CS 556: Computer Vision

Lecture 15

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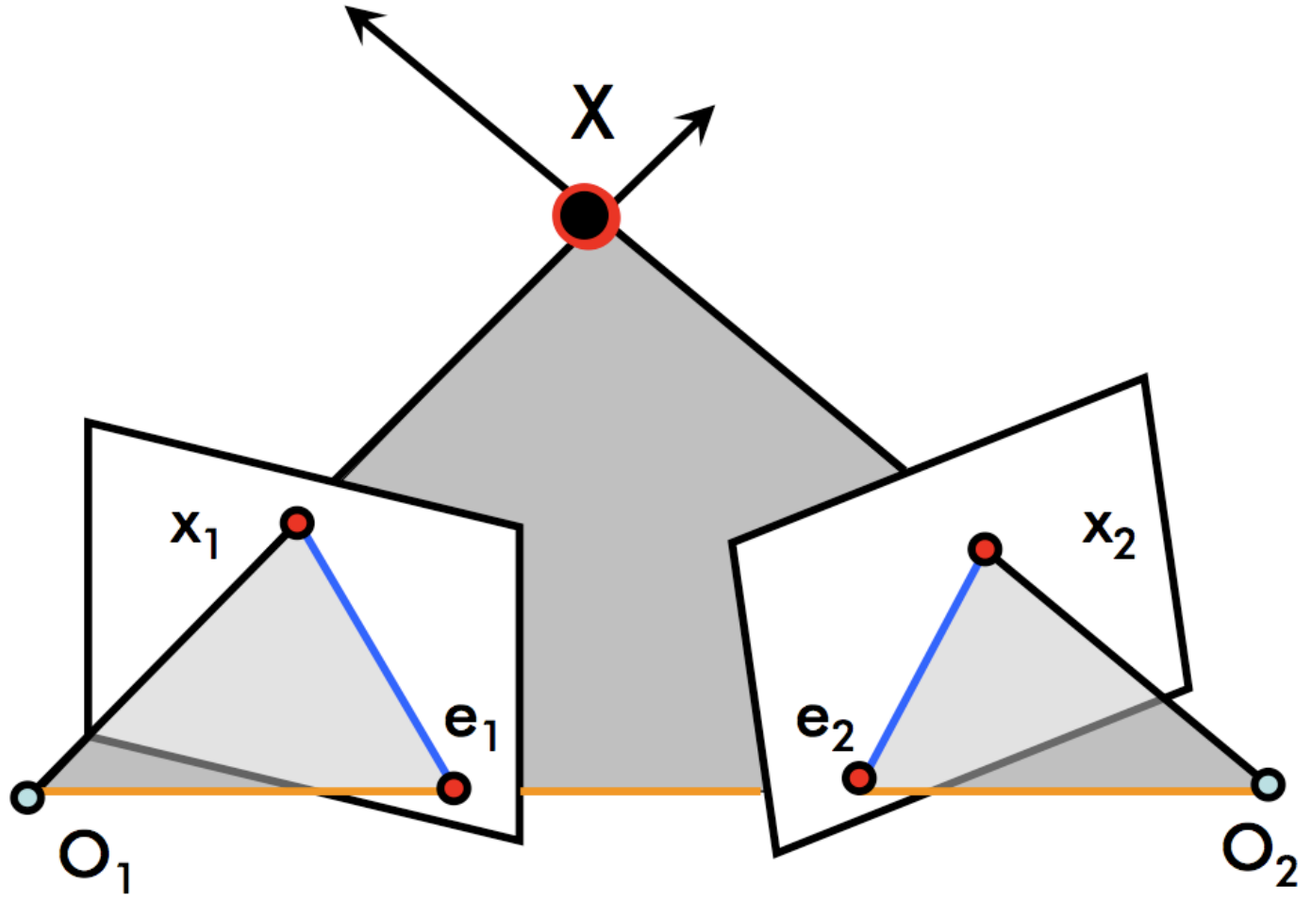
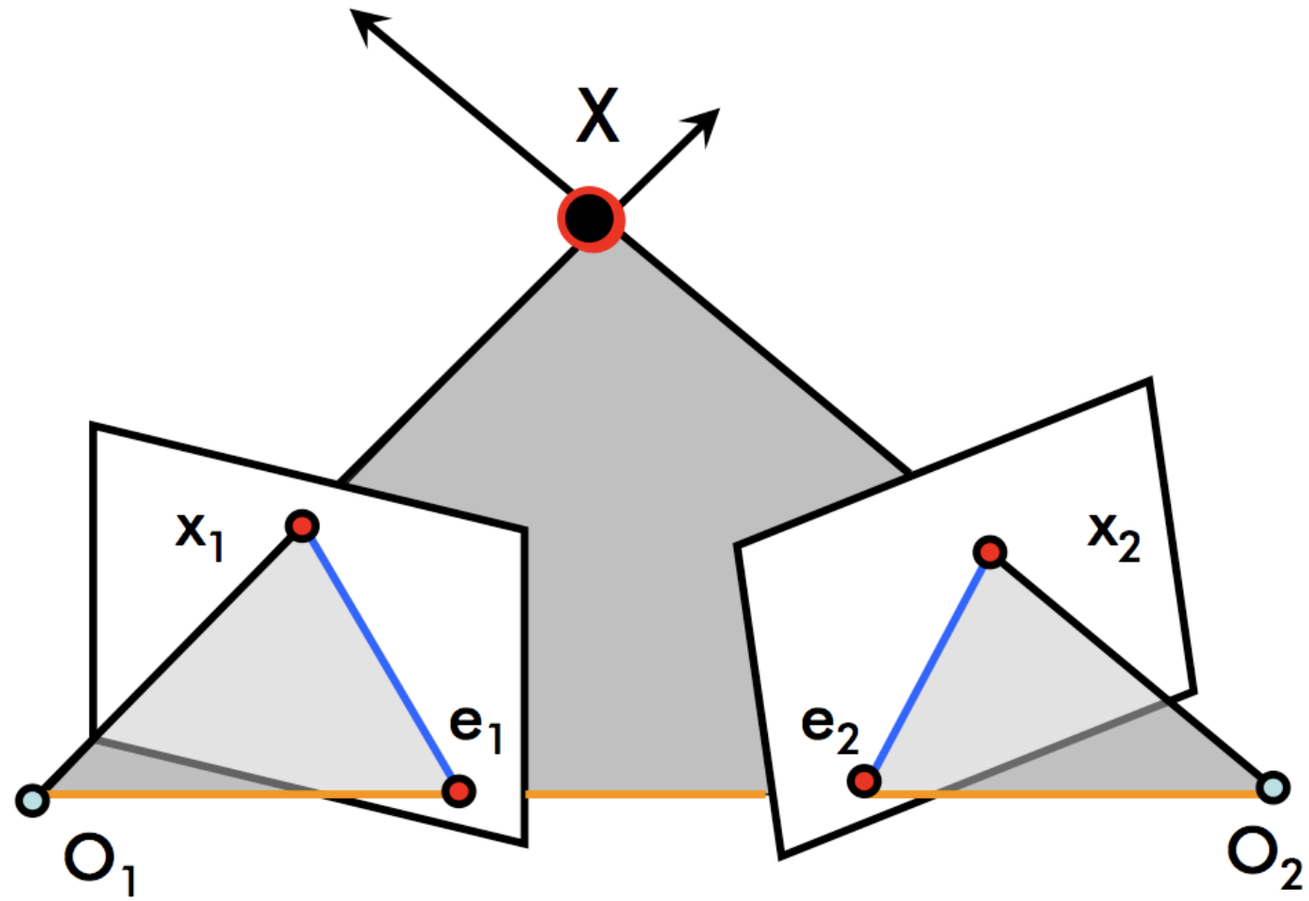
OSU Oregon State University

Epipolar Geometry -- Three Problems

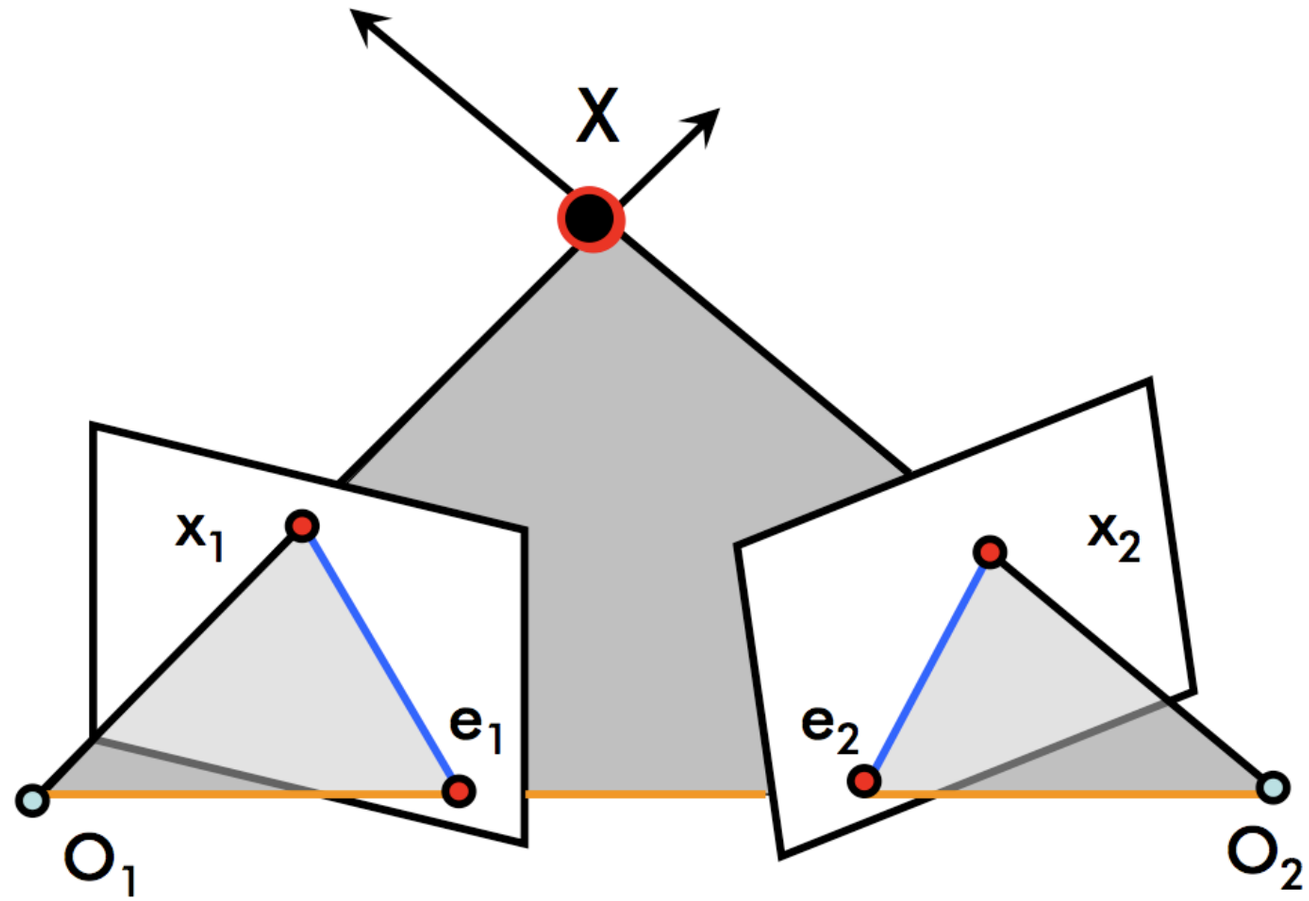


Epipolar Geometry -- Three Problems



- *Geometry* -- Given point pairs (x_{1n}, x_{2n}) , find the transform F that holds between all other point pairs

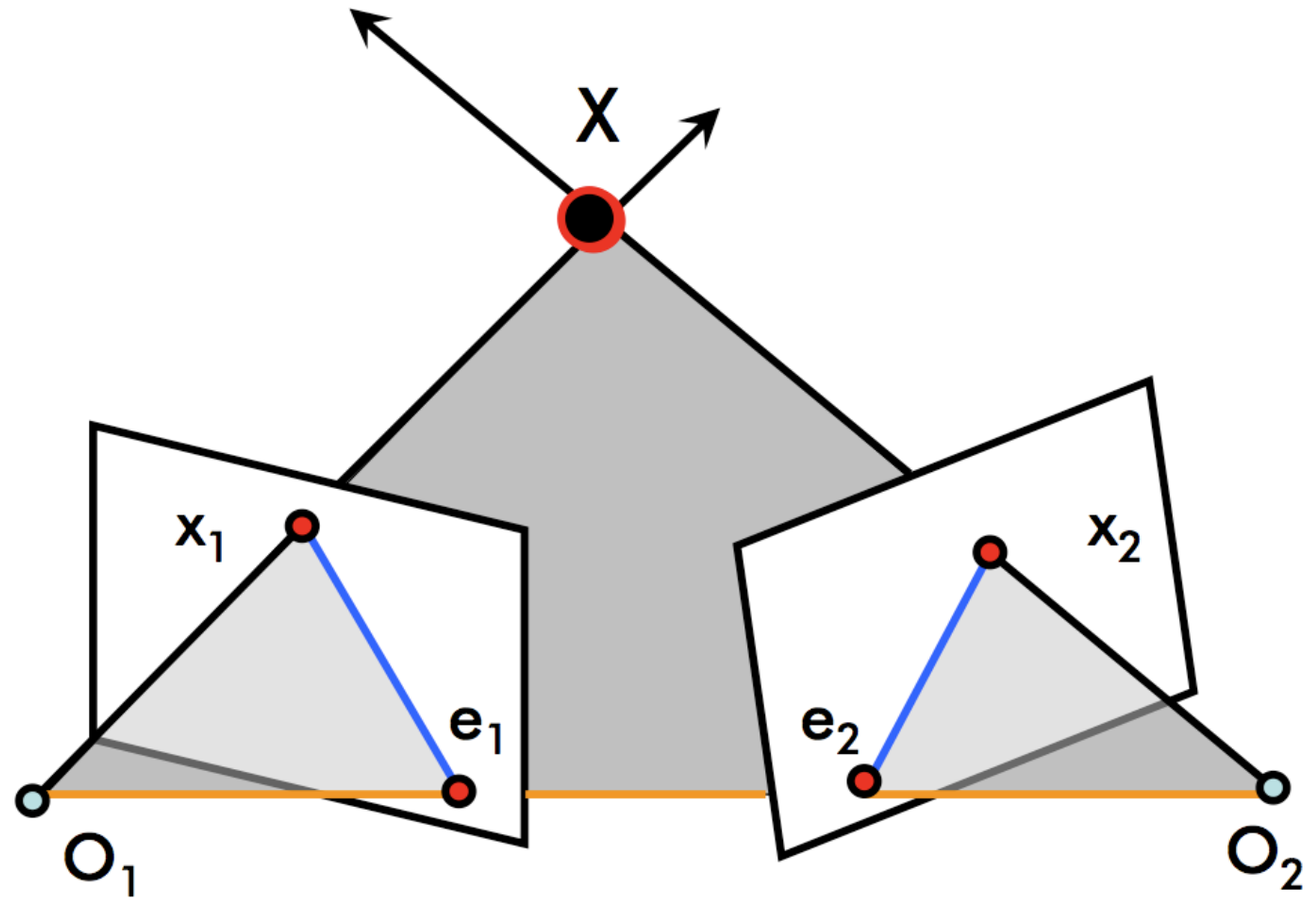
Epipolar Geometry -- Three Problems



- *Geometry* -- Given point pairs (x_{1n}, x_{2n}) , find the transform F that holds between all other point pairs

