CS 556: Computer Vision

Lecture 19

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Computational Framework of Perceptual Grouping

Perceptual Grouping: Image Segmentation

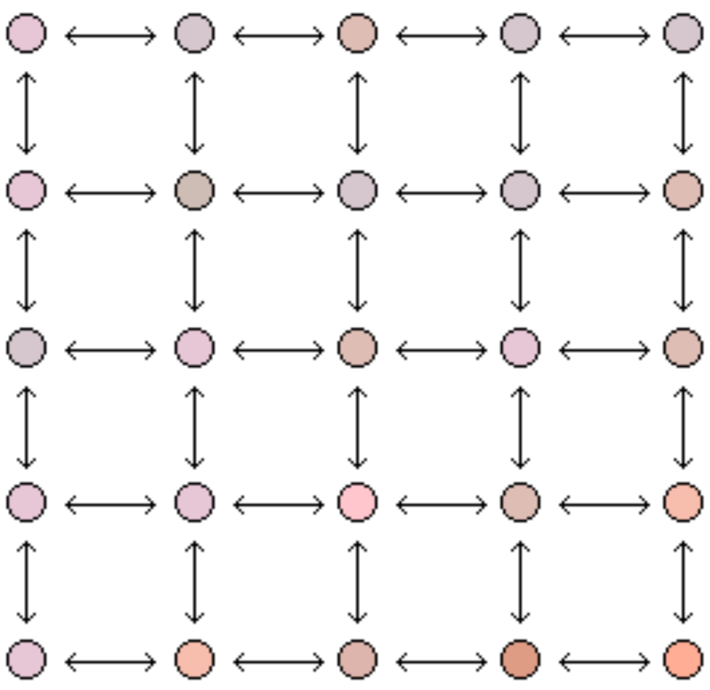
- Find contiguous clusters of pixels, so pixels within each cluster are
  - closer
  - more similar in terms of
    - color
    - texture
- Than pixels from different clusters

