CS 556: Computer Vision

Lecture 7

Prof. Sinisa Todorovic

sinisa@eecs.oregonstate.edu

OSU Oregon State University

Matching Cost of Two Features

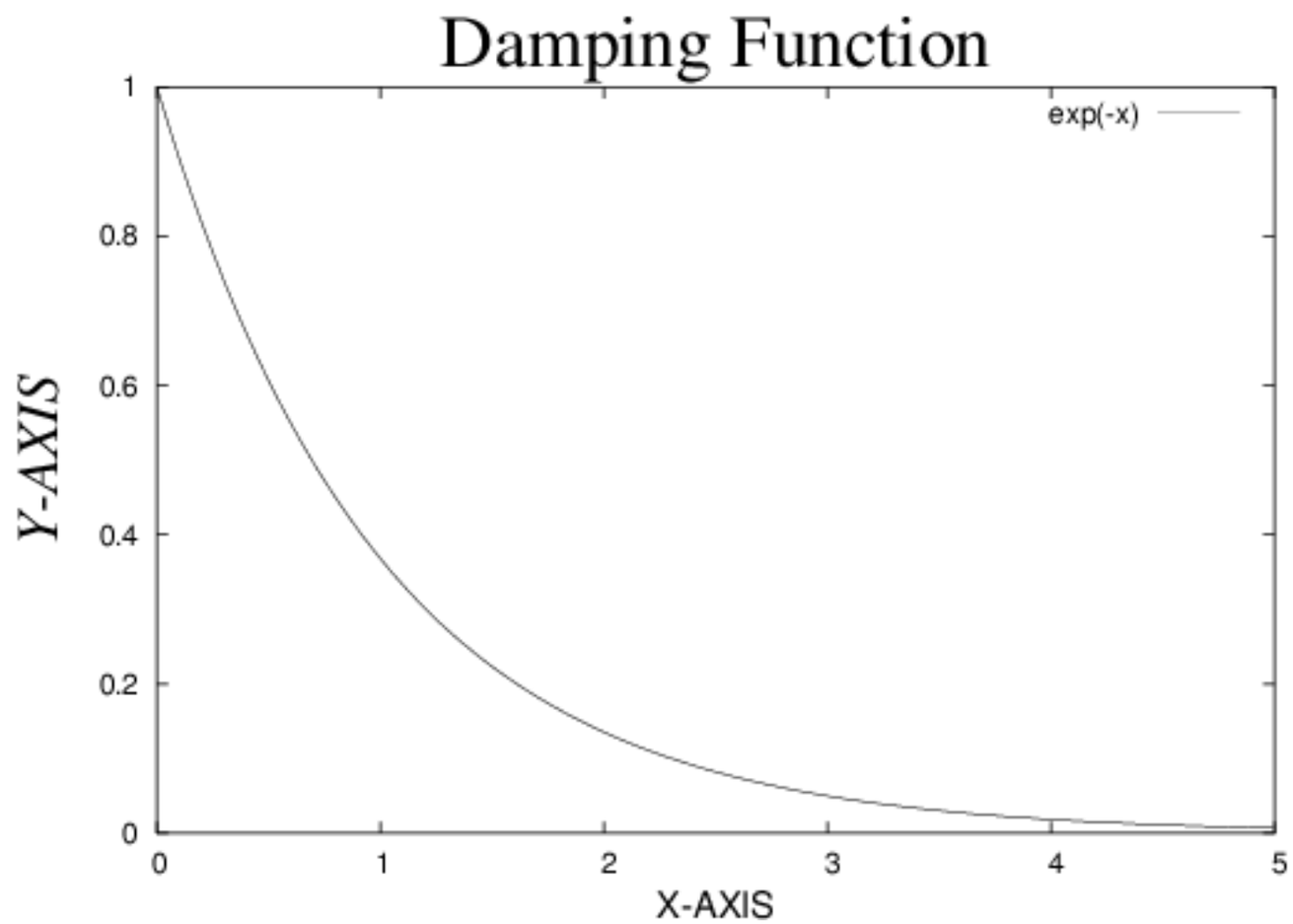
- Euclidean distance: $d(\mathbf{f_1}, \mathbf{f_2}) = \|\mathbf{f_1} - \mathbf{f_2}\|^2$

- Chi-squared distance: $d(\mathbf{f_1}, \mathbf{f_2}) = \sum_{\mathbf{i}} \frac{(\mathbf{f_1}(\mathbf{i}) - \mathbf{f_2}(\mathbf{i}))^2}{\mathbf{f_1}(\mathbf{i}) + \mathbf{f_2}(\mathbf{i})}$

- Hamming distance (only for binary features): $d(\mathbf{f_1}, \mathbf{f_2}) = \sum_{\mathbf{i}} \mathbf{1}(\mathbf{f_1}(\mathbf{i}) \neq \mathbf{f_2}(\mathbf{i}))$

Matching Similarity of Two Features

$$s(\mathbf{f}, \mathbf{f}') = \exp(-d(\mathbf{f}, \mathbf{f}'))$$



Matching Formulation

Given two sets of descriptors to be matched

$V = \{\mathbf{f_1}, \mathbf{f_2}, \ldots, \mathbf{f_n}\}$ and $V' = \{\mathbf{f'_1}, \mathbf{f'_2}, \ldots, \mathbf{f'_{n'}}\}$

