This homework is about shape retrieval – a basic problem in computer vision. By shape retrieval we mean the following problem. Given a dataset of distinct object classes and a query object, find $K$ objects in the dataset whose shapes are most similar to the query’s shape. Example results of an approach to shape retrieval are presented in Fig. 1.

For this assignment, you will use the MPEG7 dataset of shapes provided at:

http://www.dabi.temple.edu/~shape/MPEG7/dataset.html

The MPEG7 dataset consists of 70 distinct objects, where each object is represented by 20 black-and-white shape images, as illustrated in Fig. 1.

Assignment

1) (30 points) Shape description.
   For every shape in the MPEG7 dataset out of 1400 shapes:
1) (10 points) Extract \((x, y)\) coordinates of all points along the shape. If you use MATLAB, tracing contour points in a black-and-white image \(BW\) can be conveniently done with the built-in function ’bwboundaries(BW)’. Submit a printout of the software that you used for this part of the homework assignment.

2) (20 points) Given the above contour points, compute a feature descriptor \(b_k\) for each point \(k\) along the contour. You may use a code for computing any point descriptor of your choice (e.g., the one you used in HW1 and HW2). Submit a printout of the software that you used for this part of the homework assignment.

2) **(30 points)** Shape matching.

For **every pair** of shapes \((i, j)\) in the MPEG7 dataset out of \(1400 \times 1400\) pairs:

2.1) (10 points) Compute a cost matrix, \(C_{ij}\), of matching points along the shapes \(i\) and \(j\). While shapes in the MPEG7 dataset are closed, in this homework we will treat them as open sequences of points produced by the MATLAB built-in function ’bwboundaries(BW)’. In case you do not use MATLAB, the starting point of a shape should be the point on the contour with the coordinate \((x, y)\) closest to the top left image corner, i.e., coordinate \((1, 1)\). Let \(k\) denote the index of points along shape \(i\), and \(l\) denote the index of points along shape \(j\). Then, each element of matrix \(C_{ij}\) can be defined as:

\[
C_{ij,kl} = \|b_k - b_l\|_2^2, \quad k = 1, \ldots, n_i, \quad l = 1, \ldots, n_j
\]

Since shapes of the same object class may be mirror-reflected, you would need to compute two versions of \(C_{ij}\) for every shape pair \((i, j)\), where the difference between \(C_{ij}^1\) and \(C_{ij}^2\) is in the starting point of \(j\). Specifically, in \(C_{ij}^1\), the bottom left element will have coordinates \(k = 1\) and \(l = 1\), and the top right element will have coordinates \(k = n_i\) and \(l = n_j\). In \(C_{ij}^2\), the bottom left element will have coordinates \(k = 1\) and \(l = n_j\), and the top right element will have coordinates \(k = n_i\) and \(l = 1\). Submit a printout of the software that you used for this part of the homework assignment.

2.2) (20 points) Compute the Dynamic Time Warping (DTW) matching of the shapes \(i\) and \(j\), and compute their total DTW cost \(d_{ij}\). This can be done by, first, constructing the accumulated matrices \(D_{ij}^1\) and \(D_{ij}^2\) from \(C_{ij}^1\) and \(C_{ij}^2\), then, finding the DTW paths in \(D_{ij}^1\) and \(D_{ij}^2\), and, finally, selecting the minimum-cost path between \(D_{ij}^1\) and \(D_{ij}^2\), \(d_{ij} = \min(d_{ij}^1, d_{ij}^2)\). Submit a printout of the software that you used for this part of the homework assignment.

3) **(40 points)** Shape retrieval.

For every image in the MPEG7 dataset out of 1400 distinct object shapes, perform \(K\)-nearest shape retrieval, where \(K = 1, 2, 5\). In each of these 1400 experiments, the selected object shape will serve as a query, and the remaining 1399 object shapes of the dataset will serve for retrieving \(K\) shapes that are the most similar to the query. For this \(K\) shape retrieval you will use the DTW minimum matching costs \(d_{ij}\) that you computed above for all shape pairs \((i, j)\). Normalized retrieval error, \(\epsilon(K)\), is defined as a percentage of shapes out of the \(K\) retrieved shapes which **do not represent** the same object class as the query:

\[
\epsilon(K) = \frac{\# \text{ of wrong shapes retrieved}}{K}.
\]

3.1) (30 points) Average \(\epsilon(K)\) for every object class of the 70 classes in the MPEG7 dataset, \(\overline{\epsilon}_o(K) = \frac{1}{20} \sum_{f=1}^{20} \epsilon_{o,f}(K)\), and report \(\overline{\epsilon}_o(K)\) for objects \(o = 1, \ldots, 70\) and \(K = 1, 2, 5\).
3.2) (10 points) Average $\epsilon(K)$ over all objects in the MPEG7 dataset and over the 20 folds of your shape retrieval, $\tilde{\epsilon}(K) = \frac{1}{70 \cdot 20} \sum_{o=1}^{70} \sum_{f=1}^{20} \epsilon_{o,f}(K)$, and report $\tilde{\epsilon}(K)$ for $K = 1, 2, 5$. Find the optimal $K^*$ for which $\tilde{\epsilon}(K^*)$ is minimum.