Example: N-Cuts Segmentation

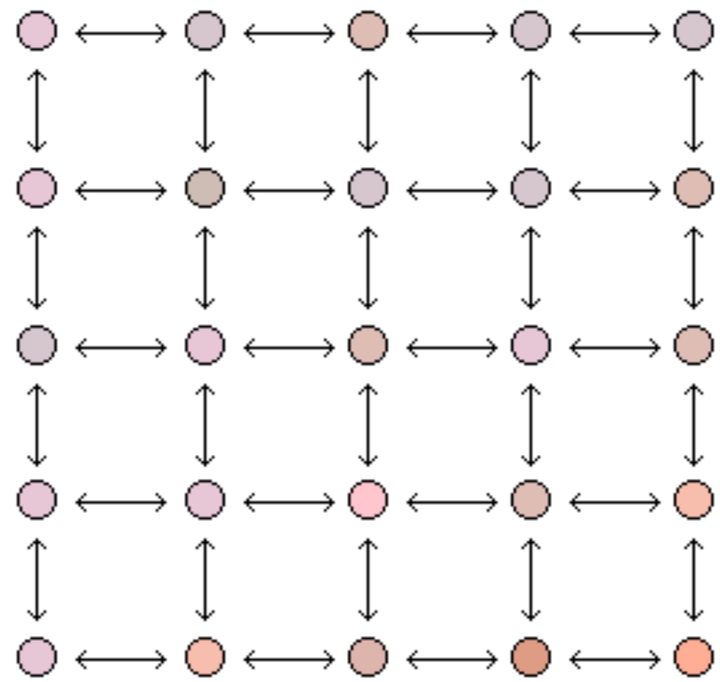
segments
or
regions

results of the Ncuts algorithm

Graph-based Clustering

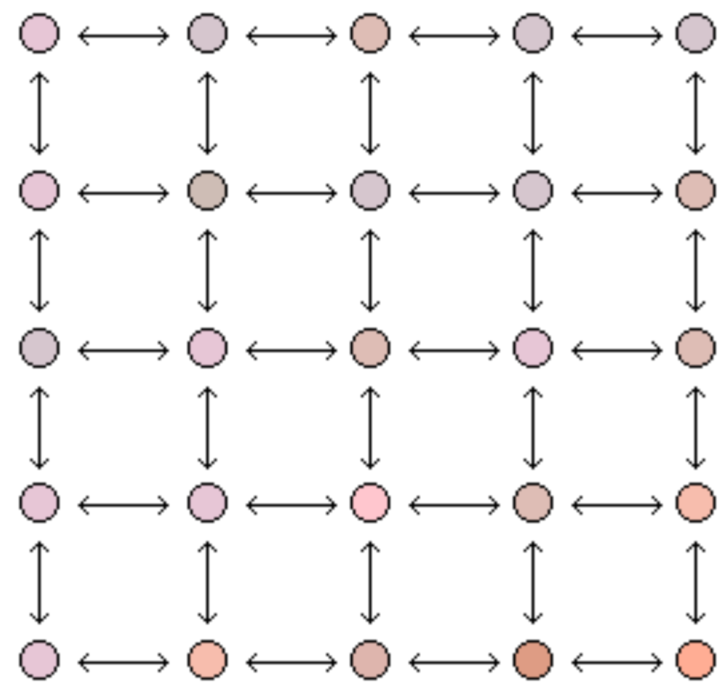


Graph-based Clustering



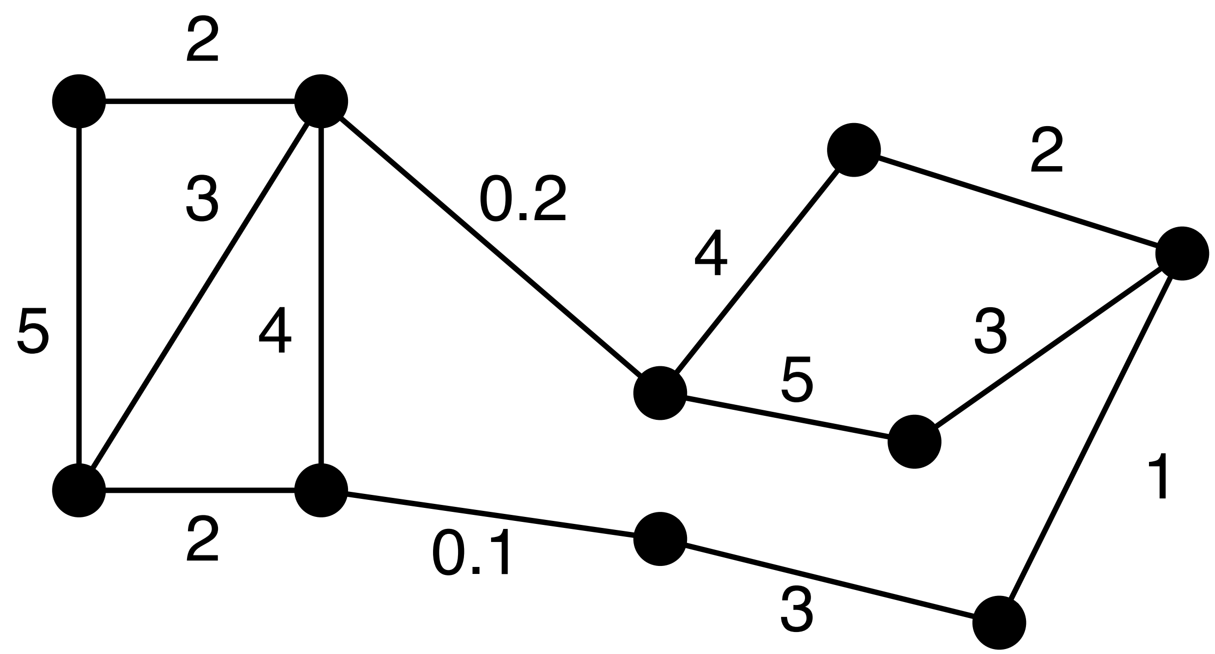
- Image elements are represented by a graph

Graph-based Clustering

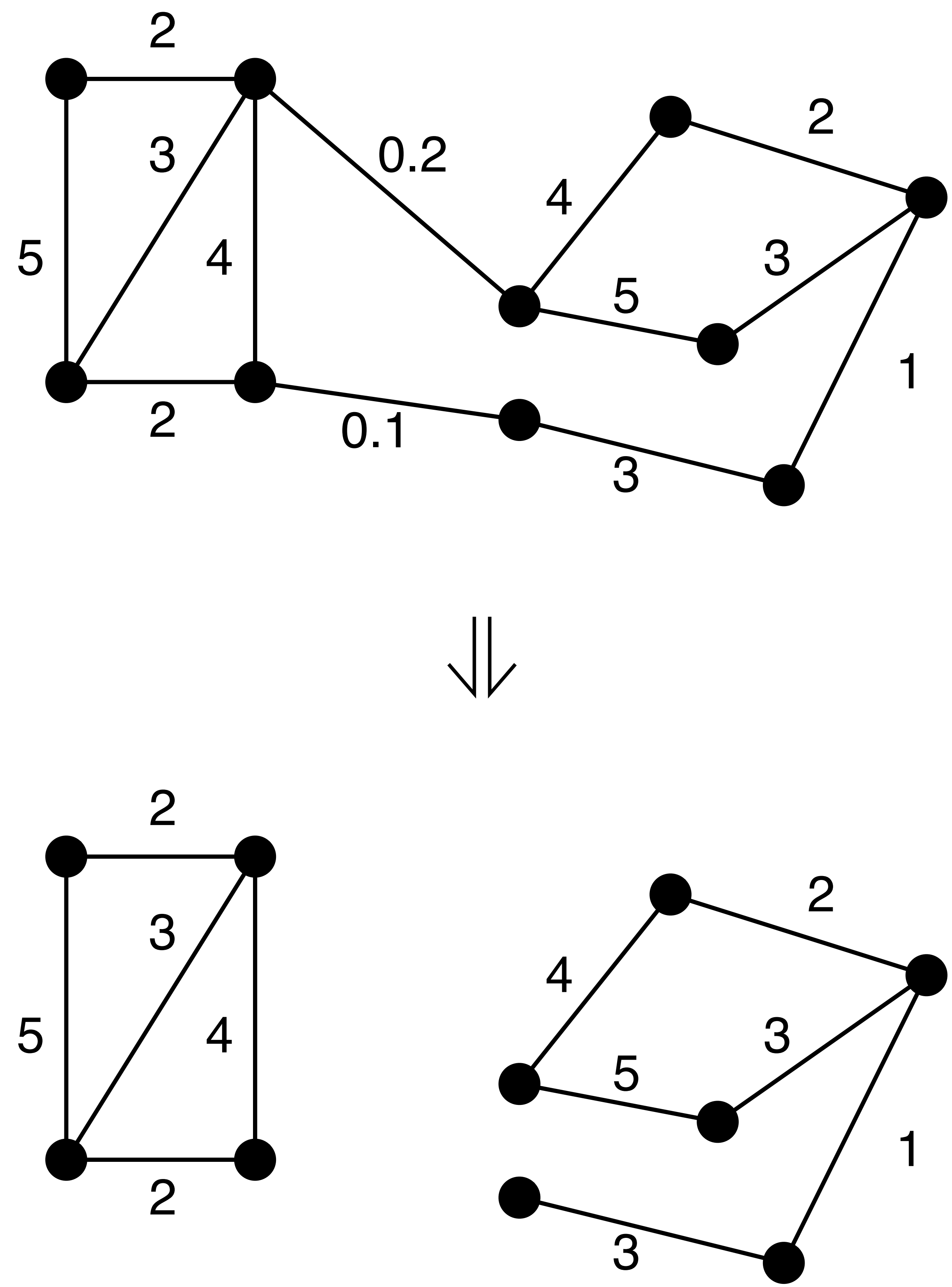


- Image elements are represented by a graph
- Clustering = Graph partitioning into subgraphs