- *Correspondence -- Given F and x_1, find x_2*

Epipolar Geometry -- Three Problems

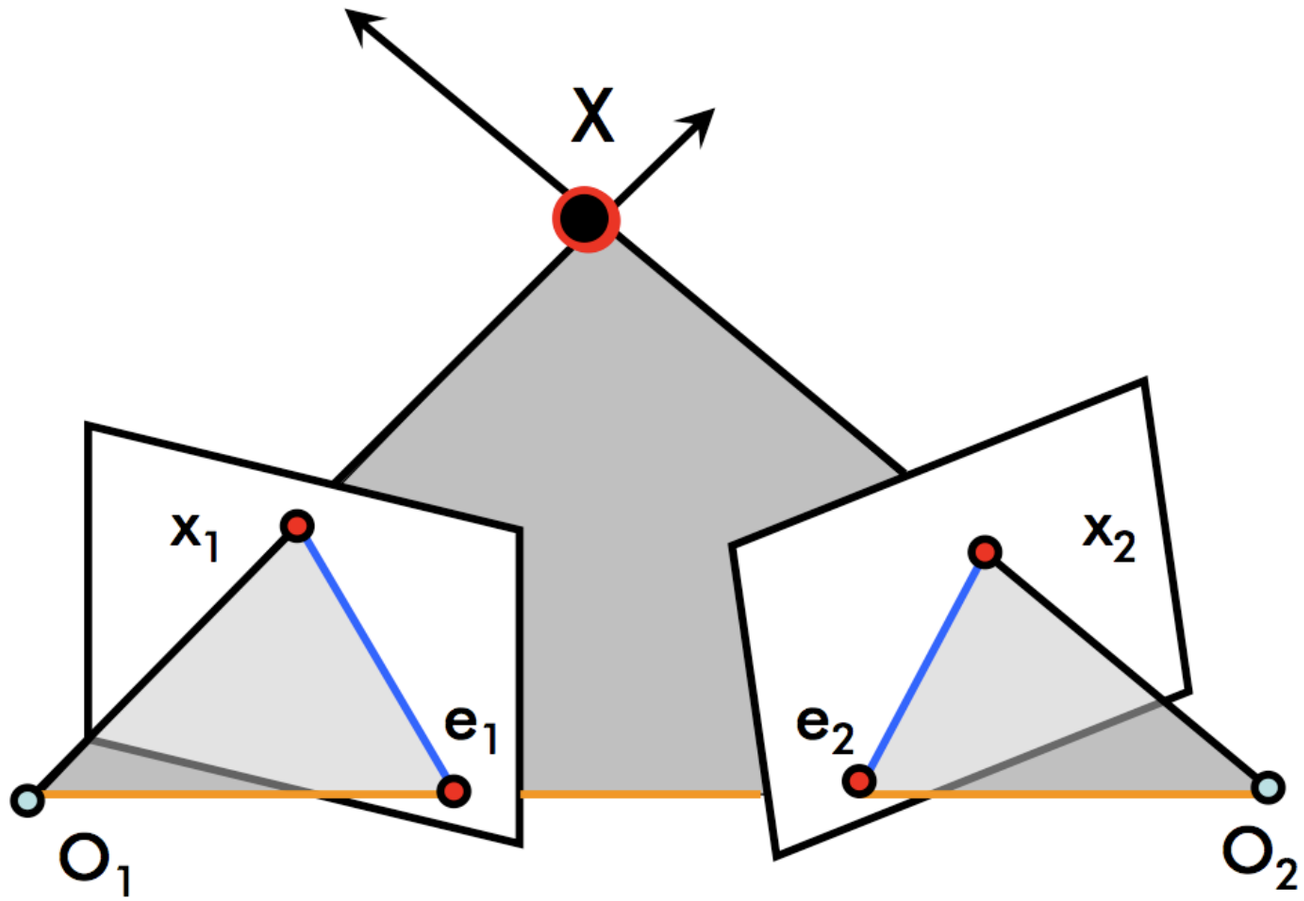


- *Geometry* -- Given point pairs (x_{1n}, x_{2n}) , find the transform F that holds between all other point pairs

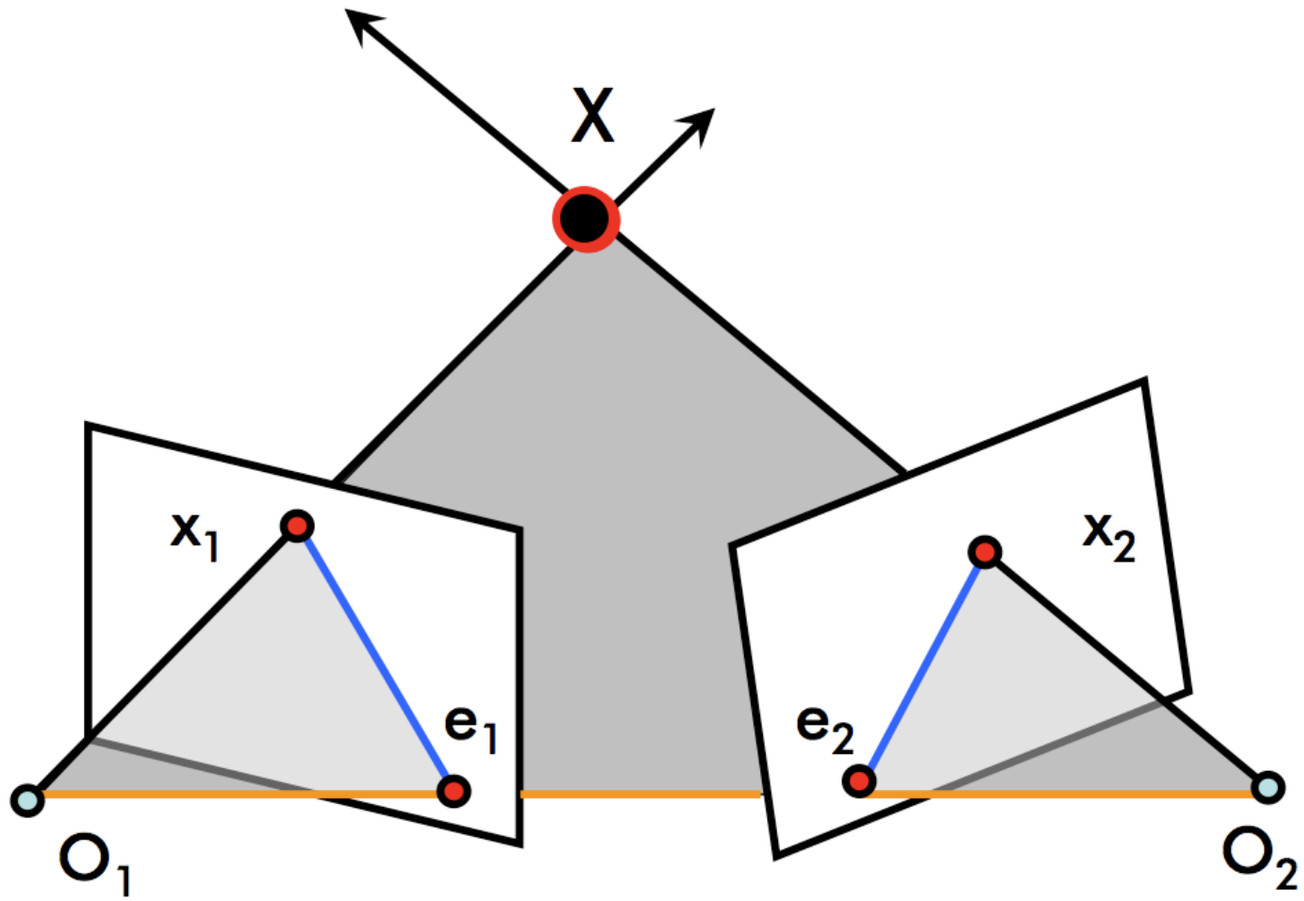
- *Correspondence -- Given F and x_1, find x_2*

- *3D Reconstruction* -- Given x_1 and x_2 , find X

Epipolar Geometry Problems



Epipolar Geometry Problems



- *Correspondence* -- Given F and x_1, find x_2

Motivation: Epipolar Constraint

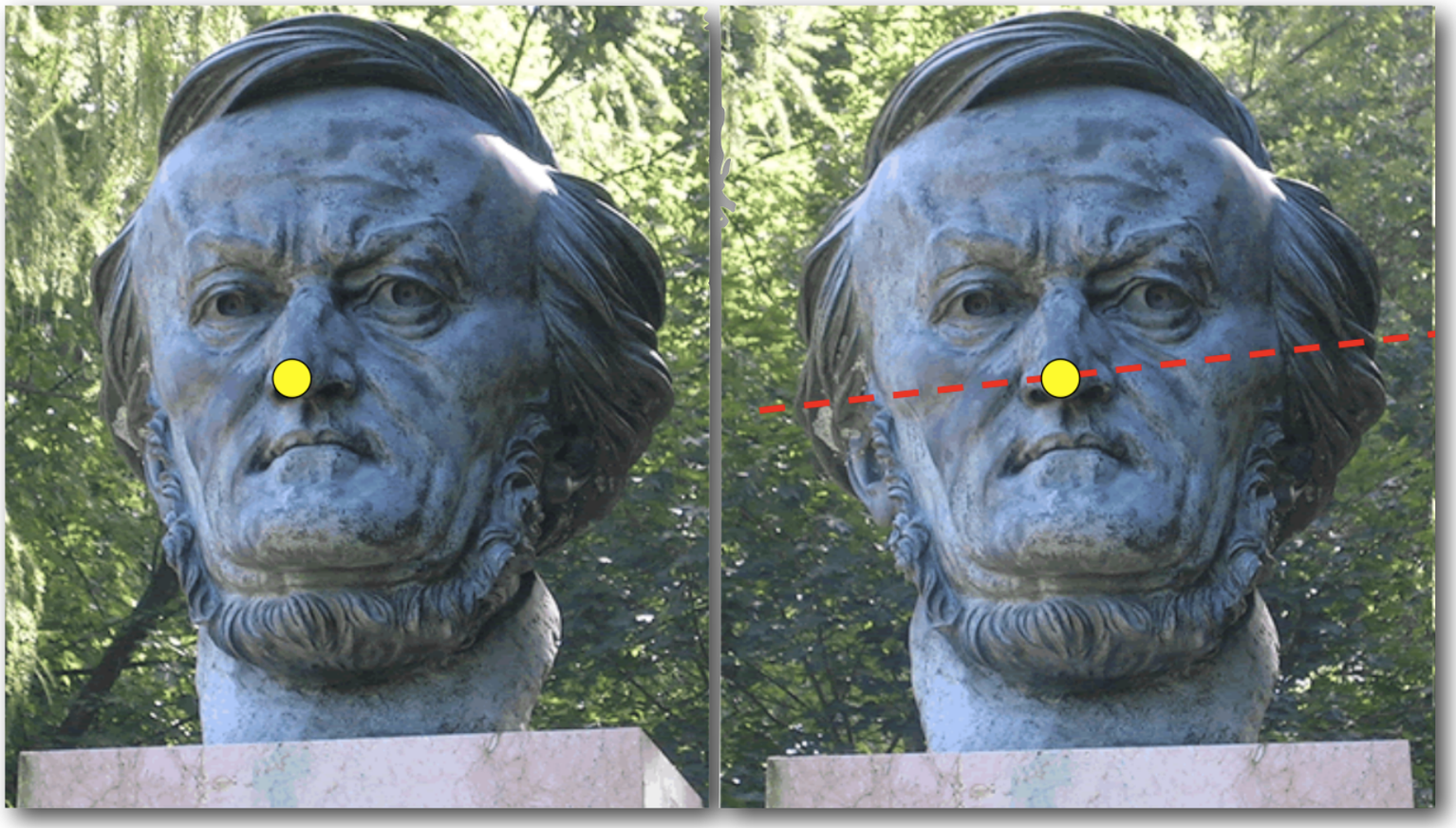
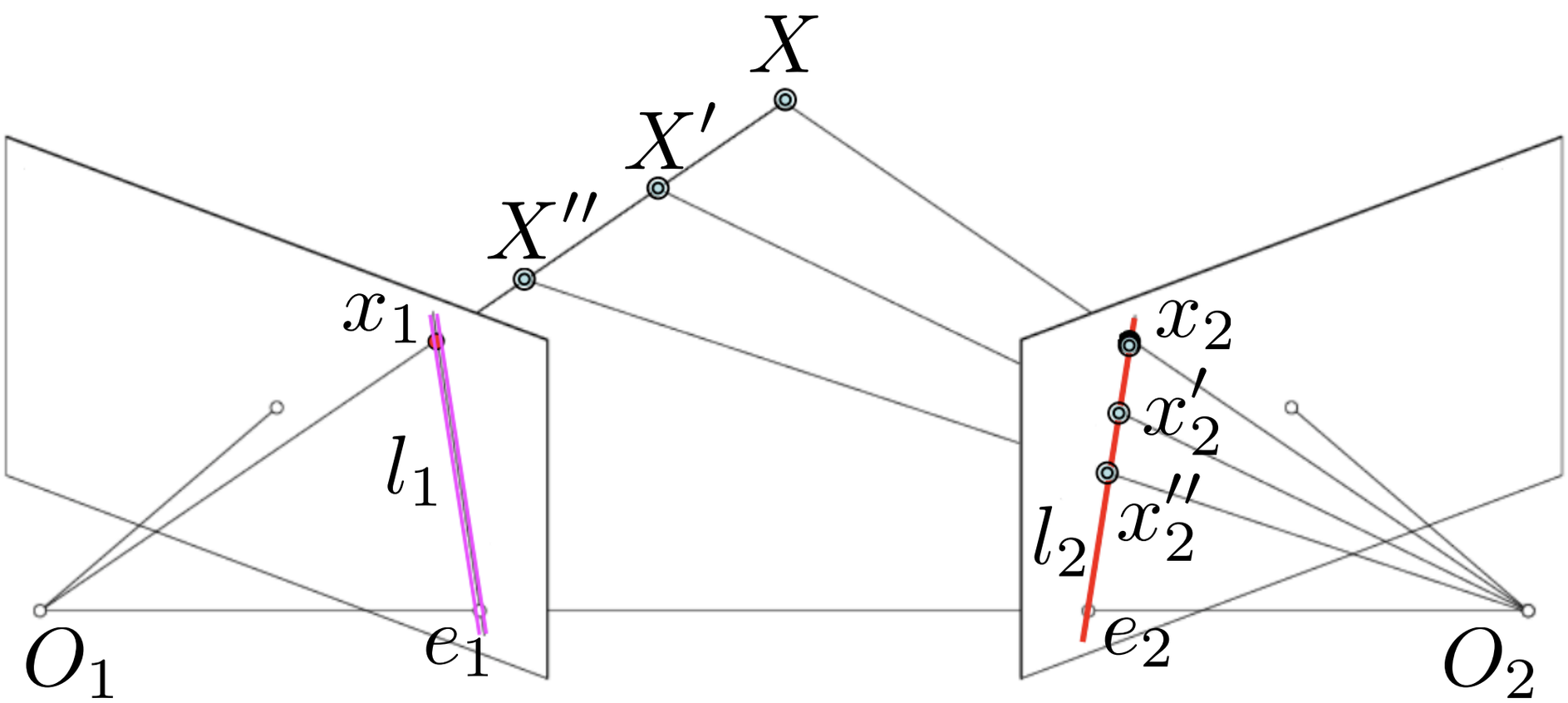


