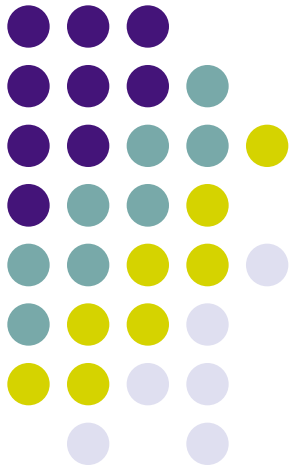
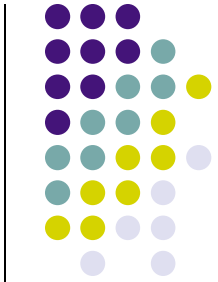
Belief Propagation on Markov Random Fields

Aggeliki Tsoli

Brown University



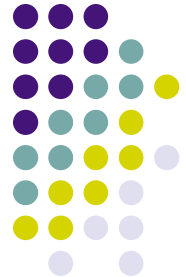
Belief Propagation



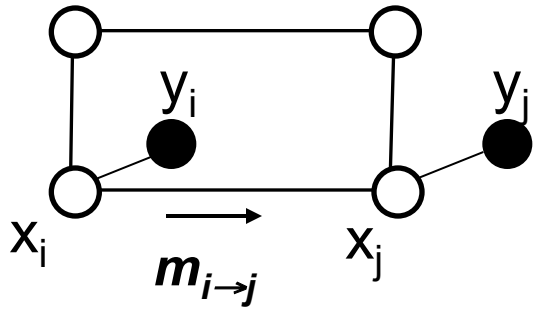
- Goal: compute marginal posteriors of latent nodes

- Algorithm:
  - Iterative
  - Message passing between latent nodes

Belief Propagation Algorithm



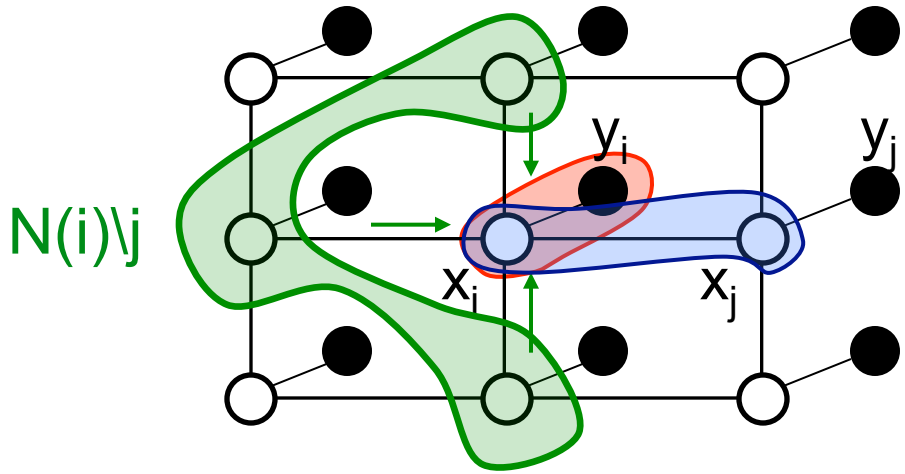
1) For each neighboring latent nodes x_i, x_j
2) Send message $m_{i \to j}$ from x_i to x_j



3) Update belief about marginal distribution at node x_j
4) Go to step 1, until convergence

Step 2: Message Passing

- **Message $m_{i \to j}$ from x_i to x_j**

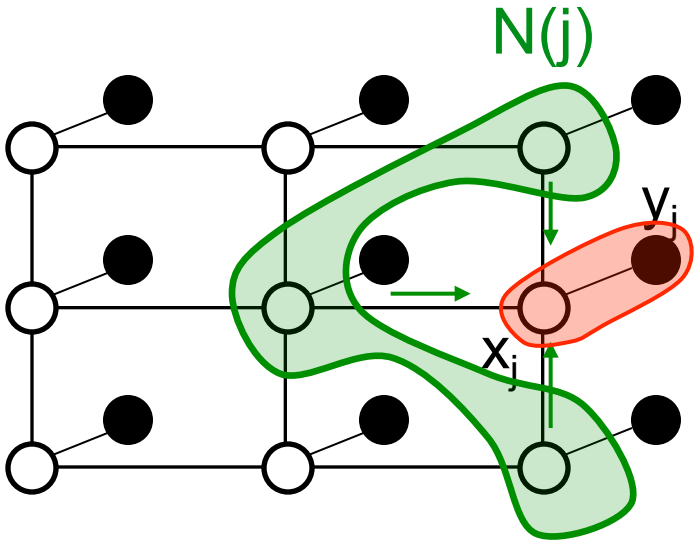


$$m_{i \to j}(x_j) = \sum_{(x^i)} \phi(x_i, y_i)\, \psi(x_j, x_j) \prod_{k \in N(i) \setminus j} m_{k \to i}(x_i)$$

- Messages initially uniformly distributed

Step 3: Belief Update

- **Belief $b(x_j)$:**



$$b(x_j) = k\, \phi(x_j, y_j) \prod_{q \in N(j)} m_{q \to j}(x_j)$$

Example

- Compute belief at node 1.