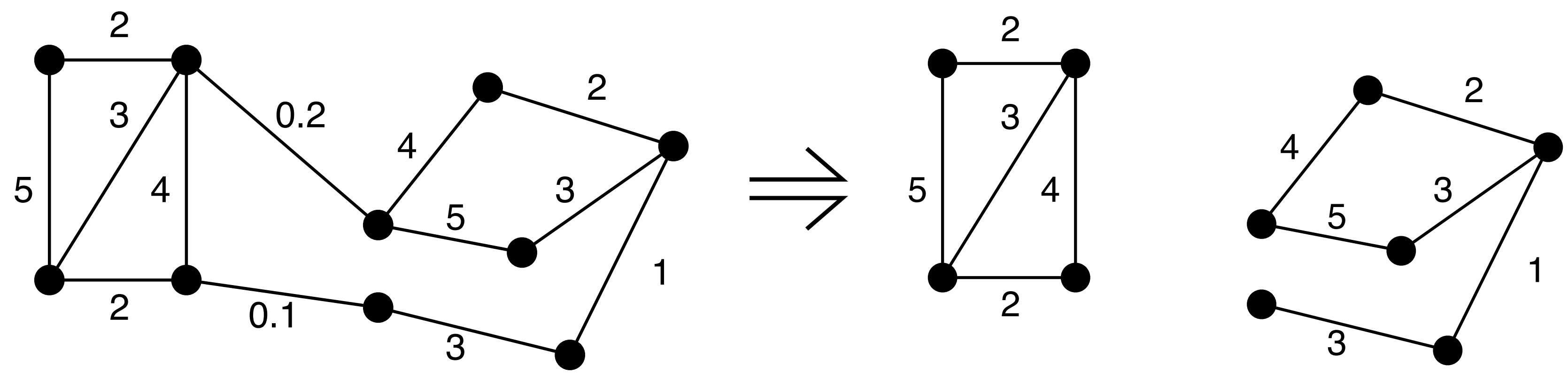
Major Goal: Graph Partitioning



Major Goal: Graph Partitioning



Example: Pixel Clustering



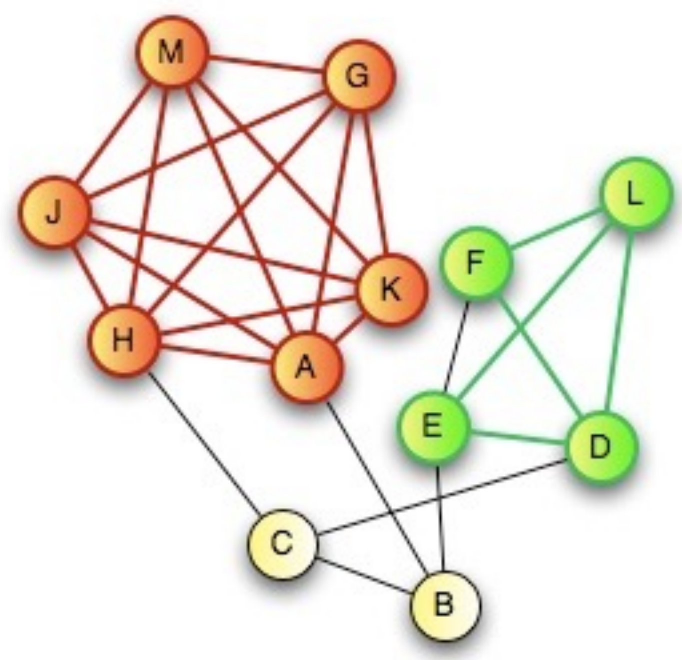
Find a cut, such that in one cluster all pixels are:

- neighbors
- more similar to one another than to pixels in the other cluster

Graph Terminology

- Clique
- Adjacency matrix
- Node degree
- Volume
- Cut
- Association

Clique



- A subgraph with all nodes fully connected
- Maximal clique
- Maximum weighted clique

Adjacency Matrix

$$A = \begin{array}{c} \\ i \end{array} \begin{bmatrix} & \overset{j}{\vdots} & \\ \cdots & a_{ij} & \\ & & \end{bmatrix}$$

$$a_{ij} = \exp\left(-\frac{1}{2\sigma^2}\|\mathrm{SIFT}_i - \mathrm{SIFT}_j\|^2\right)$$

Adjacency Matrix

$$A = \begin{array}{c} \\ \\ i \\ \\ \end{array} \left[\begin{array}{ccc} & \overset{j}{\vdots} & \\ \cdots & a_{ij} & \\ & & \end{array} \right]$$

- Affinities between pairs of nodes (i,j) in the graph

$$a_{ij} = \exp\left(-\frac{1}{2\sigma^2} \|\text{SIFT}_i - \text{SIFT}_j\|^2 \right)$$