image 1 image 2

- Two views of the same object
- Can any point in image 2 correspond to the yellow point in image 1?

source: S. Savarese

Motivation: Epipolar Constraint

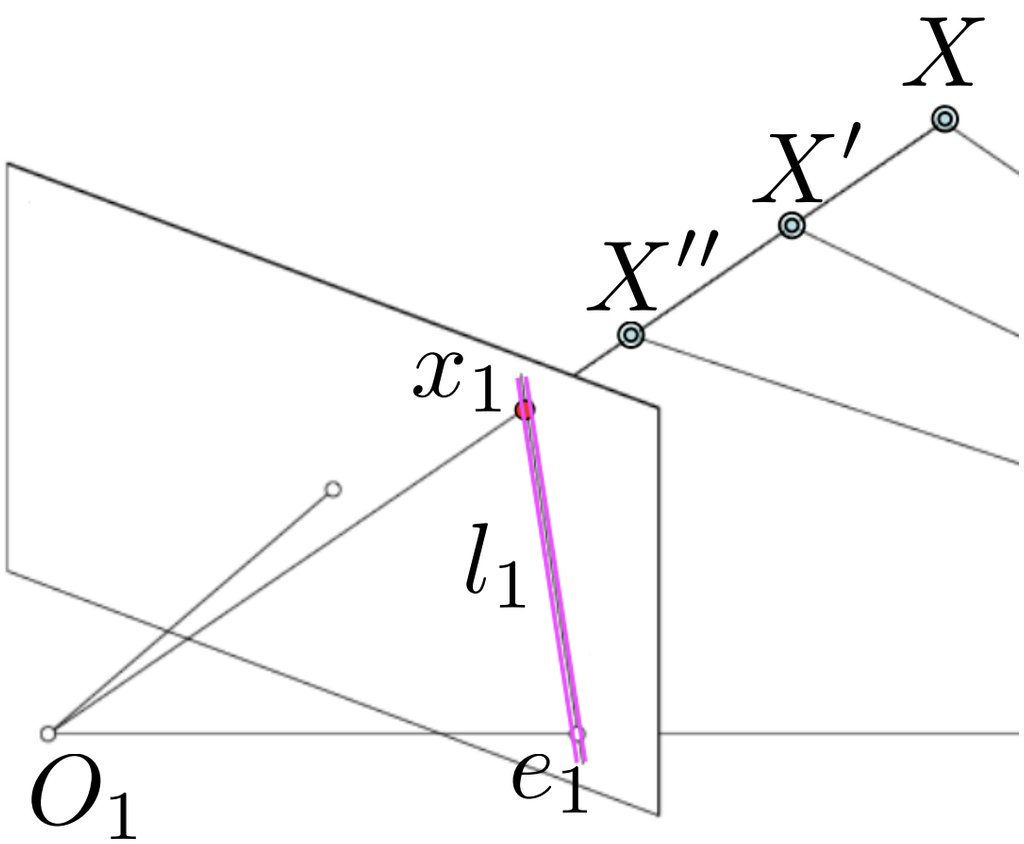


Points x_2 corresponding to point x_1
must lie along the epipolar line l_2

and

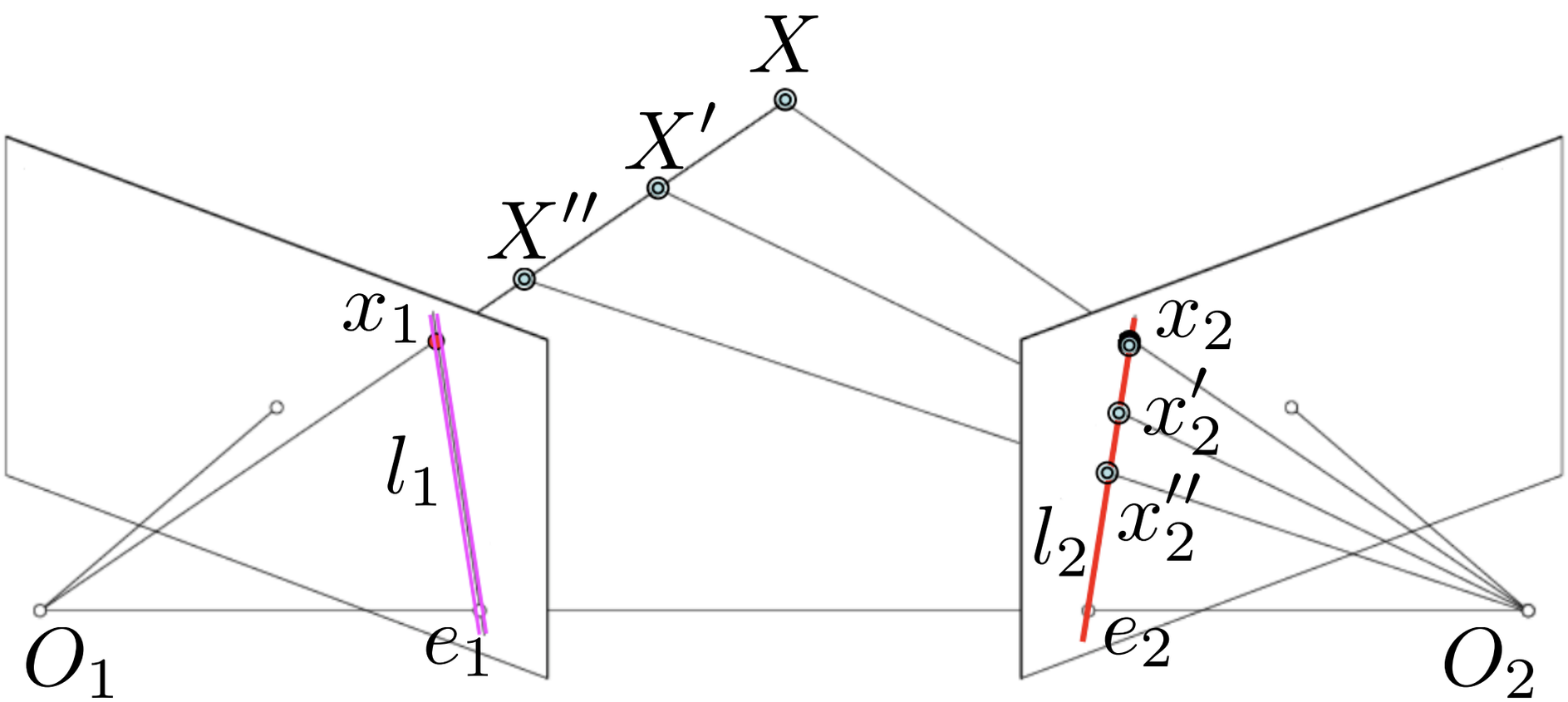
Points x_1 corresponding to point x_2
must lie along the epipolar line l_1

Epipolar Constraint



Point x_1 can be a projection of multiple 3D points

Epipolar Constraint



corresponding points in image 2
must lie along the epipolar line

Stereo Matching

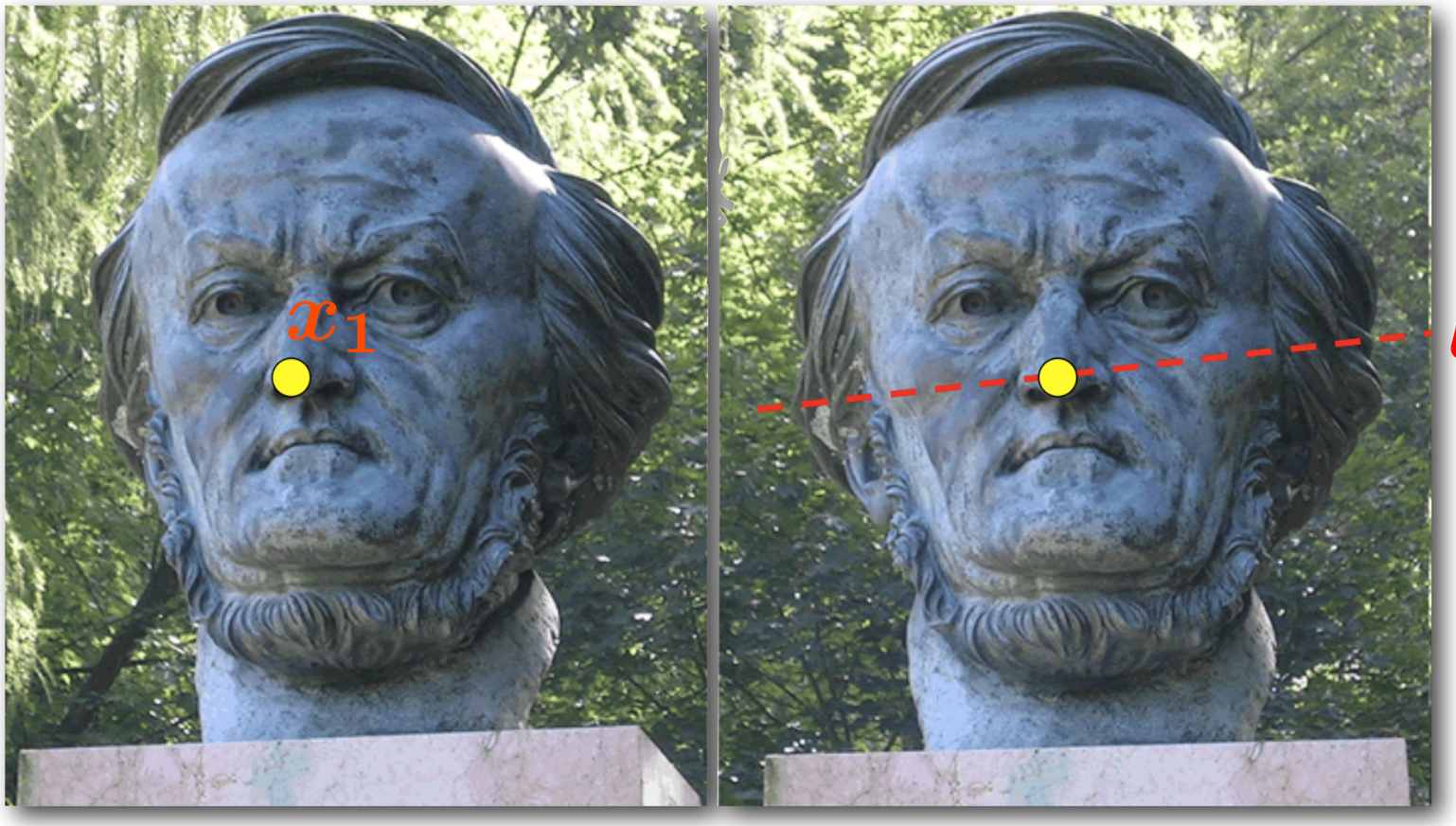
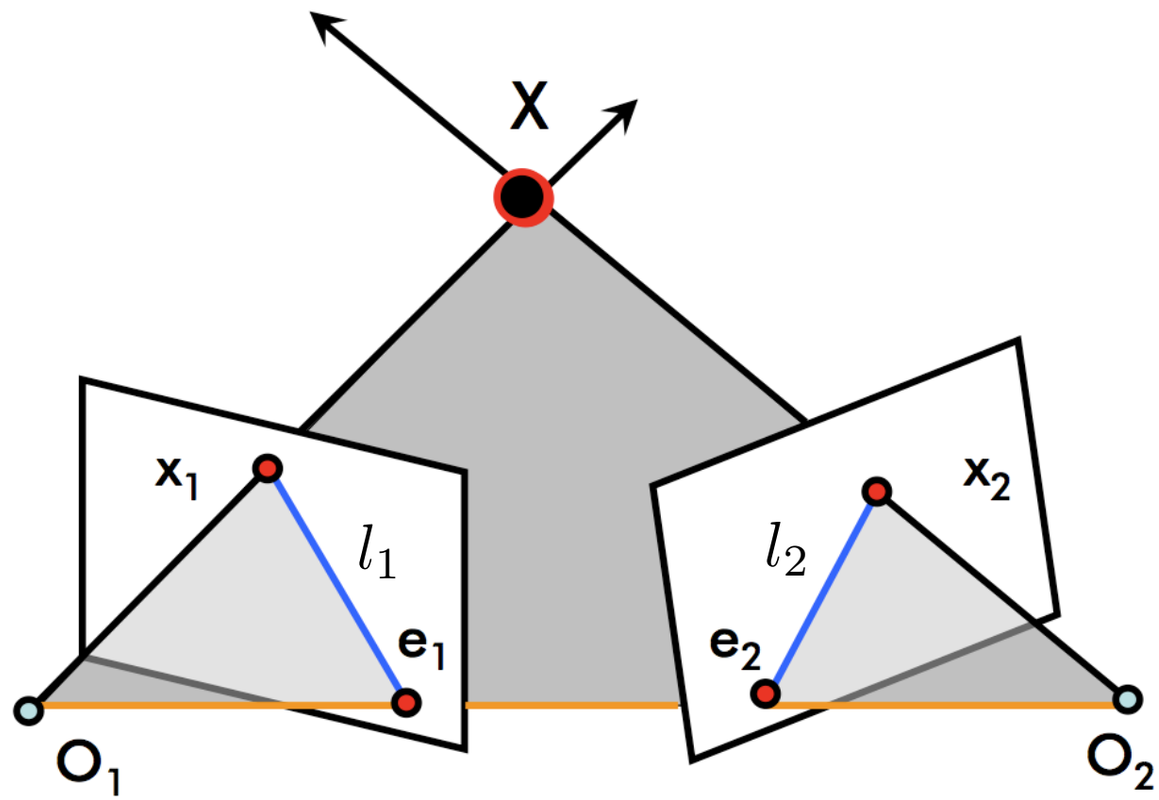


image 1 image 2

- Given F and a set of points in image 1
- Find the corresponding points in image 2