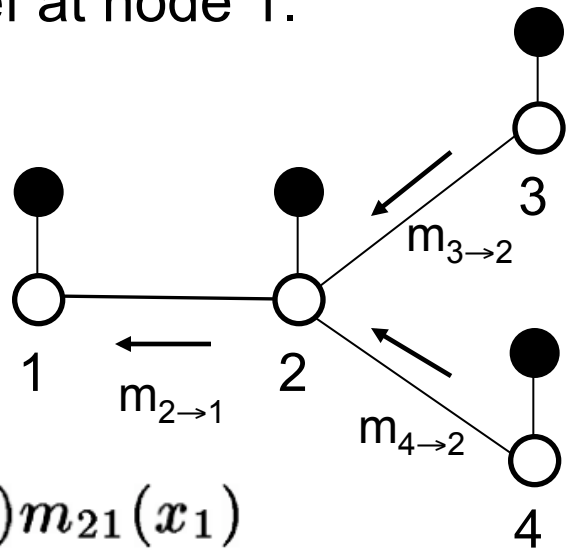


Fig. 12 (Yedidia et al.)

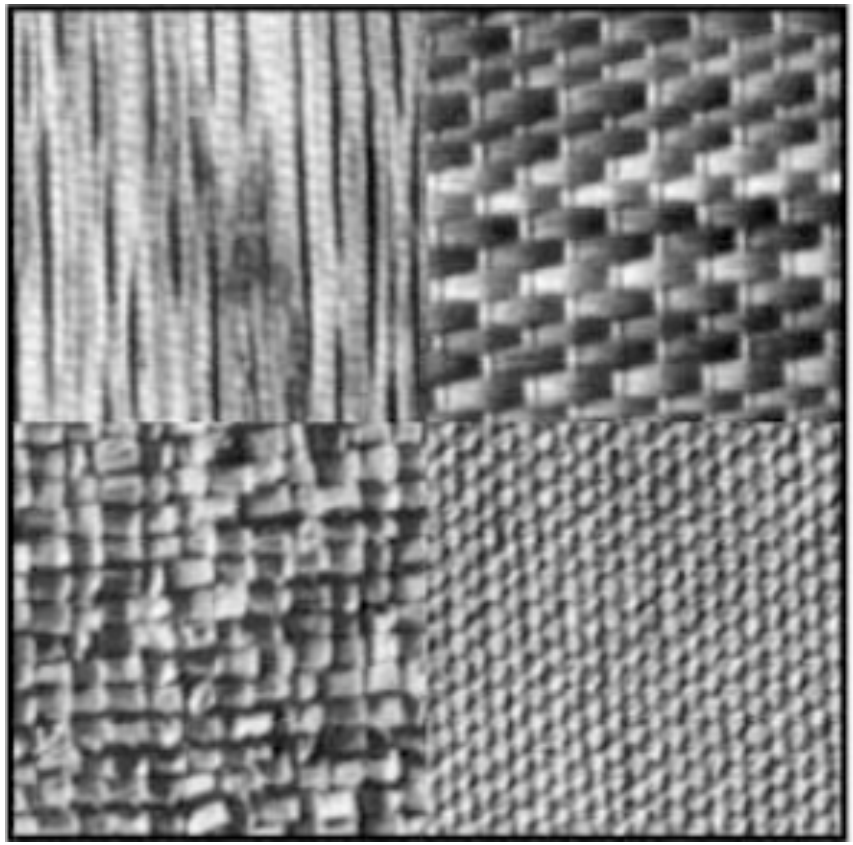
$$b_1(x_1) = k\phi_1(x_1)m_{21}(x_1)$$

$$b_1(x_1) = k\phi_1(x_1)\sum_{x_2}\psi_{12}(x_1,x_2)\phi_2(x_2)m_{32}(x_2)m_{42}(x_2)$$

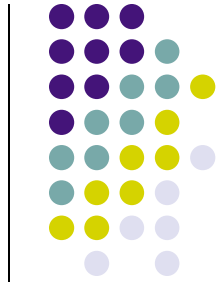
$$b_1(x_1) = k\phi_1(x_1)\sum_{x_2}\psi_{12}(x_1,x_2)\phi_2(x_2)\sum_{x_3}\phi_3(x_3)\psi_{23}(x_2,x_3)\sum_{x_4}\phi_4(x_4)\psi_{24}(x_2,x_4)$$

MRF-based Image Segmentation

$$\beta = 1$$



MRF-based Image Segmentation



$$\beta = 1$$