Adjacency Matrix

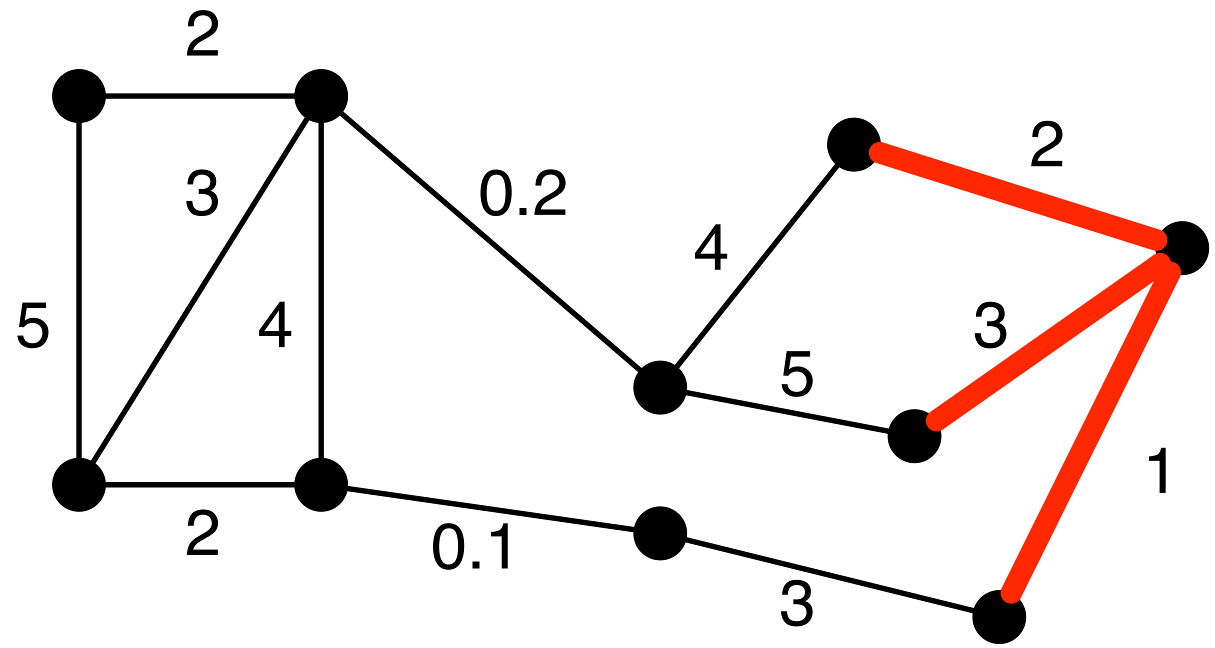
$$A = \; i \begin{bmatrix} & & \overset{j}{\vdots} & \\ & \cdots & a_{ij} & \\ & & & \end{bmatrix}$$

- Affinities between pairs of nodes (i,j) in the graph

- Example: Nodes = Pixels ⇒

$$a_{ij} = \exp\left(-\frac{1}{2\sigma^2}\|\mathrm{SIFT}_i - \mathrm{SIFT}_j\|^2\right)$$