Epipolar Constraint



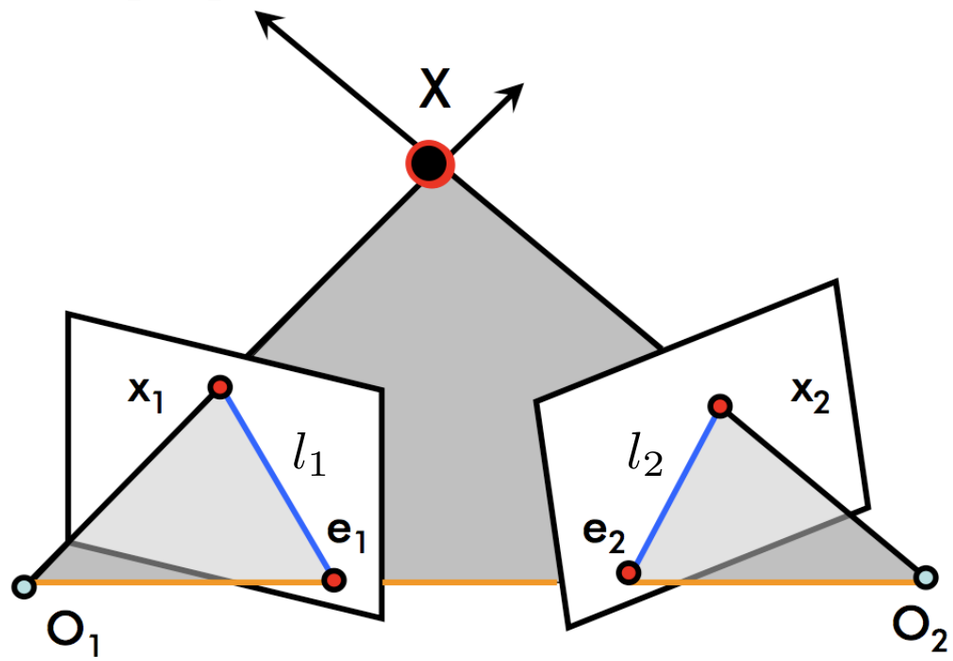
a line passing through two points:

$$l_2 = e_2 \times x_2 = [e_{2\times}]x_2$$

epipolar line

epipole

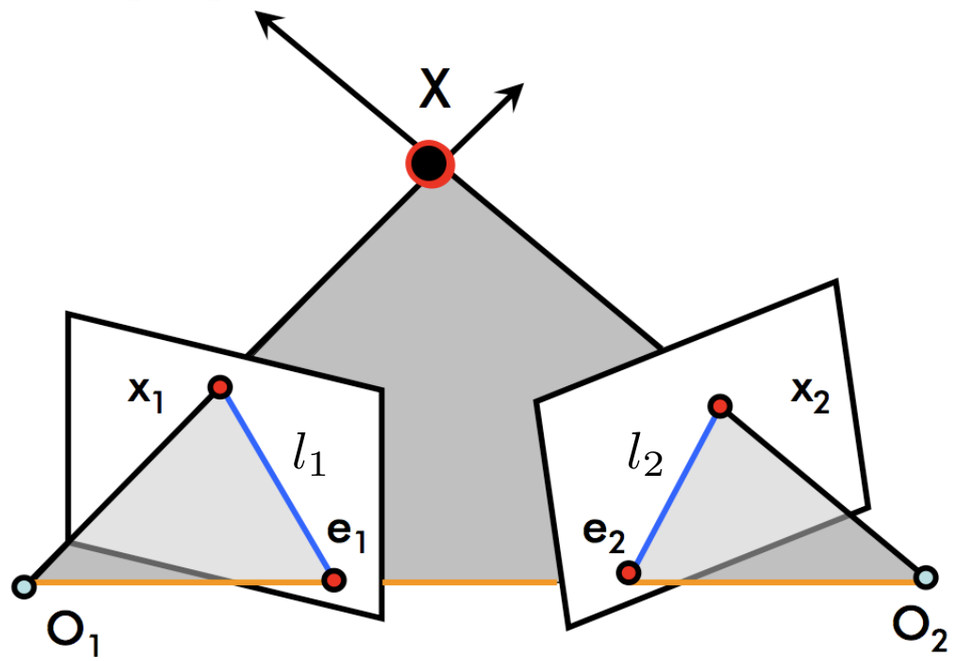
Epipolar Constraint



Geometry:

$$x_2 = M_2 X \quad \wedge \quad X = M_1^{-1} x_1 \quad \Rightarrow \quad x_2 = M_2 M_1^{-1} x_1$$

Epipolar Constraint



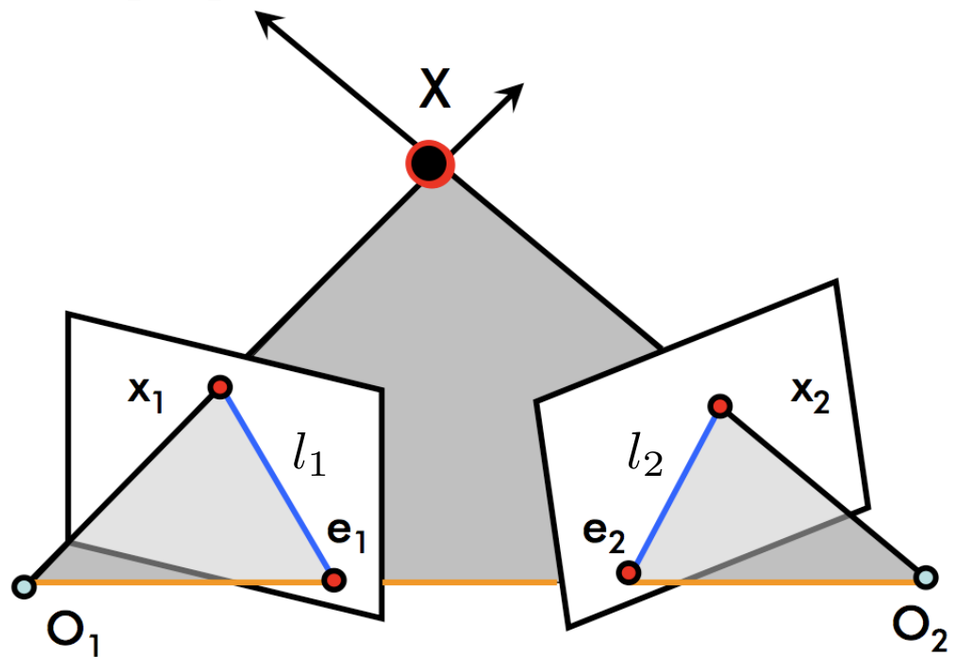
Geometry:

$$x_2 = M_2 X \quad \wedge \quad X = M_1^{-1} x_1 \quad \Rightarrow \quad x_2 = M_2 M_1^{-1} x_1$$

Epipolar constraint:

$$l_2 = e_2 \times x_2 = [e_{2\times}] x_2$$

Epipolar Constraint



Geometry:

$$x_2 = M_2 X \quad \wedge \quad X = M_1^{-1} x_1 \quad \Rightarrow \quad x_2 = M_2 M_1^{-1} x_1$$

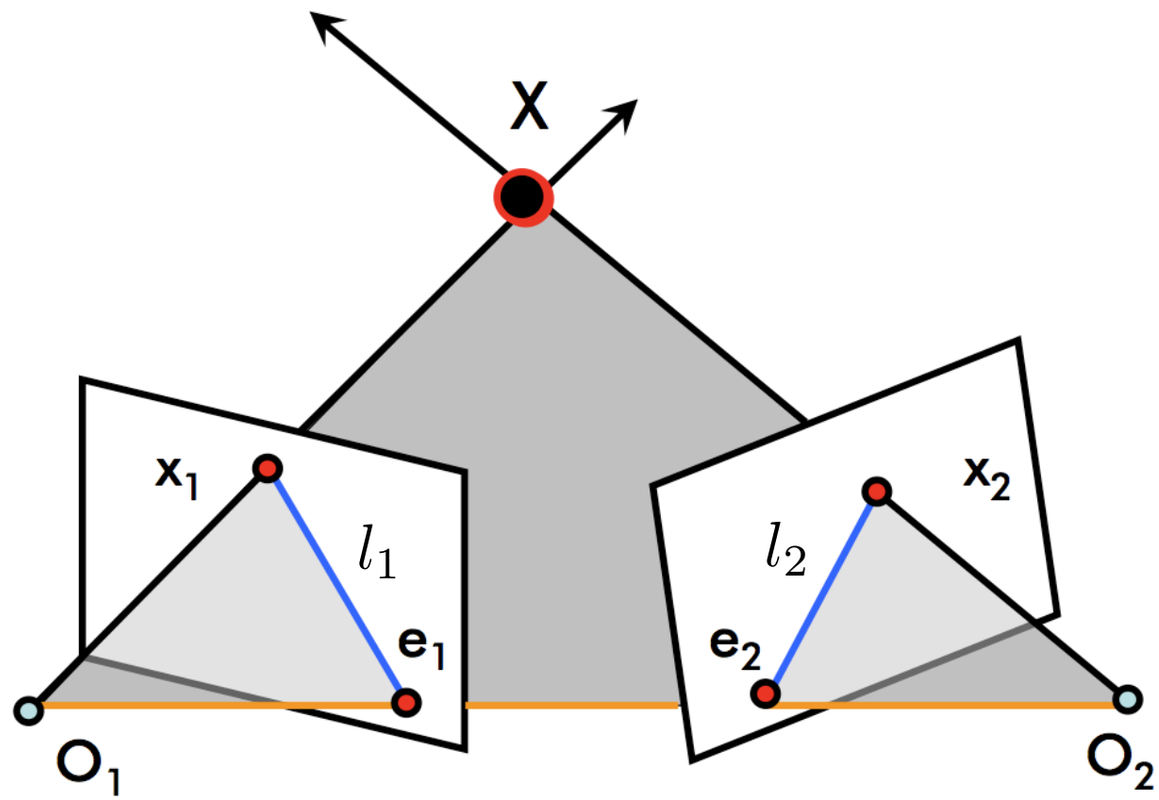
Epipolar constraint:

$$l_2 = e_2 \times x_2 = [e_{2\times}] x_2$$

$$\Rightarrow l_2 = \underbrace{[e_{2\times}] M_2 M_1^{-1}}_{F} x_1 = F x_1$$

fundamental matrix

Finding Epipolar Lines



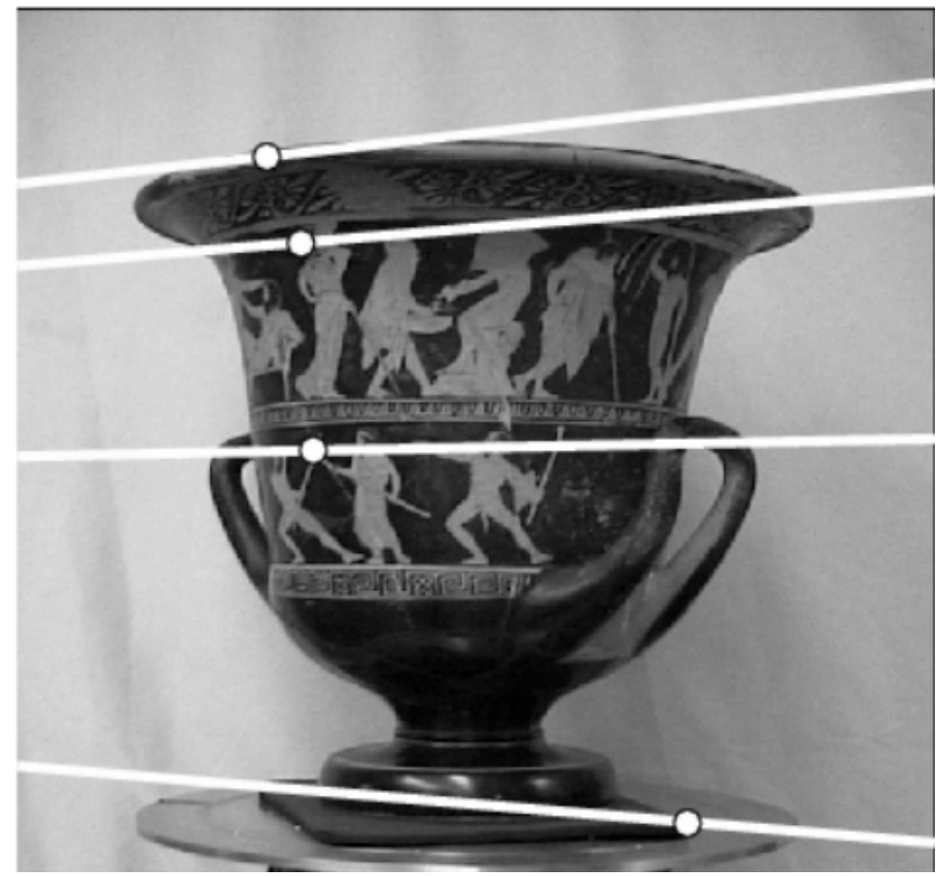
Given F and x_1, find the epipolar line of x_1:

$$l_2 = F\ x_1$$

epipolar line in image 2

Finding Epipolar Lines -- Example

$$l_{1n} = F^T x_{2n}$$



$$l_{2n} = F\ x_{1n}$$

Epipolar lines for a few sample points

Improved Matching Formulation: Linear Assignment

$$\min_{Y} \ \mathrm{Trace}(A^\top Y)$$

subject to:

$$\forall i, j \ \ y_{ij} \in [0, 1]$$

$$\forall i, \ \sum_j y_{ij} = 1 \qquad \forall j, \ \sum_i y_{ij} = 1$$

Improved Matching Formulation: Linear Assignment

indicator matrix
of matching pairs

$$\min_{Y} \ \mathrm{Trace}(A^\top Y)$$

subject to:

$$\forall i, j \ \ y_{ij} \in [0, 1]$$

$$\forall i, \ \sum_j y_{ij} = 1 \qquad \forall j, \ \sum_i y_{ij} = 1$$

Improved Matching Formulation: Linear Assignment

distance matrix

$$\min_{Y} \ \mathrm{Trace}(A^\top Y)$$

subject to:

$$\forall i, j \ \ y_{ij} \in [0, 1]$$

$$\forall i, \ \sum_j y_{ij} = 1 \qquad \forall j, \ \sum_i y_{ij} = 1$$

Improved Matching Formulation: Linear Assignment

$$\min_{Y} \ \mathrm{Trace}(A^\top Y)$$

subject to:

$$\forall i, j \ \ y_{ij} \in [0, 1]$$

$$\forall i, \ \sum_j y_{ij} = 1 \qquad \forall j, \ \sum_i y_{ij} = 1$$

one-to-one matching

Improved Matching Formulation: Linear Assignment

Hungarian Algorithm

$$\min_{Y} \ \mathrm{Trace}(A^\top Y)$$

subject to:

$$\forall i, j \ \ y_{ij} \in [0, 1]$$

$$\forall i, \ \sum_j y_{ij} = 1 \qquad \forall j, \ \sum_i y_{ij} = 1$$

Improved Matching Formulation: Linear Assignment

Hungarian Algorithm

$$\min_{Y} \ \mathrm{Trace}(A^\top Y)$$

subject to:

$$\forall i, j \ \ y_{ij} \in [0, 1]$$

$$\forall i, \ \sum_j y_{ij} = 1 \qquad \forall j, \ \sum_i y_{ij} = 1$$

additional constraints:

$$\forall j, \ \ x_j \in l_j = F x_i$$

2D point in image 2

epipolar line in image 2

Improved Matching Formulation: Linear Assignment

Hungarian Algorithm

$$\min_{Y} \ \mathrm{Trace}(A^\top Y)$$

subject to:

$$\forall i, j \ \ y_{ij} \in [0, 1]$$

$$\forall i, \ \sum_j y_{ij} = 1 \qquad \forall j, \ \sum_i y_{ij} = 1$$

additional constraints:

$$\forall j : \quad x_j^\top F x_i = 0$$

2D point in image 2

2D point in image 1

How to Incorporate the Additional Stereo Constraints?

additional constraints:

$$\forall j: \quad x_j^\top F x_i = 0$$

2D point in image 2

2D point in image 1

How to Incorporate the Additional Stereo Constraints?

additional constraints:

$$\sum_{(i,j)} [x_j^\top F x_i] \cdot y_{ij} = 0$$

indicator of matching pairs

$$\forall j: \quad x_j^\top F x_i = 0$$

2D point in image 2

2D point in image 1

How to Incorporate the Additional Stereo Constraints?

additional constraints:

$$\sum_{(i,j)} \boxed{[x_j^\top F x_i]} \cdot y_{ij} = 0$$

indicator of matching pairs

b_{ij}

$$\mathrm{tr}(B^\top Y)$$

Improved Matching Formulation: Linear Assignment

Hungarian Algorithm

$$\min_{Y} \ \mathrm{Trace}(A^\top Y)$$

subject to:

$$\forall i, j \ \ y_{ij} \in [0, 1]$$

$$\forall i, \ \sum_j y_{ij} = 1 \qquad \forall j, \ \sum_i y_{ij} = 1$$

Improved Matching Formulation: Linear Assignment

$$\min_{Y} \; \left[\mathrm{Trace}(A^\top Y) + \mathrm{Trace}(B^\top Y)\right]$$

subject to:

$$\forall i, j \;\; y_{ij} \in [0, 1]$$

$$\forall i, \;\; \sum_j y_{ij} = 1 \qquad\qquad \forall j, \;\; \sum_i y_{ij} = 1$$

Improved Matching Formulation: Linear Assignment

Hungarian Algorithm

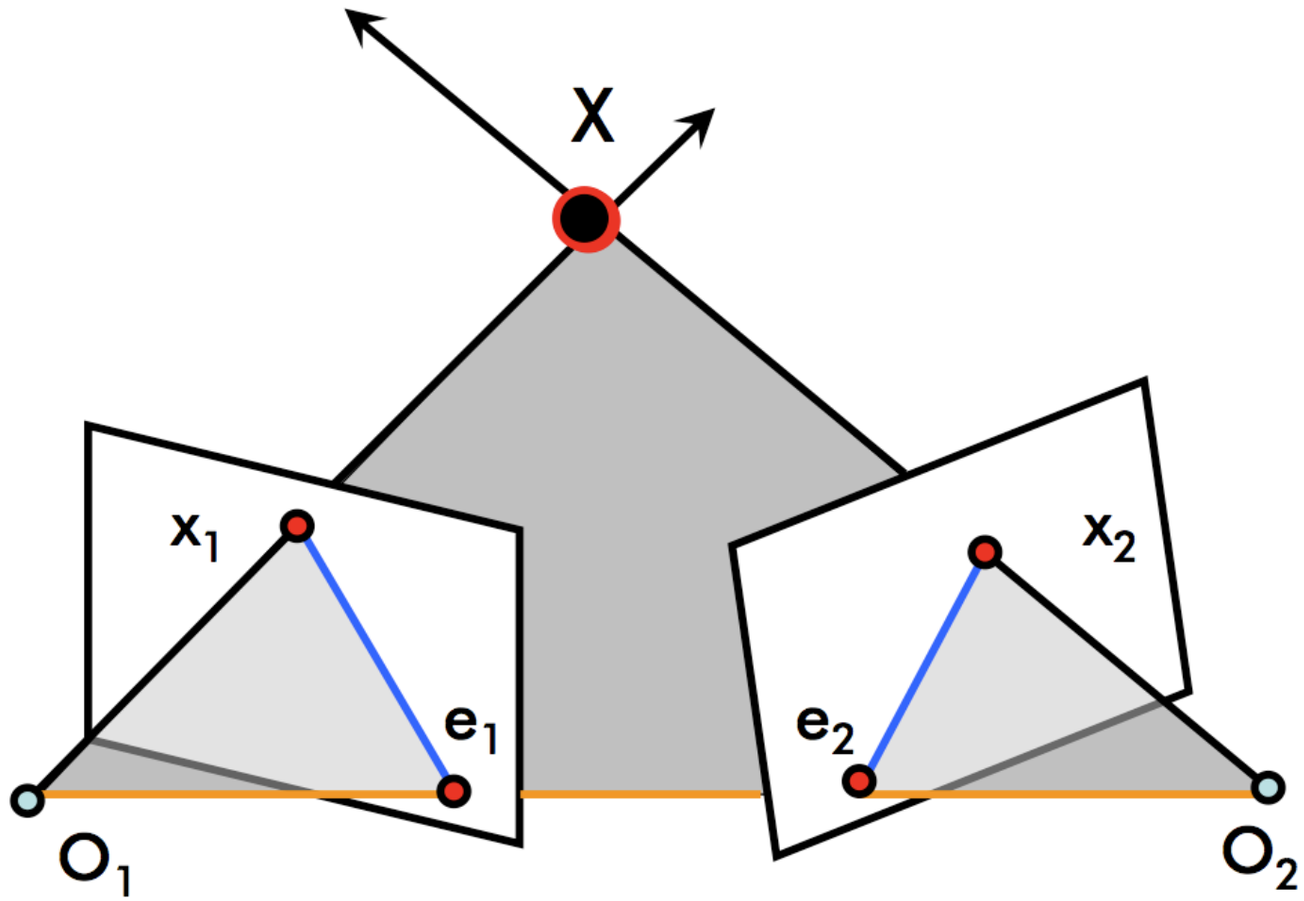
$$\min_{Y} \ \mathrm{Trace}((A+B)^\top Y)$$

subject to:

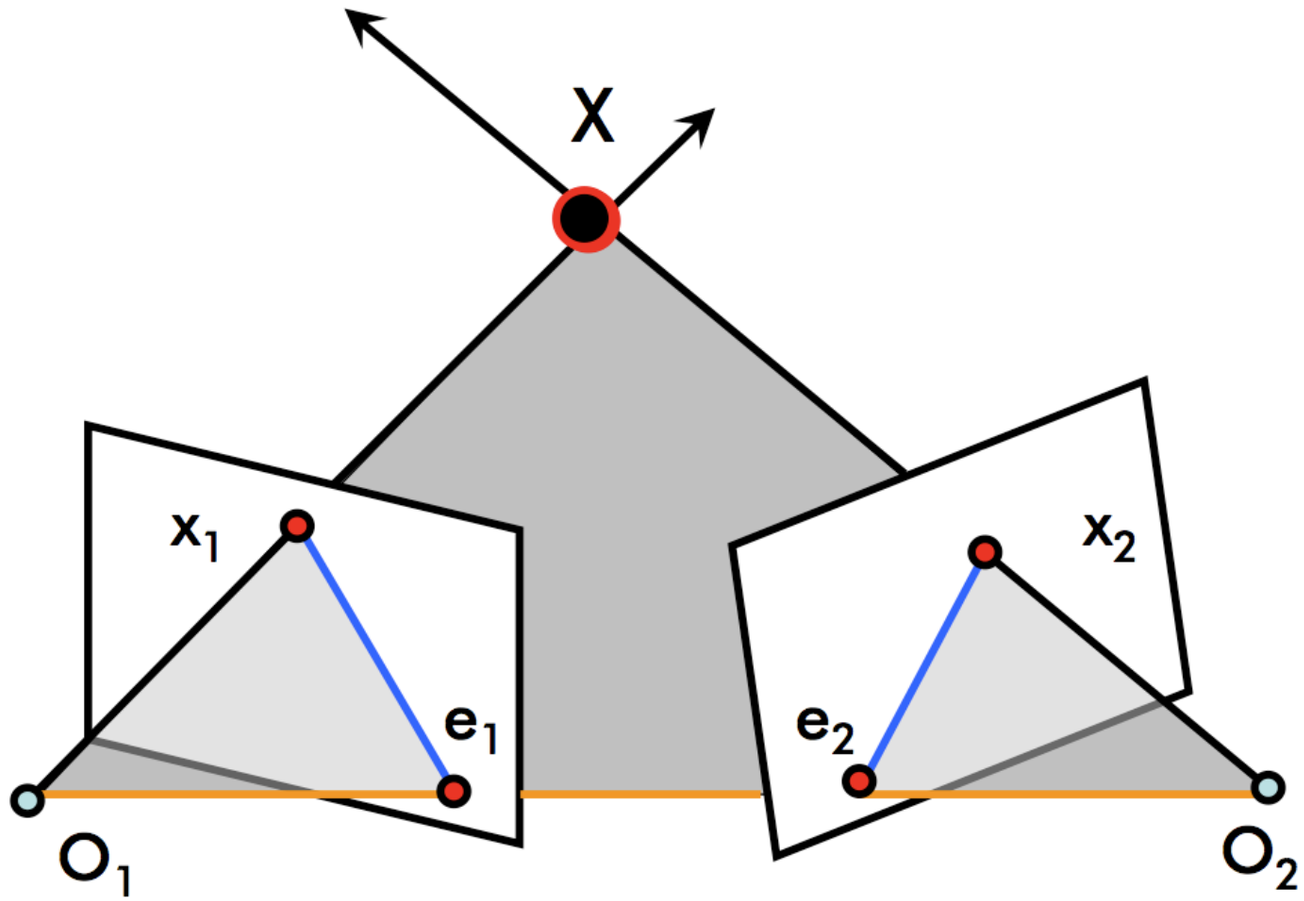
$$\forall i, j \ \ y_{ij} \in [0, 1]$$

$$\forall i, \ \sum_j y_{ij} = 1 \qquad \forall j, \ \sum_i y_{ij} = 1$$

Epipolar Geometry Problems

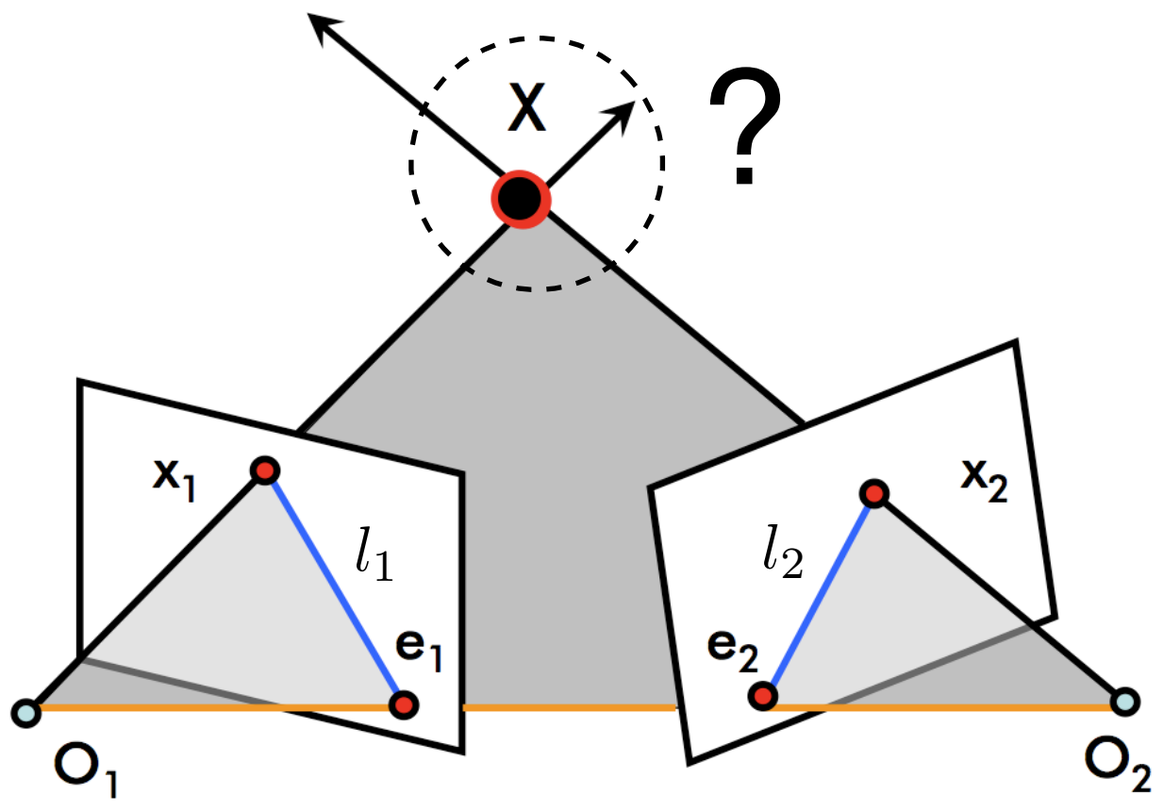


Epipolar Geometry Problems



- *3D Reconstruction* -- Given x_1 and x_2, find X

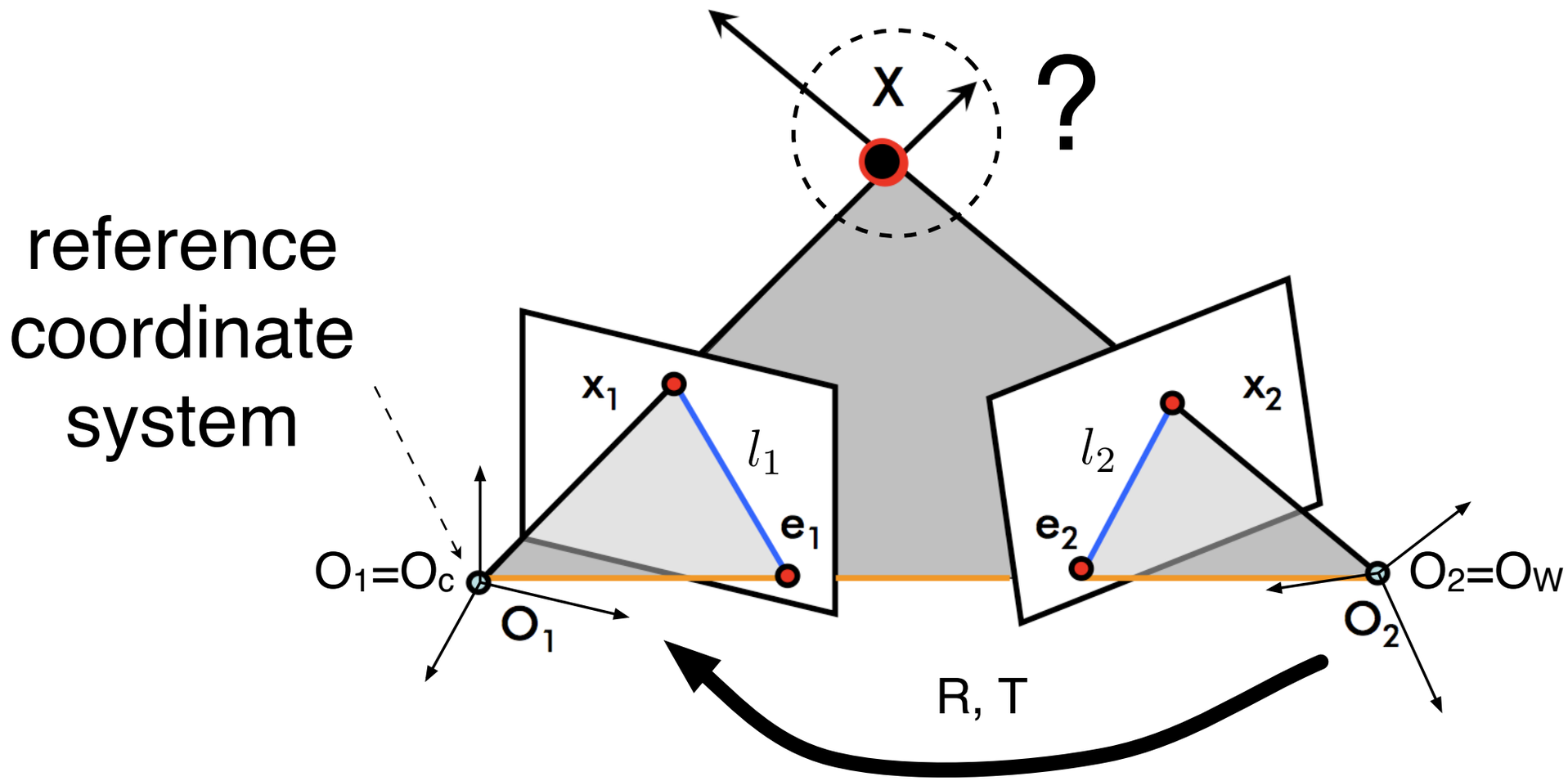
3D Structure Reconstruction from Stereo Images



<u>Goal:</u>

Estimate X, given two observations of X in stereo images and camera parameters $M_1 = K_1\ [I\ \ \mathbf{0}]$ and $M_2 = K_1\ [R\ \ \mathbf{t}]$

3D Structure Reconstruction from Stereo Images



<u>Goal:</u>

Estimate X, given two observations of X in stereo images

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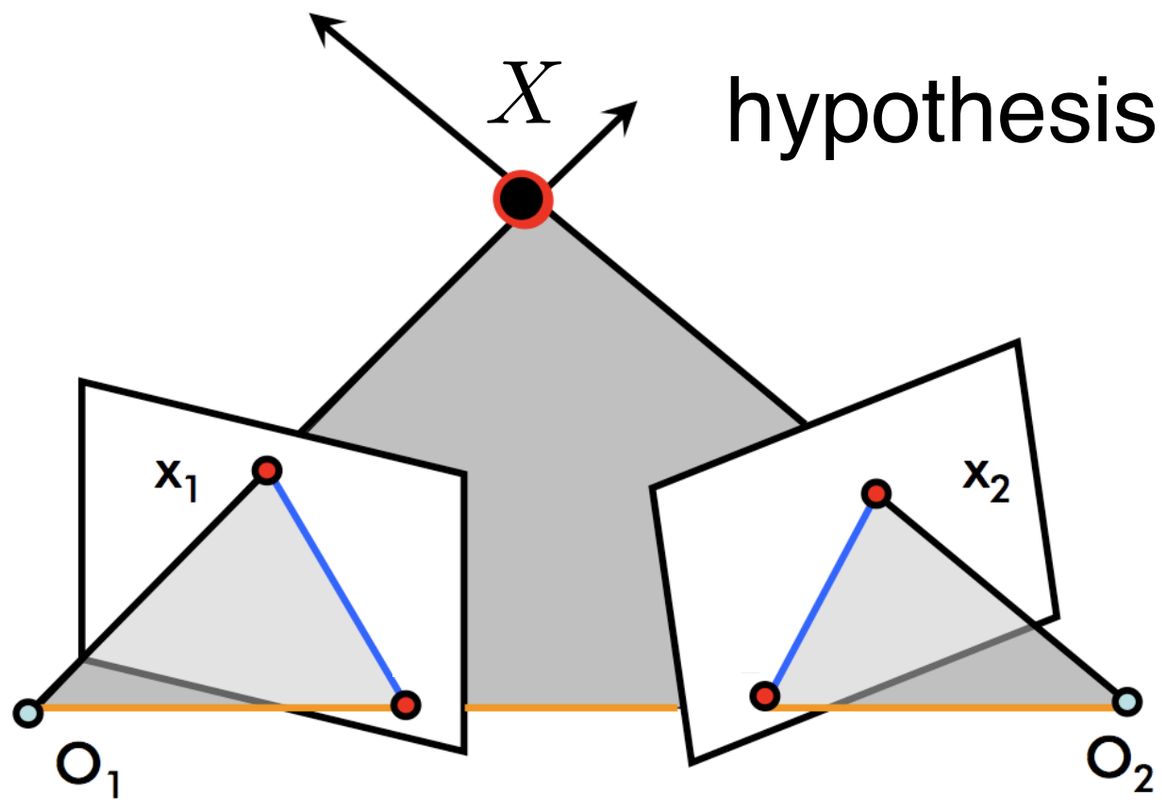
Minimization of Geometric Error

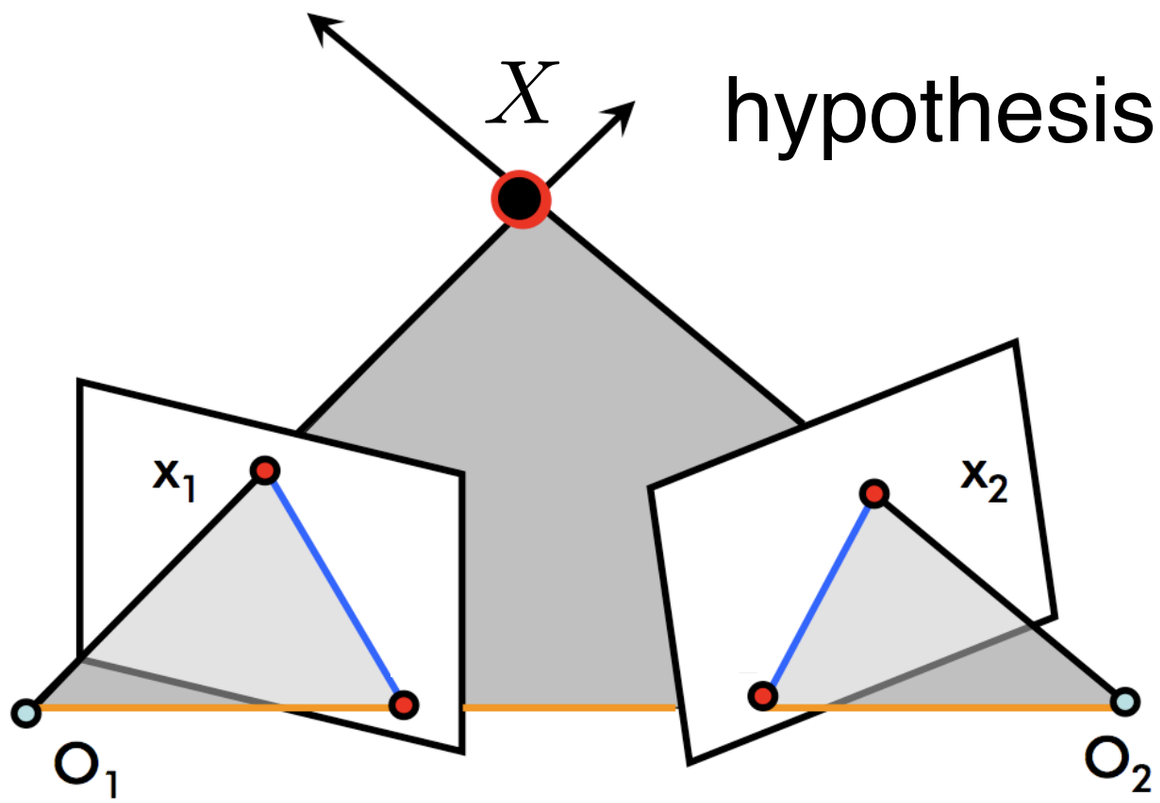


$$x_1 - M_1 X = \mathbf{0} \qquad\qquad x_2 - M_2 X = \mathbf{0}$$

$$\Downarrow \qquad\qquad\qquad \Downarrow$$

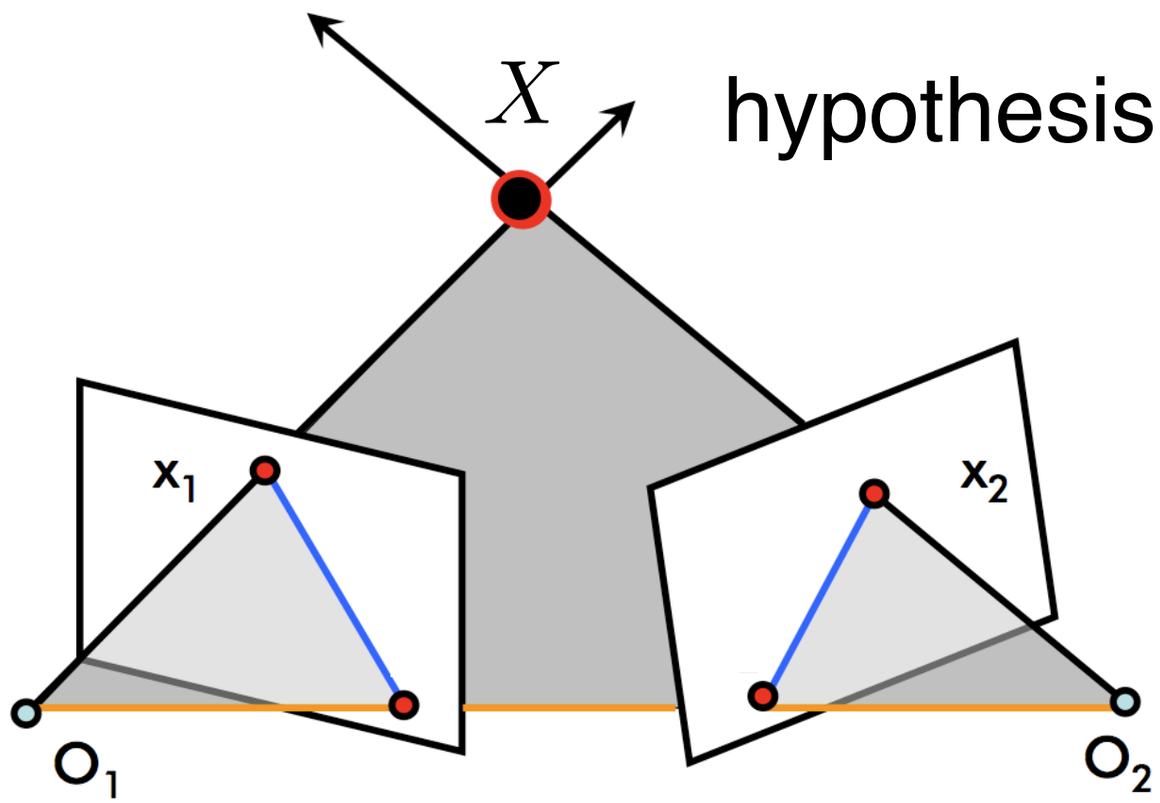
$$\|x_1 - M_1 X\|_2^2 = 0 \qquad\qquad \|x_2 - M_2 X\|_2^2 = 0$$

