Matching Cost of Two Features

- Euclidean distance:
  \[ d(f_1, f_2) = \| f_1 - f_2 \|^2 \]

- Chi-squared distance:
  \[ d(f_1, f_2) = \sum_i \frac{(f_1(i) - f_2(i))^2}{f_1(i) + f_2(i)} \]

- Hamming distance (only for binary features):
  \[ d(f_1, f_2) = \sum_i 1(f_1(i) \neq f_2(i)) \]
Matching Similarity of Two Features

\[ s(f, f') = \exp(-d(f, f')) \]
Matching Formulation

Given two sets of descriptors to be matched

\[ V = \{ f_1, f_2, \ldots, f_n \} \quad \text{and} \quad V' = \{ f'_1, f'_2, \ldots, f'_n \} \]

Find the legal mapping \( M \in \mathcal{M} \)

\[ M = \{ (f, f') : (f, f') \subseteq V \times V' \} \]

Which maximizes the total similarity of matching

\[ \hat{M} = \max_{M \in \mathcal{M}} \sum_{(f, f') \in M} s(f, f') \]
Linearization

Linearization by introducing an indicator

\[ x(f, f') = \begin{cases} 
1 & , \quad (f, f') \in M \\
0 & , \quad (f, f') \notin M 
\end{cases} \]

Total similarity of matching:

\[
\sum_{(f, f') \in M} s(f, f') = \sum_{(f, f') \in M} s(f, f') x(f, f') = \sum_{(f, f') \in V \times V'} s(f, f') x(f, f')
\]
Linearization

Instead of:  \[ \hat{M} = \max_{M \in \mathcal{M}} \sum_{(f, f') \in M} s(f, f') \]

we get:

\[ \hat{x} = \max_{x \in \{0,1\}^{n \cdot n'}} \sum_{(f, f') \in V \times V'} s(f, f') x(f, f') = \max_{x \in \{0,1\}^{n \cdot n'}} s^\top x \]
Matching Formulation

\[ \hat{x} = \max_{x \in \{0,1\}^{n \cdot n'}} s^\top x \]

\[ \hat{x} = 1 \quad \text{trivial solution} \]

we need to constrain the formulation
Relaxation

\[ \hat{x} = \max_{x \in \{0,1\}^{n \cdot n'}} s^\top x \]
Relaxation

\[ \hat{x} = \max_{x \in [0,1]^{n \cdot n'}} s^\top x \]
Matching Formulation

\[ \hat{x} = \max_{x \in [0,1]^{n \cdot n'}} s^\top x \]

subject to:

\[ \|x\|_\ell = 1 \]

\[ \ell = \{0, 1, 2, \infty\} \]
Matching Formulation

\[ \hat{x} = \max_{\mathbf{x} \in [0,1]^{n \cdot n'}} \mathbf{s}^\top \mathbf{x} \]

subject to:

\[ \| \mathbf{x} \|_\ell = 1 \]

\[ \| \mathbf{x} \|_\ell = \left[ \sum_i |x_i|^\ell \right]^{1/\ell}, \quad \ell = 1, 2, \ldots \]
Matching Formulation

\[ \hat{x} = \max \ s^\top x \]
\[ x \in [0,1]^{n \cdot n'} \]

subject to:

\[ \|x\|_\ell = 1 \]

\[ \|x\|_\ell = \left[ \sum_{i} |x_i|^\ell \right]^{1/\ell} \]
\[ \ell = 1, 2, \ldots \]
Matching Formulation

\[ \hat{x} = \max_{x \in [0,1]^{n \times n'}} s^T x \]

subject to:

\[ \|x\|_\ell = 1 \]

\[ \ell = \{0, 1, 2, \infty\} \]
Matching Formulation

\[ \hat{x} = \max \ s^\top x \]

subject to:

\[ \text{every } x(f, f') \geq 0 \]

\[ \|x\|_\ell = 1 \]

\[ \ell = \{0, 1, 2, \infty\} \]
Matching Formulation: Special Case

\[
\hat{x} = \max s^\top x, \quad \text{every } s(f, f') \geq 0
\]

subject to:

\[
\text{every } x(f, f') \geq 0
\]

\[
\|x\|_2 = 1
\]

closed-form solution

for \( s \geq 0 \) \( \Rightarrow \)

\[
\hat{x} = \frac{s}{\|s\|_2^2}
\]
Matching Formulation: Special Case

\[ \hat{x} = \max s^\top x, \text{ every } s(f, f') \geq 0 \]

subject to:

\[ \text{every } x(f, f') \geq 0 \]

\[ \|x\|_2 = 1 \]

closed-form solution

\[ \text{for } s \geq 0 \quad \Rightarrow \quad \hat{x} = \frac{s}{\|s\|_2^2} \]
MATLAB -- Solving Linear Programming Problems


\[
\min_{x} \quad f^T x
\]

such that:

\[
Ax \leq b
\]

\[
A_{eq}x = b_{eq}
\]

\[
b_l \leq x \leq b_u
\]
MATLAB -- Solving Linear Programming Problems


\[
x = \text{linprog}(f, A, b);
\]

\[
x = \text{linprog}(f, A, b, Aeq, beq);
\]

\[
x = \text{linprog}(f, A, b, Aeq, beq, lb, ub);
\]

\[
x = \text{linprog}(f, A, b, Aeq, beq, lb, ub, x0);
\]