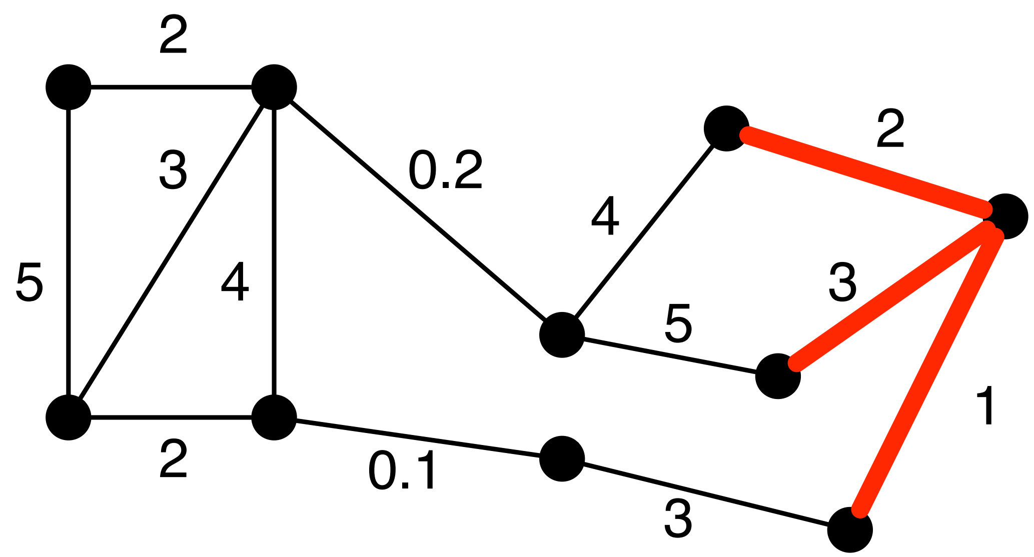
Node Degree



$$d_i = 6$$

$$d_i = \sum_j a_{ij}$$

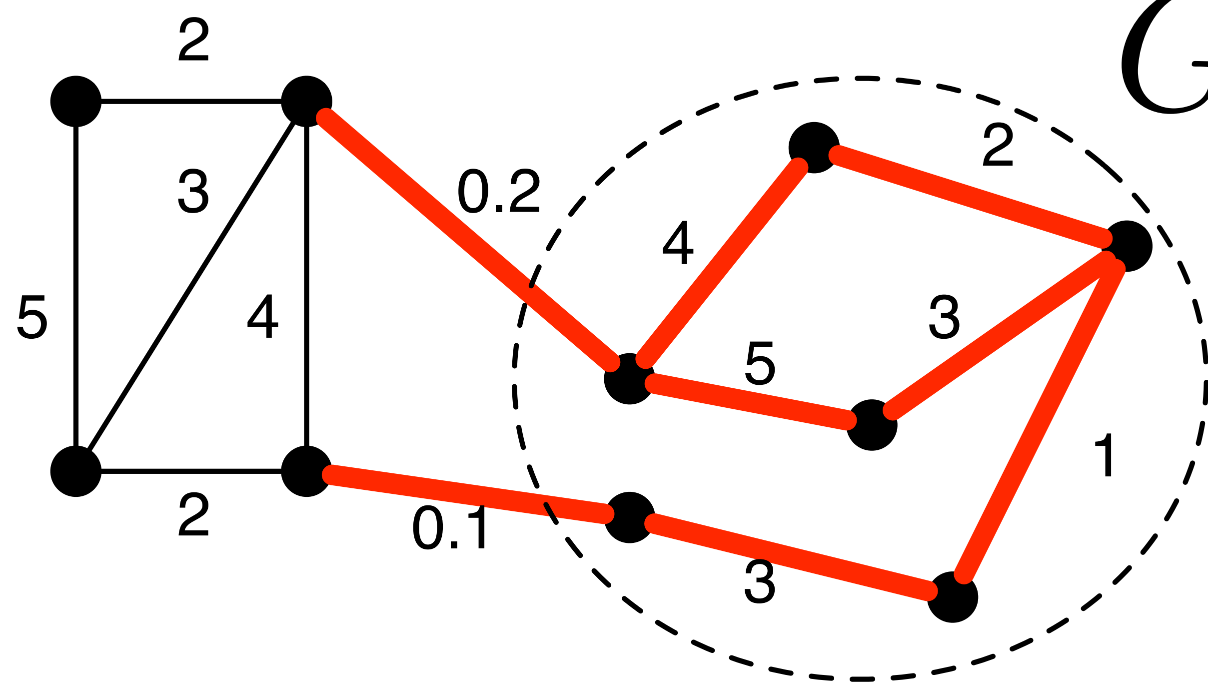
Node Degree



$$D = \begin{bmatrix} d_1 & 0 & 0 & \dots \\ 0 & d_2 & 0 & \\ \vdots & & & \\ \dots & 0 & 0 & d_N \end{bmatrix} \qquad \mathbf{1}_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} \leftarrow i$$

$$\Rightarrow \quad d_i = \mathbf{1}_i^{\mathrm{T}} \, D \, \mathbf{1}_i$$

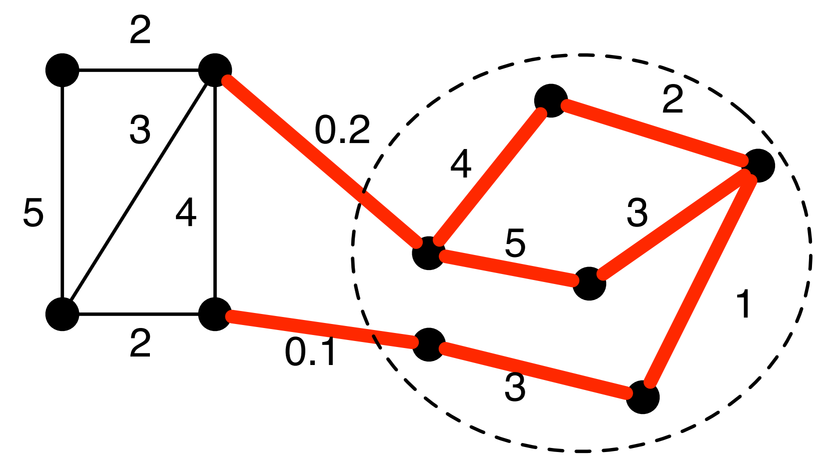
Volume of a Subgraph



$$G_1 \subset G$$

$$\mathrm{vol}(G_1) = \sum_{i \in G_1} d_i$$

Volume of a Subgraph

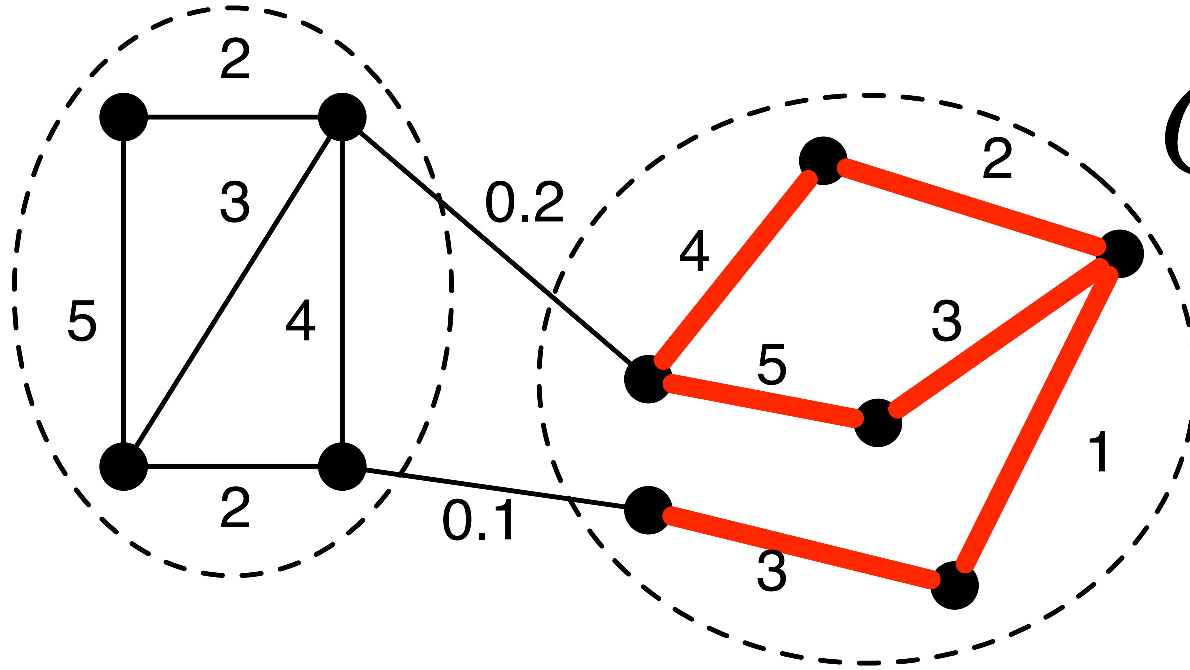


$$D = \begin{bmatrix} d_1 & 0 & 0 & \dots \\ 0 & d_2 & 0 & \\ \vdots & & & \\ \dots & 0 & 0 & d_N \end{bmatrix} \qquad \mathbf{1}_{G_1} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix}$$