Minimization of Geometric Error



$$\min_{X} \quad \left(\|x_1 - M_1 X\|^2 + \|x_2 - M_2 X\|^2\right)$$

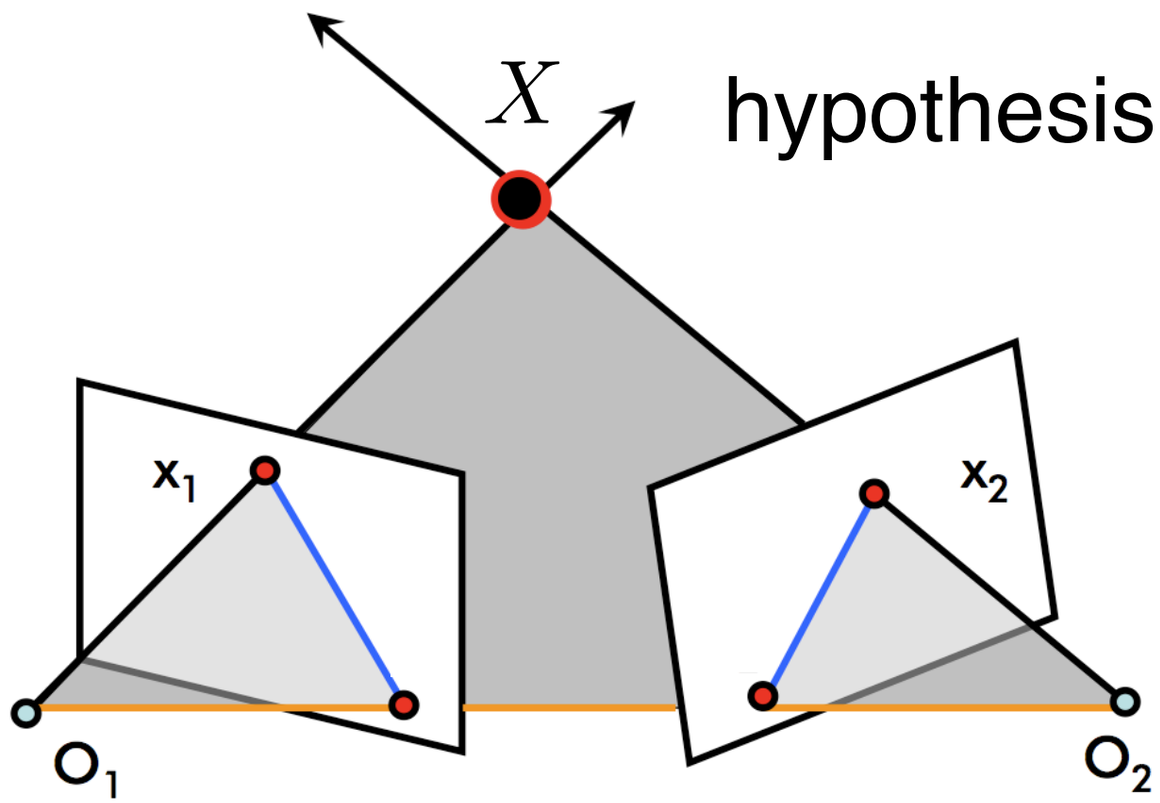
Minimization of Geometric Error



$$\min_{X} \quad \left(\|x_1 - M_1 X\|^2 + \|x_2 - M_2 X\|^2\right)$$

subject to: $x_2^{\mathrm{T}} F x_1 = 0$ ← epipolar constraint

Minimization of Geometric Error



$$\min_{X} \quad \left(\|x_1 - M_1 X\|^2 + \|x_2 - M_2 X\|^2\right)$$

subject to: $X^{\mathrm{T}}(M_2^{\mathrm{T}}\ F\ M_1)\ X = 0$ epipolar constraint

Minimization of Geometric Error -- Solution

$$\min_X \quad \left(\|x_1 - M_1 X\|^2 + \|x_2 - M_2 X\|^2\right)$$

subject to: $\quad X^{\mathrm{T}}(M_2^{\mathrm{T}}\ F\ M_1)\ X = 0$

$$\Downarrow$$

$$\min_X \quad \left(\|x_1 - M_1 X\|^2 + \|x_2 - M_2 X\|^2 + \lambda X^T (M_2^T F M_1) X)\right)$$

Minimization of Geometric Error -- Solution

$$\min_X \quad \left(\|x_1 - M_1 X\|^2 + \|x_2 - M_2 X\|^2\right)$$

subject to: $\quad X^{\mathrm{T}}(M_2^{\mathrm{T}}\ F\ M_1)\ X = 0$

$$\Downarrow$$

$$\min_X \quad \left(\|x_1 - M_1 X\|^2 + \|x_2 - M_2 X\|^2 + \lambda X^T (M_2^T F M_1) X)\right)$$

$$A^T A$$

$$\Downarrow$$

$$\min_X \quad \left(\|x_1 - M_1 X\|^2 + \|x_2 - M_2 X\|^2 + \|AX\|_2^2\right)$$

Minimization of Geometric Error -- Solution

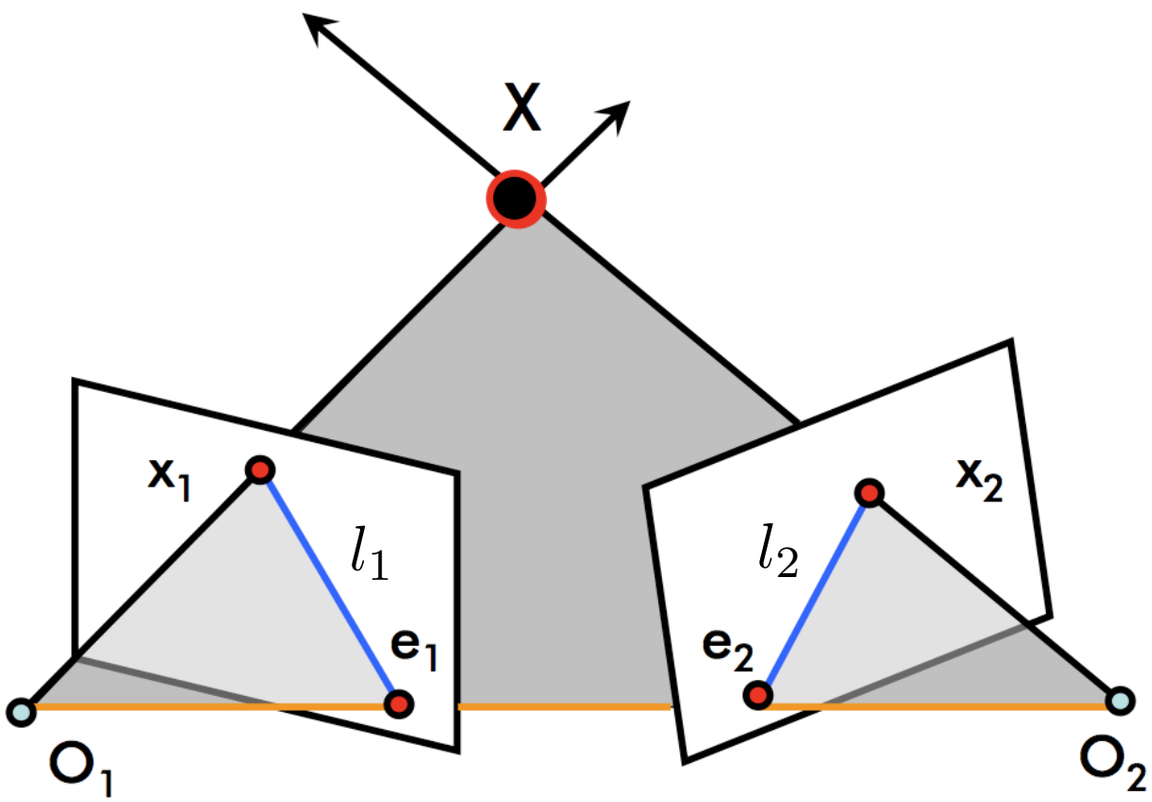
$$\min_X \ \left(\|x_1 - M_1 X\|^2 + \|x_2 - M_2 X\|^2 + \|AX\|_2^2\right)$$

$$\Downarrow$$

$$\min_X \ \sum_i \|a_i - A_i X\|^2$$

Algorithm: <u>Levenberg-Marquardt</u>

Summary



epipolar line in image 2:

$$l_2 = F x_1$$

epipolar line in image 1:

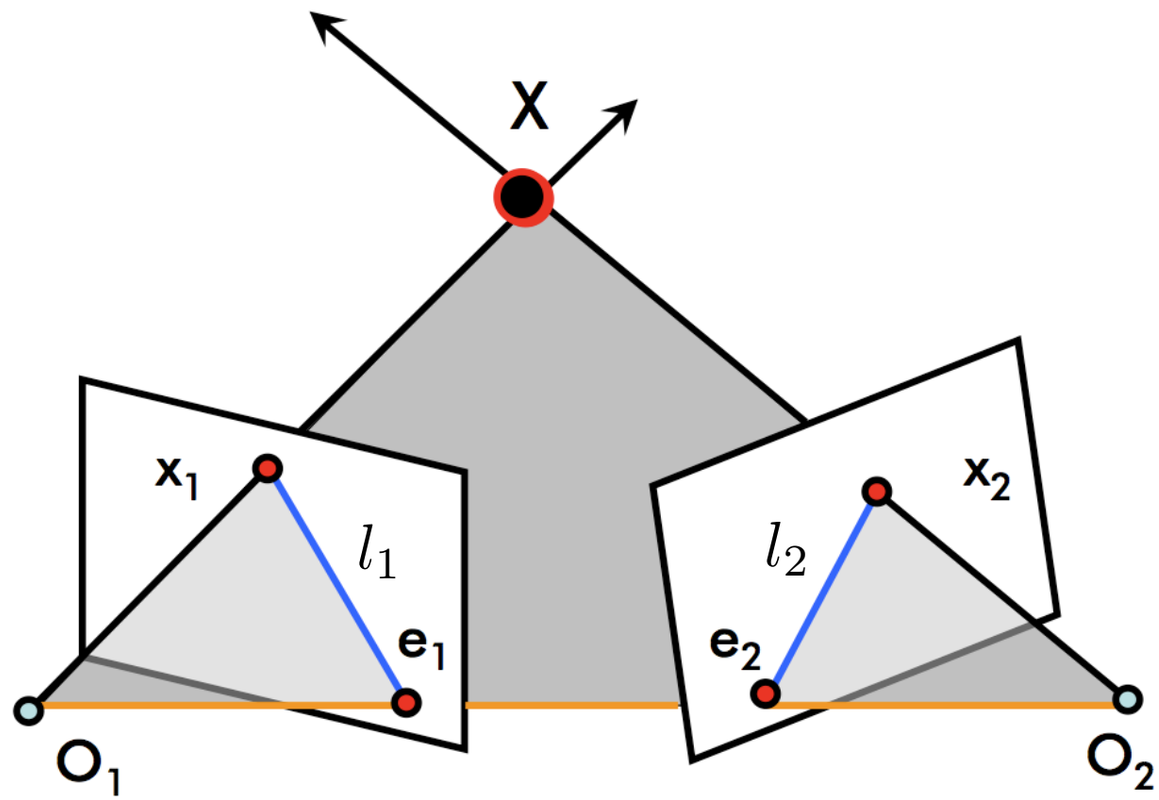
$$l_1 = F^T x_2$$

fundamental equation:

$$x_2^{\mathrm{T}} \; F \; x_1 \; = \; 0$$

No explicit relationship between x_1 and x_2, because F is singular

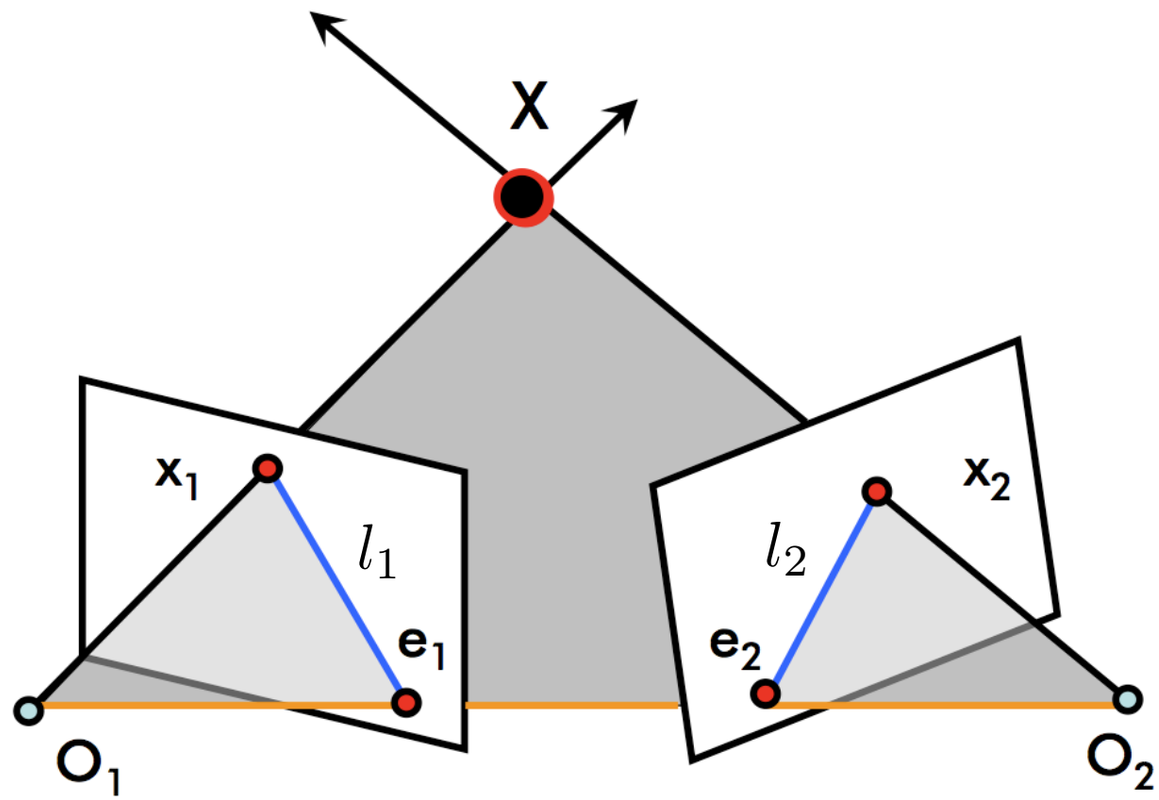
Example Problem: Finding Epipoles



Given the fundamental matrix, find the epipoles

$$F = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \Rightarrow \; e_1 = \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}, \; e_2 = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}$$

