathbf{p}_i, \mathbf{q}_{i'}): \quad z_{ii'} = 1 \quad \text{and} \quad \mathbf{q}_{i'}^\top \ F^{(0)} \ \mathbf{p}_i < \epsilon \approx 0 \tag{3}$$

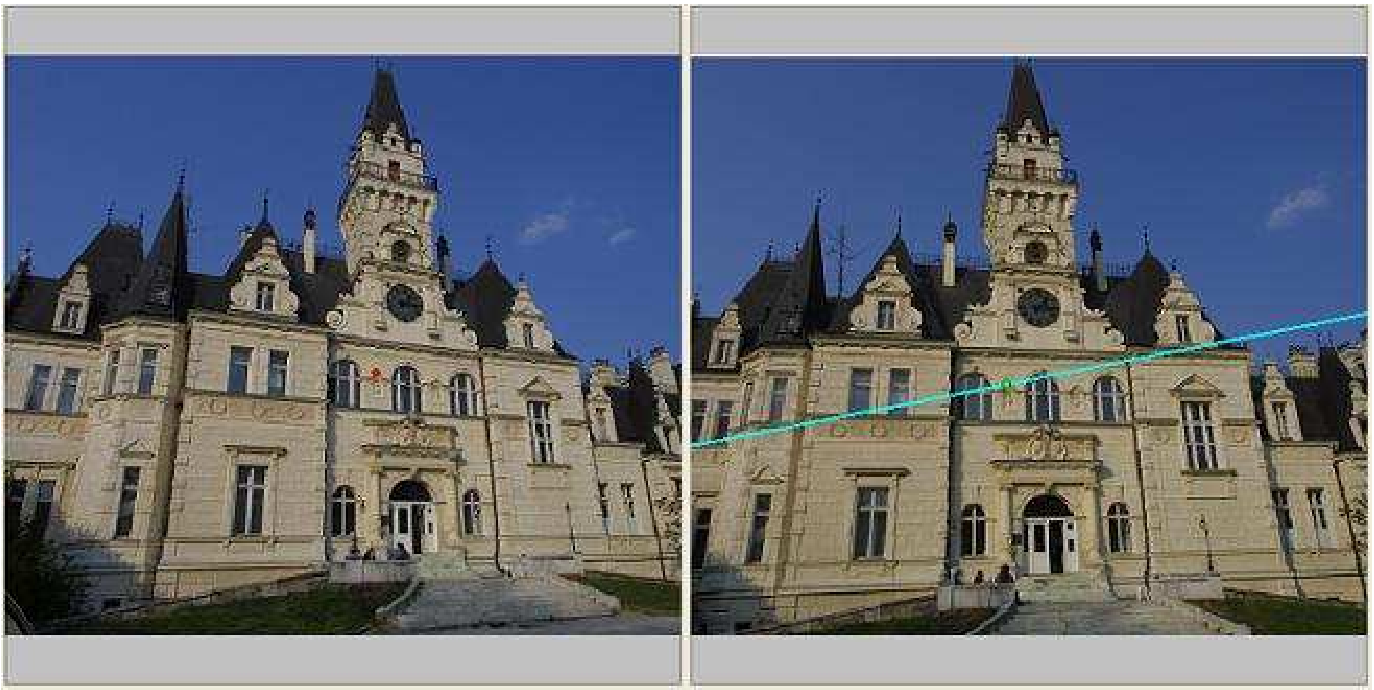

Fig. 1. An example how to show Figure $1a$ and Figure $1b$ in your PDF report: one point in image 1 is marked red, and its corresponding epipolar line is marked cyan in image 2. As can be seen the red point in image 1 corresponds to the cyan point in image 2, and the cyan point must lie on the epipolar line in image 2.

where $F^{(0)}$ is the fundamental matrix that you estimated using the $N^{(0)}$ manually selected pairs of points. Select the value of ϵ so that $N = 30$.

2.5) Merge the two sets of $N^{(0)} = 10$ manually selected and $N = 30$ automatically matched point pairs, and thus obtain a new set of $N^{(1)} = N^{(0)} + N = 40$ point pairs. Re-estimate the fundamental matrix $F^{(1)}$ based on these $N^{(1)}$ point pairs, and **present $F^{(1)}$ in your PDF report**.

3) ($5 \times 2 \times 2$ points) Show in Figure $1a$ one example point in image 1, and its epipolar line in image 2 (superimpose the epipolar line over image 2 using a distinct color), computed for $F^{(0)}$. Show in Figure $1b$ the same point in image 1, and its epipolar line in image 2 (superimpose the epipolar line over image 2 using a distinct color), computed for $F^{(1)}$. In the captions of Figures $1a$ and $1b$, specify the row and column of the point you selected in image 1, and parameters of the epipolar line in image 2. Note that if your estimates of $F^{(0)}$ and $F^{(1)}$ are wrong then the point in image 2 that truly corresponds to the point you selected in image 1 will not lie on your epipolar line.
4) ($5 \times 2 \times 2$ points) Show in Figure $2a$ another example point in image 2 (different from above), and its epipolar line in image 1 (superimpose the epipolar line over image 1 using a distinct color), computed for $F^{(0)}$. Show in Figure $2b$ the same point in image 2, and its epipolar line in image 1 (superimpose the epipolar line over image 1 using a distinct color), computed for $F^{(1)}$. In the captions of Figures $2a$ and $2b$, specify the row and column of the point you selected in image 2, and parameters of the epipolar line in image 1. Note that if your estimates of $F^{(0)}$ and $F^{(1)}$ are wrong then the point in image 1 that truly corresponds to the point you selected in image 2 will not lie on your epipolar line.
5) (5×2 points) Show in Figure 3 clearly marked epipoles of image 1 and image 2. In the caption of Figure 3, specify the coordinates of the two epipoles.
6) (10 points) Printout of the software that you used for this homework assignment.

Summary of Your Report

- 5 fundamental matrices $F^{(0)}$ based on the $N^{(0)} = 10$ manually selected point pairs.
- 5 fundamental matrices $F^{(1)}$ based on the $N^{(1)} = 40$ point pairs, where 10 pairs are manually selected and 30 pairs are automatically matched.
- 25 figures with captions: Figure $1a$, $1b$, $2a$, $2b$, 3.
- Printout of the software.