G_1

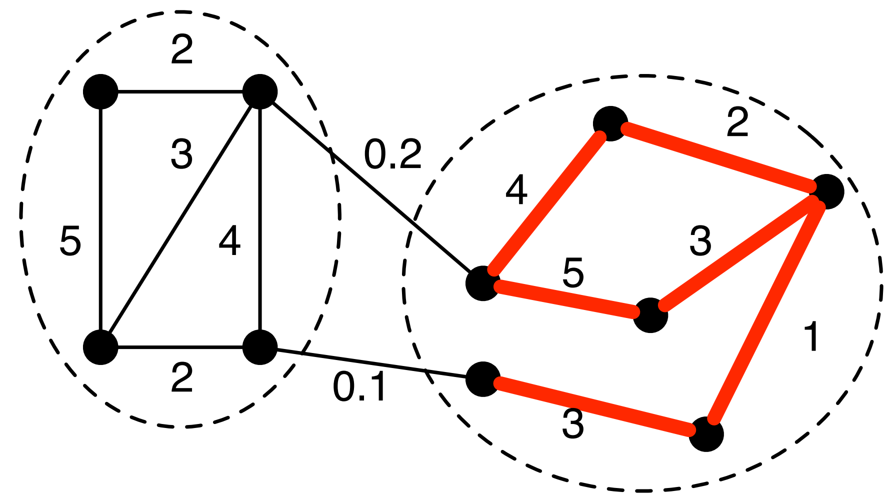
$$\Rightarrow \quad \mathrm{vol}(G_1) = \mathbf{1}_{G_1}^{\mathrm{T}} \; D \; \mathbf{1}_{G_1}$$

Association within a Subgraph



$$\text{assoc}(G_1) = \sum_{i,j \in G_1} a_{ij}$$

Association within a Subgraph



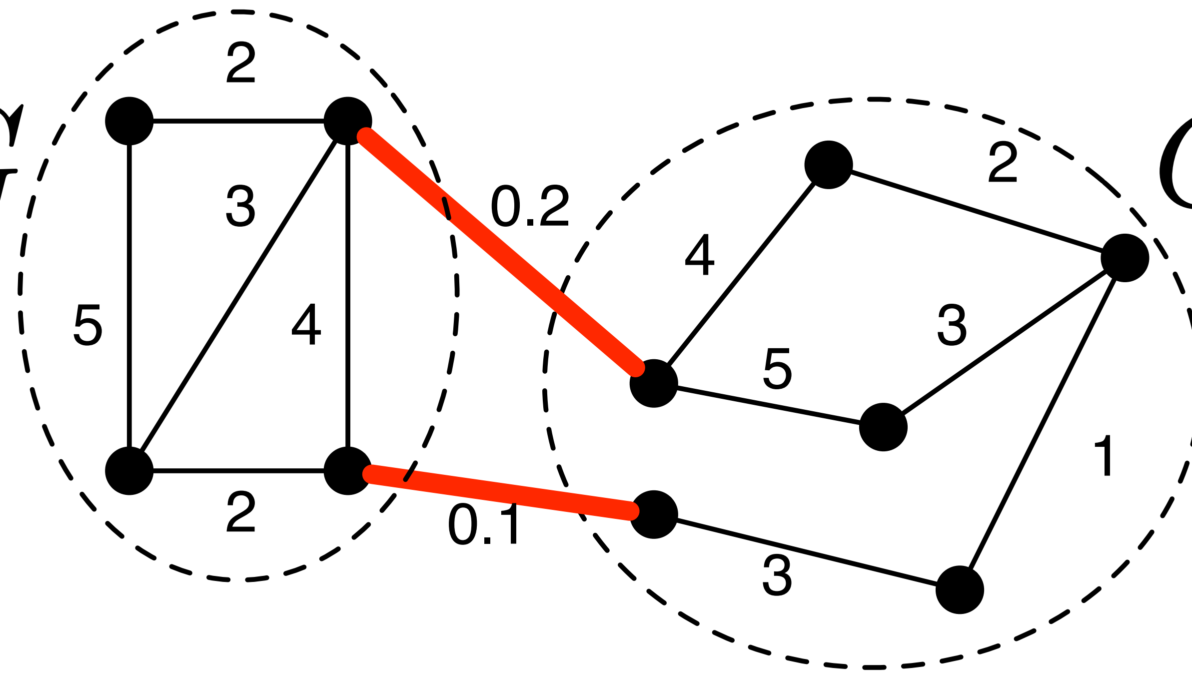
adjacency matrix

$$A = \begin{array}{c} \\ \\ i \\ \\ \end{array} \begin{array}{c} j \\ \left[\begin{array}{cc} & \vdots \\ \cdots & a_{ij} \\ & \end{array} \right] \end{array}$$

$$\mathrm{assoc}(G_1) = \sum_{i,j \in G_1} a_{ij} = \mathbf{1}_{G_1}^{\mathrm{T}} \; A \; \mathbf{1}_{G_1}$$