Find the legal mapping $M \in \mathcal{M}$

$$M = \{(\mathbf{f}, \mathbf{f}') : (\mathbf{f}, \mathbf{f}') \subseteq V \times V'\}$$

Which maximizes the total similarity of matching

$$\hat{M} = \max_{M \in \mathcal{M}} \sum_{(\mathbf{f}, \mathbf{f}') \in M} \boldsymbol{s}(\mathbf{f}, \mathbf{f}')$$

Linearization

Linearization by introducing an indicator

$$x(\mathbf{f}, \mathbf{f}') = \begin{cases} 1 & , \quad (\mathbf{f}, \mathbf{f}') \in M \\ 0 & , \quad (\mathbf{f}, \mathbf{f}') \notin M \end{cases}$$

Total similarity of matching:

$$\sum_{(\mathbf{f},\mathbf{f}')\in M} s(\mathbf{f}, \mathbf{f}') = \sum_{(\mathbf{f},\mathbf{f}')\in M} s(\mathbf{f}, \mathbf{f}')x(\mathbf{f}, \mathbf{f}') = \sum_{(\mathbf{f},\mathbf{f}')\in V\times V'} s(\mathbf{f}, \mathbf{f}')x(\mathbf{f}, \mathbf{f}')$$

Linearization

Instead of: $$\hat{M} = \max_{M \in \mathcal{M}} \sum_{(\mathbf{f},\mathbf{f}') \in M} s(\mathbf{f}, \mathbf{f}')$$

we get:

$$\hat{\mathbf{x}} = \max_{\mathbf{x} \in \{0,1\}^{n \cdot n'}} \sum_{(\mathbf{f},\mathbf{f}') \in V \times V'} s(\mathbf{f}, \mathbf{f}') x(\mathbf{f}, \mathbf{f}') = \max_{\mathbf{x} \in \{0,1\}^{n \cdot n'}} \mathbf{s}^\top \mathbf{x}$$

Matching Formulation

$$\hat{\mathbf{x}} = \max_{\mathbf{x} \in \{0,1\}^{n \cdot n'}} \mathbf{s}^\top \mathbf{x}$$

$$\Downarrow$$

$$\hat{\mathbf{x}} = \mathbf{1}$$ trivial solution

$$\Downarrow$$

we need to constrain the formulation

Relaxation

$$\hat{\mathbf{x}} = \max_{\boxed{\mathbf{x} \in \{0,1\}^{n \cdot n'}}} \mathbf{s}^\top \mathbf{x}$$

Relaxation

$$\hat{\mathbf{x}} = \max_{\boxed{\mathbf{x} \in [0,1]^{n \cdot n'}}} \mathbf{s}^\top \mathbf{x}$$

Matching Formulation

$$\hat{\mathbf{x}} = \max_{\mathbf{x} \in [0,1]^{n \cdot n'}} \mathbf{s}^\top \mathbf{x}$$

subject to:

$$\|\mathbf{x}\|_\ell = 1$$

$$\ell = \{0, 1, 2, \infty\}$$

Matching Formulation

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$$\|\mathbf{x}\|_\ell = \left[\sum_i |x_i|^\ell \right]^{1/\ell}, \quad \ell = 1, 2, \ldots$$

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Matching Formulation

$$\hat{\mathbf{x}} = \max \quad \mathbf{s}^\top \mathbf{x}$$

subject to:

$$\text{every} \quad x(\mathbf{f}, \mathbf{f}') \geq 0$$

$$\|\mathbf{x}\|_\ell = 1$$

$$\ell = \{0, 1, 2, \infty\}$$

Matching Formulation: Special Case

$$\hat{\mathbf{x}} = \max \;\; \mathbf{s}^\top \mathbf{x}, \;\; \text{every} \;\; s(\mathbf{f}, \mathbf{f}') \geq 0$$

subject to:

$$\text{every} \;\; x(\mathbf{f}, \mathbf{f}') \geq 0$$

$$\|\mathbf{x}\|_2 = 1$$

closed-form solution

for $\mathbf{s} \geq \mathbf{0} \quad \Rightarrow \quad \hat{\mathbf{x}} = \dfrac{\mathbf{s}}{\|\mathbf{s}\|_2^2}$

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$\Downarrow$ closed-form solution

for $\mathbf{s} \geq \mathbf{0}$ $\Rightarrow$ $\hat{\mathbf{x}} = \dfrac{\mathbf{s}}{\|\mathbf{s}\|_2^2}$

MATLAB -- Solving Linear Programming Problems

http://www.mathworks.com/help/optim/ug/linprog.html

$$\min_{\mathbf{x}} \quad \mathbf{f}^T\mathbf{x}$$

such that:

$$A\mathbf{x} \leq \mathbf{b}$$

$$A_{eq}\mathbf{x} = \mathbf{b}_{eq}$$

$$\mathbf{b}_l \leq \mathbf{x} \leq \mathbf{b}_u$$

MATLAB -- Solving Linear Programming Problems

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```
x = linprog(f,A,b);

x = linprog(f,A,b,Aeq,beq);

x = linprog(f,A,b,Aeq,beq,lb,ub);

x = linprog(f,A,b,Aeq,beq,lb,ub,x0);
```