Problem: Finding Epipoles



Since *e* lies on the epipolar lines:

image 1: $$e_1^{\mathrm{T}}\ l_1 = 0$$

image 2: $$e_2^{\mathrm{T}}\ l_2 = 0$$

Problem: Finding Epipoles

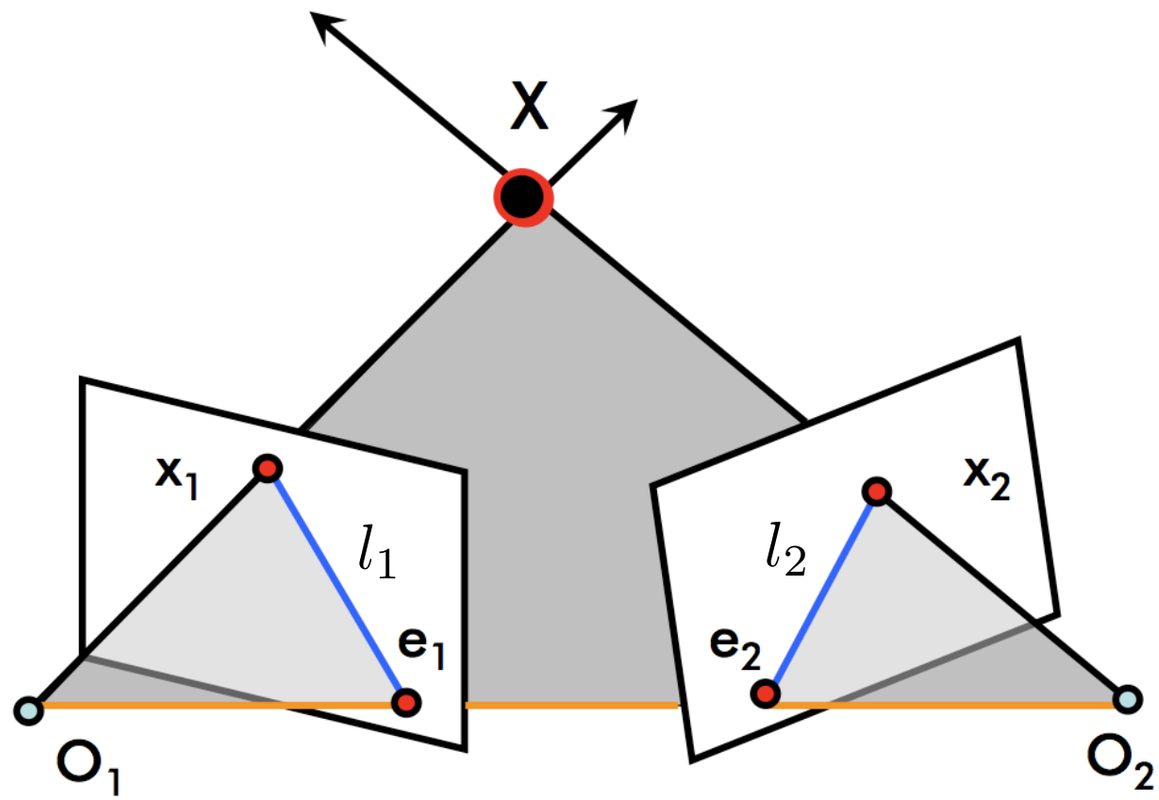
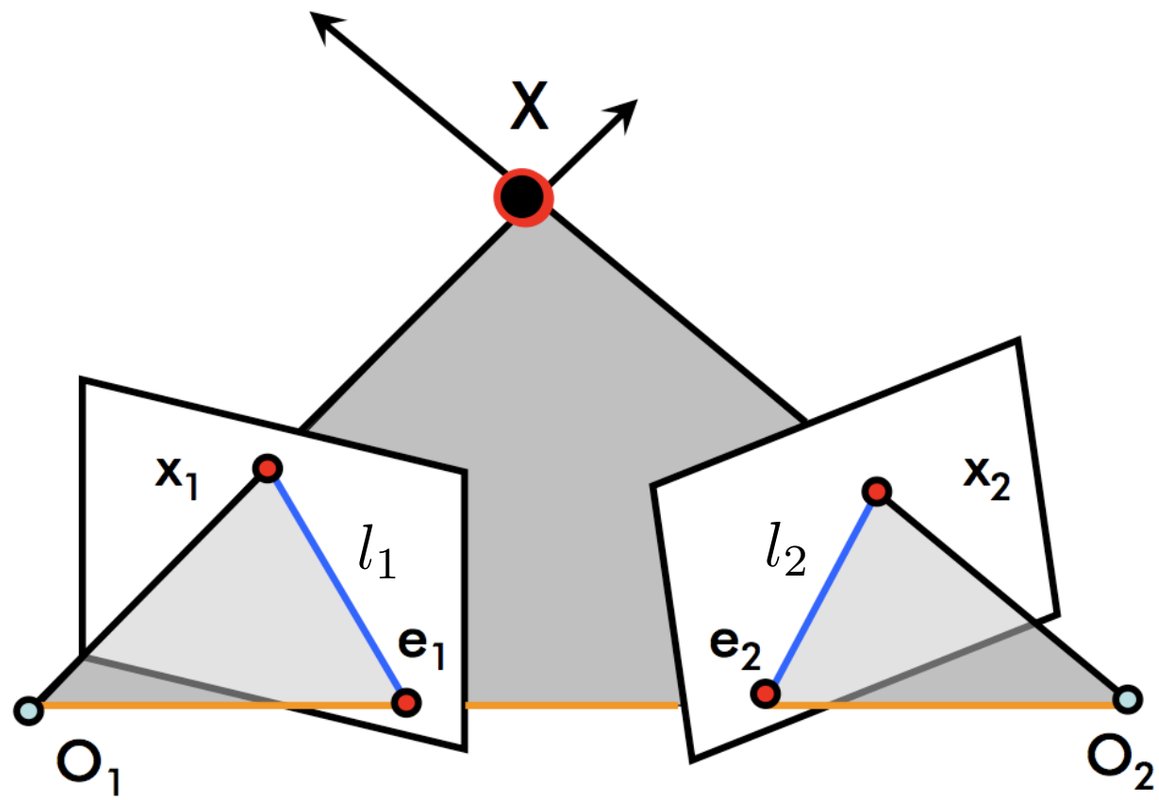


image 1: $$e_1^{\mathrm{T}} \left(F^{\mathrm{T}} x_2\right) = 0$$

image 2: $$e_2^{\mathrm{T}} \left(F x_1\right) = 0$$

Problem: Finding Epipoles

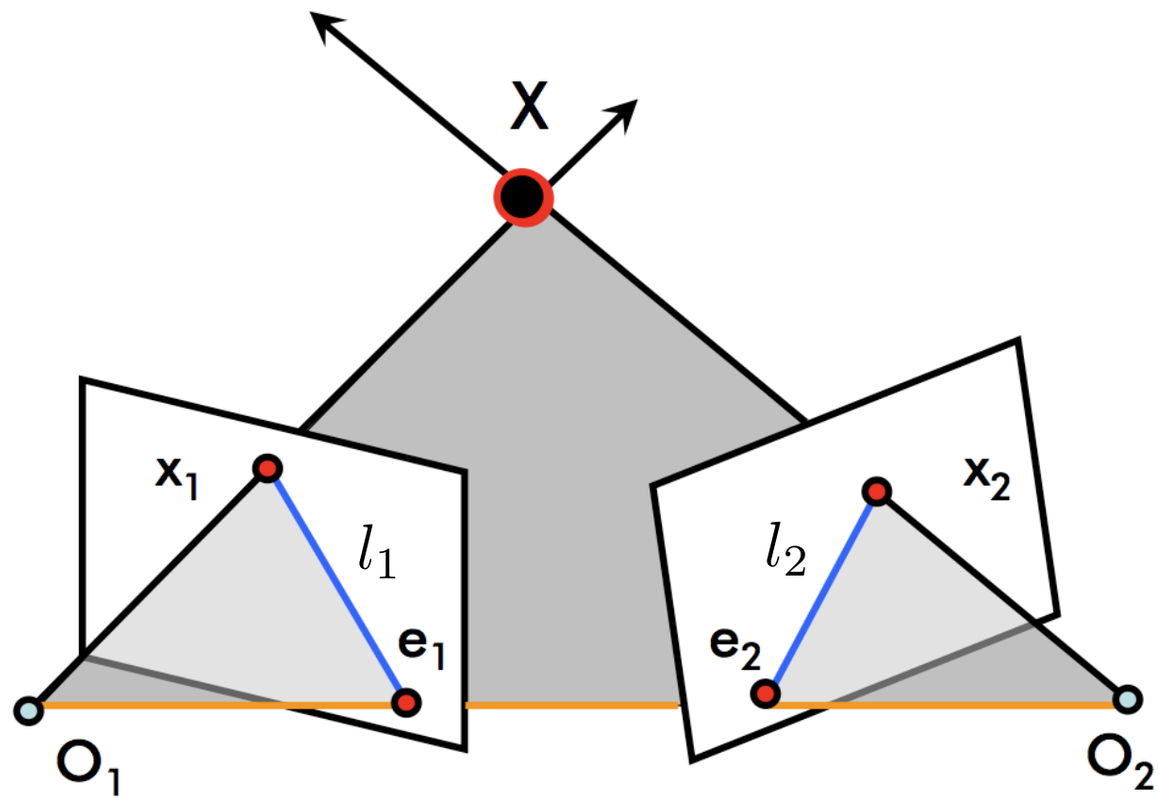


homogeneous system of equations:

image 1: $$Fe_1 = 0$$

image 2: $$F^{\mathrm{T}} e_2 = 0$$

Problem: Finding Epipoles

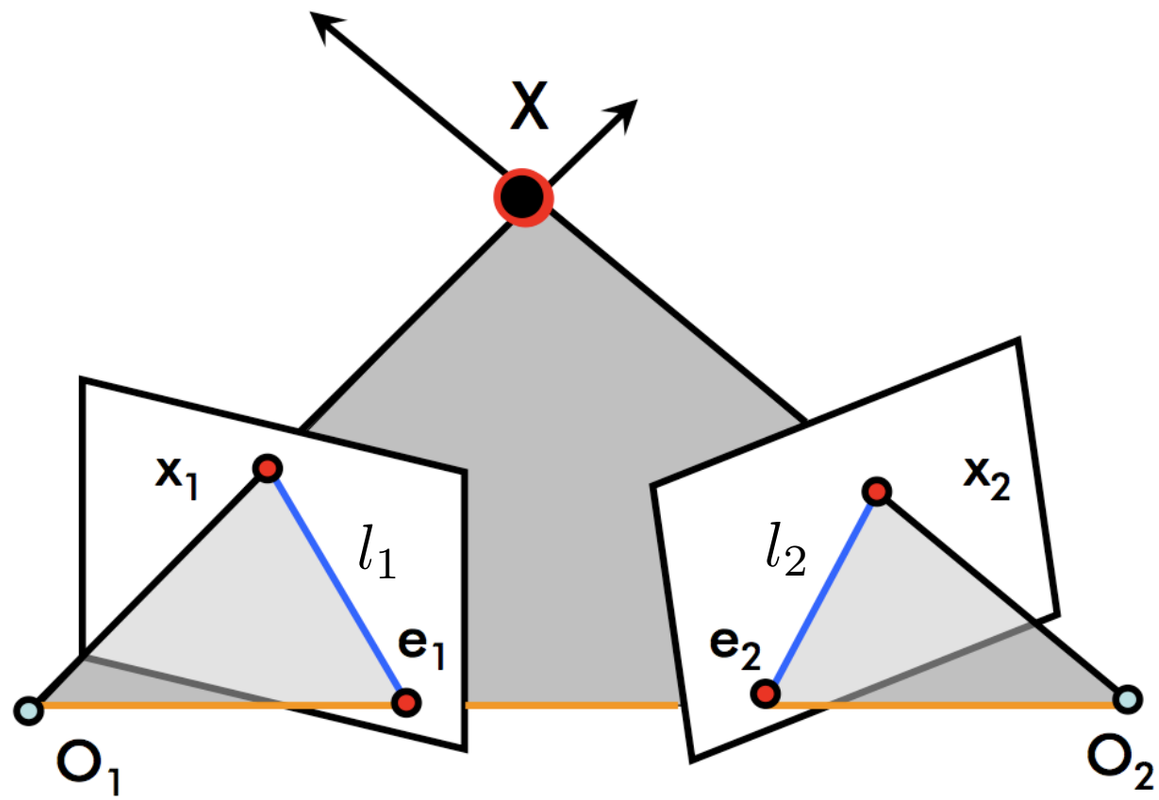


homogeneous system of equations:

image 1: $Fe_1 = 0 \quad \Rightarrow \quad \min_{e_1} \; Fe_1 \quad \text{s.t.} \quad \|e_1\|_2^2 = 1$

image 2: $F^{\mathrm{T}} e_2 = 0 \quad \Rightarrow \quad \min_{e_2} \; F^T e_2 \quad \text{s.t.} \quad \|e_2\|_2^2 = 1$

Problem: Finding Epipoles



homogeneous system of equations:

image 1: e_1 is the eigenvector of F

image 2: e_2 is the eigenvector of F^T