Cut between Two Graph Partitions

$G_2 \subset G$



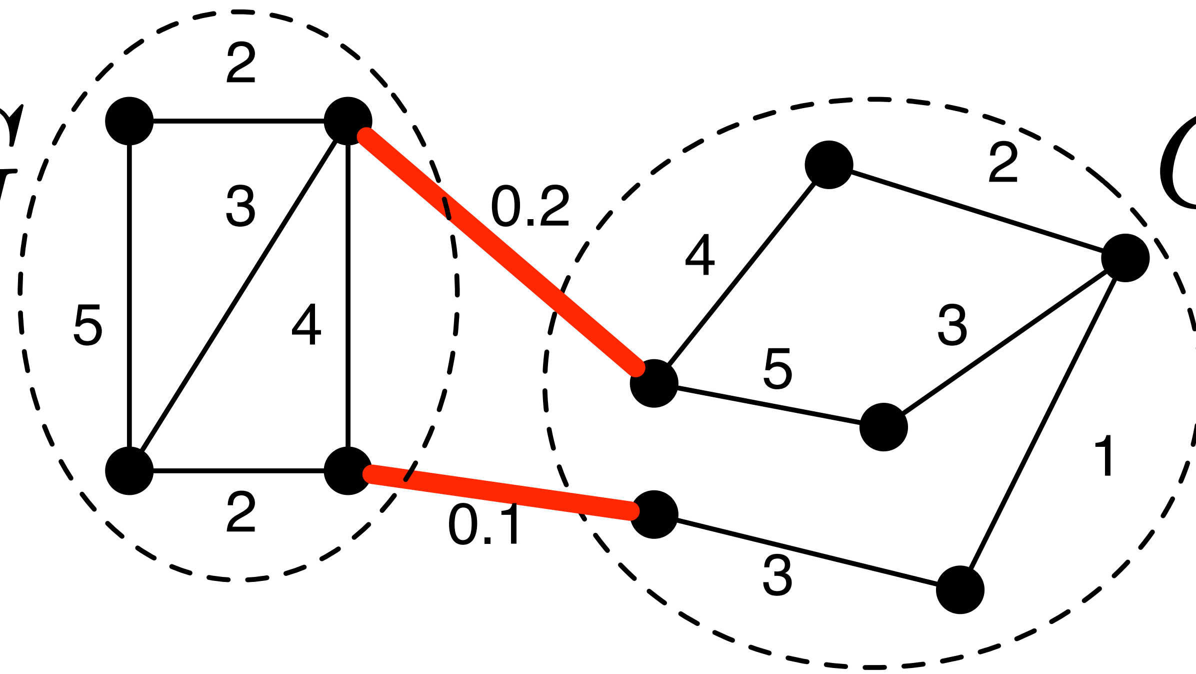
$G_1 \subset G$

$$G_1 \cap G_2 = \emptyset$$

$$\mathrm{cut}(G_1, G_2) = \sum_{i \in G_1, j \in G_2} a_{ij}$$

Cut between Two Graph Partitions

$G_2 \subset G$

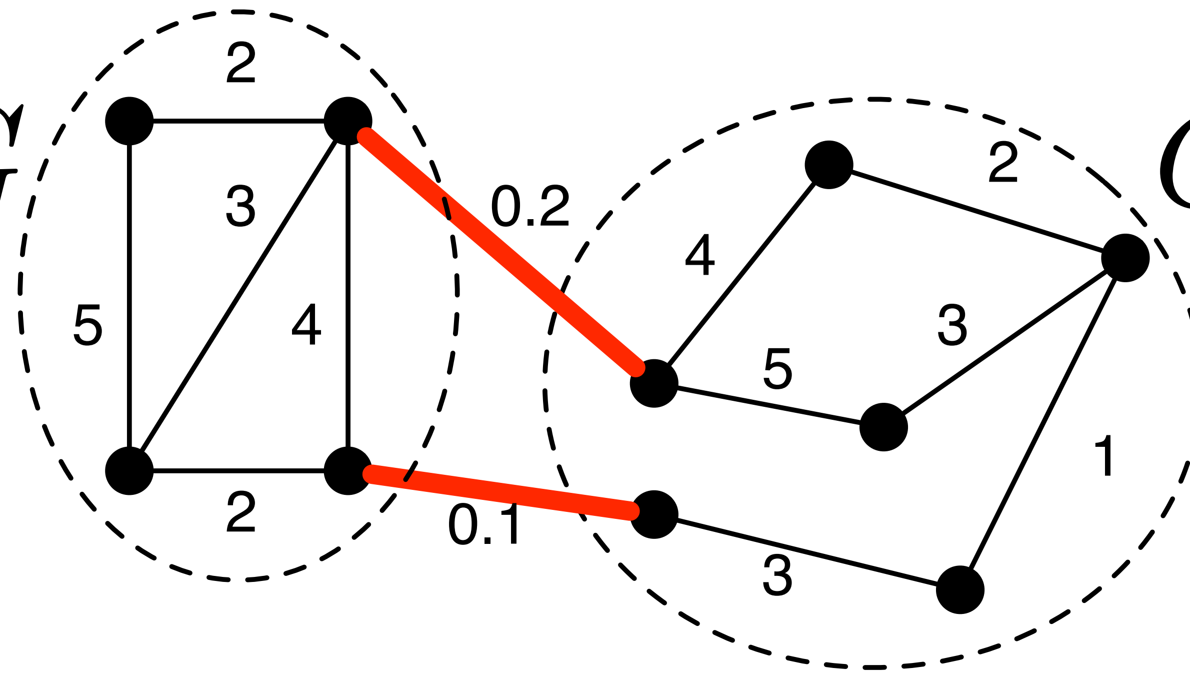


$G_1 \subset G$

$$G_1 \cap G_2 = \emptyset$$

$$\mathrm{cut}(G_2, G_1) = \sum_{i \in G_2, j \in G_1} a_{ij}$$

Cut between Two Graph Partitions

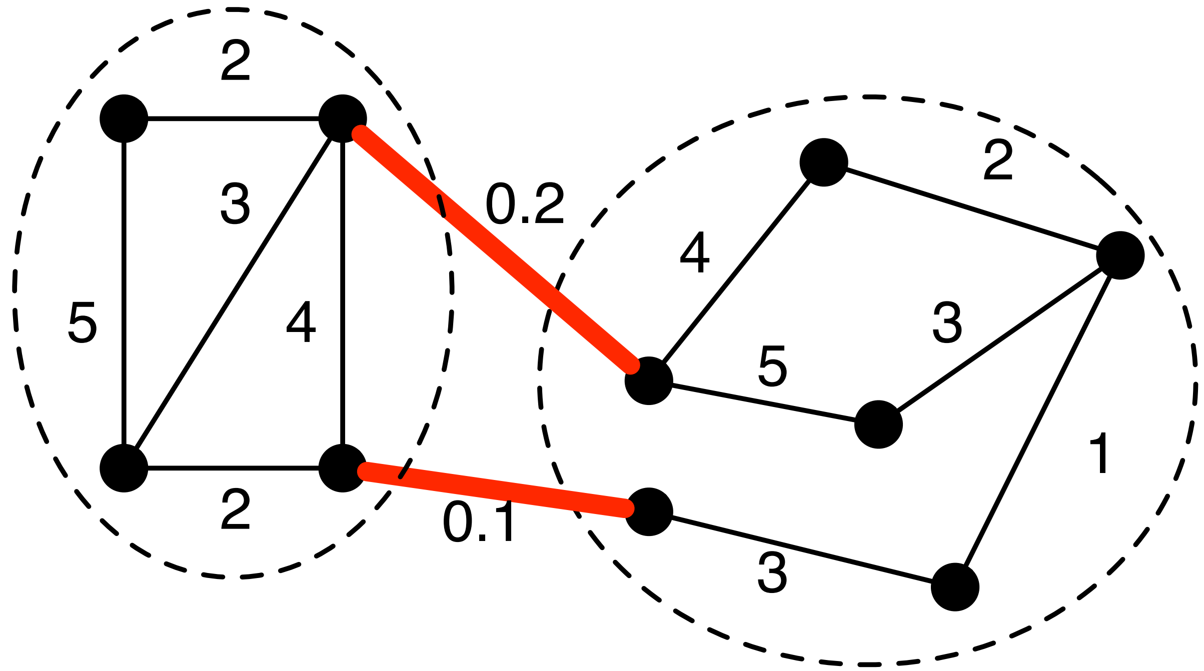
$G_2 \subset G$  $G_1 \subset G$

$$G_1 \cap G_2 = \emptyset$$

$$\mathrm{cut}(G_1, G_2) \;\neq\; \mathrm{cut}(G_2, G_1)$$

in general

Cut between Two Graph Partitions

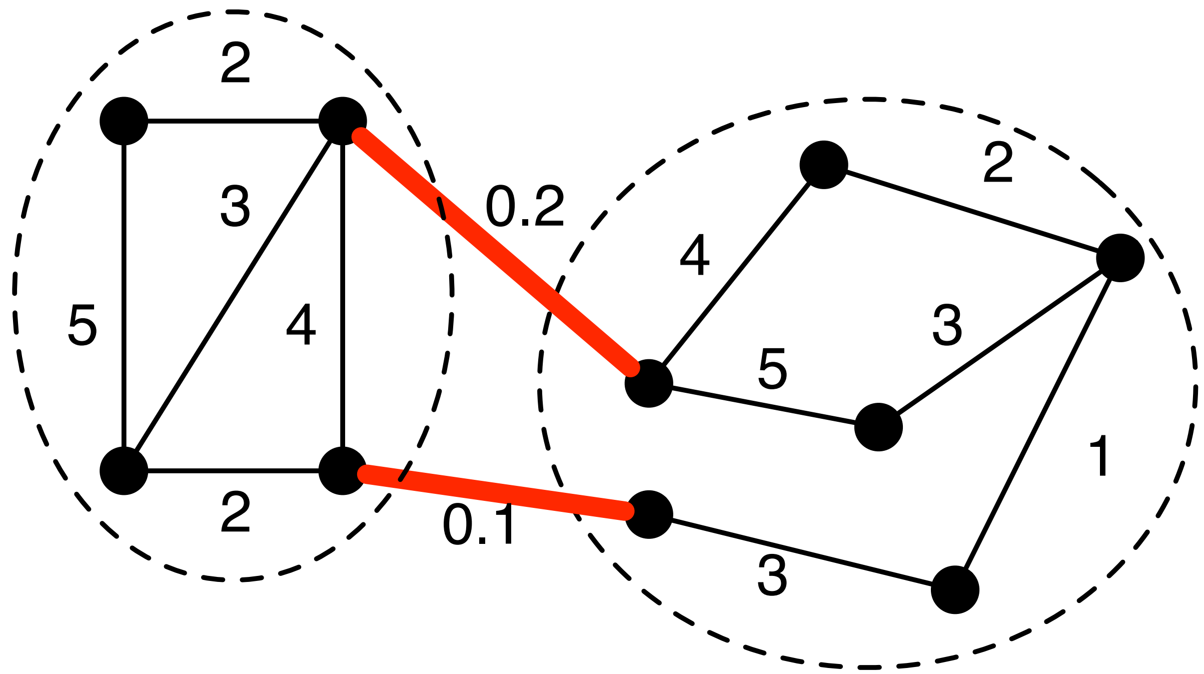


$$\mathrm{cut}(G_1, G_2) = \mathrm{vol}(G_1) - \mathrm{assoc}(G_1)$$

$$= \mathbf{1}_{G_1}^{\mathrm{T}} \; D \; \mathbf{1}_{G_1} - \mathbf{1}_{G_1}^{\mathrm{T}} \; A \; \mathbf{1}_{G_1}$$

$$= \mathbf{1}_{G_1}^{\mathrm{T}} \; (D - A) \; \mathbf{1}_{G_1}$$

Cut between Two Graph Partitions



$$\mathrm{cut}(G_2, G_1) = \mathbf{1}_{G_2}^{\mathrm{T}} \, (D - A) \, \mathbf{1}_{G_2}$$

$$= (\mathbf{1} - \mathbf{1}_{G_1})^{\mathrm{T}} \, (D - A) \, (\mathbf{1} - \mathbf{1}_{G_1})$$