ECE468: EXAM 1

NAME: ____

INSTRUCTIONS

- This is a 45 minute exam containing **THREE** problems: 1.1–1.5; 2.1–2.4; 3.1–3.2
- For the exam, you may use the textbook, one letter-size crib sheet, calculator, and pens/pencils
- Cheating during the exam will result in a failing grade for the entire course

Problem	Max points	Earned points
1.1	5pts	
1.2	30pts	
1.3	10pts	
1.4	20pts	
1.5	5pts	
2.1	20pts	
2.2	20pts	
2.3	5pts	
3.1	10pts	
3.2	15pts	
TOTAL	140pts	

$$f(x,y) \longrightarrow \boxed{H(u,v)} \longrightarrow g(x,y)$$

Fig. 1. A discrete-space system.

Problem 1 – (65pts)

Consider a discrete-space system, shown in Fig. 1, where $f(x,y)$ is input, $g(x,y)$ is output, and $H(u,v)$ is the filter transfer function in the frequency domain, defined as

$$H(u,v) = \frac{1}{2}\left[\cos(2\pi u/M) + \cos(2\pi v/N)\right], \quad u = 0, 1, \ldots, M-1, \quad v = 0, 1, \ldots, N-1\ .$$

The input image is corrupted by additive, stationary noise, $\eta(x,y)$, $f(x,y) = f_0(x,y) + \eta(x,y)$. The expected value and variance of noise are μ and σ^2, respectively. The goal of this discrete-space system is to filter out noise, and obtain the best estimate of $f_0(x,y)$.

1.1. (5pts)

Is $H(u,v)$ a lowpass, highpass, bandpass, or bandreject filter? Explain why?

(Hint: Roughly estimate how $|H(u,v)|$ changes for $u = 0, 1, \ldots, M-1,\ \ v = 0, 1, \ldots, N-1$.)

1.2. (30pts)

Compute nine elements of the 3×3 spatial filter $h(x, y)$ that represents a spatial-domain equivalent to $H(u, v)$. The convolution of this spatial filter and input image $f(x, y)$ produces output image $g(x, y)$=$f(x, y) * h(i, j)$.

$$h(x,y) = \begin{bmatrix} h(-1,-1) & h(-1,0) & h(-1,1) \\ h(0,-1) & h(0,0) & h(0,1) \\ h(1,-1) & h(1,0) & h(1,1) \end{bmatrix} =?$$

1.3. (10pts)

Compute the expected value and variance of the input image $f(x, y)$.

(Hint: $E\{f(x, y)\} = E\{f_0(x, y) + \eta(x, y)\}$.)

1.4. (20pts)

Compute the expected value and variance of the output image $g(x, y)$.

(Hint: Noise $\eta(x, y)$ is stationary, and $E\{g(x, y)\} = E\{\sum_{i=-1}^{1} \sum_{j=-1}^{1} h(i, j) f(i - x, j - y)\}$.)

1.5. (5pts)

Explain formally whether filtering with

$$H(u,v) = \frac{1}{2}\left[\cos(2\pi u/M) + \cos(2\pi v/N)\right], \quad u = 0, 1, \ldots, M-1, \quad v = 0, 1, \ldots, N-1$$

reduces additive noise in the image, by comparing your results from 1.3 and 1.4. Simple answers, like "yes" or "no", without any additional explanation, will be interpreted as random guessing, and will NOT earn you points.

Problem 2 – (45pts)

Consider the following 3×4 image $f(x, y)$ whose pixels take values in $\{0, 1, 2, \ldots, 7\}$:

$$f(x,y) = \begin{bmatrix} 2 & 2 & 1 & 3 \\ 0 & 2 & 1 & 2 \\ 1 & 2 & 0 & 1 \end{bmatrix}$$

2.1. (20pts)

Find transformation $g_1(x, y) = T_1(f(x, y))$ that will equalize the pixel histogram of $f(x, y)$. Plot $T_1(i)$, $i = 0, 1, \ldots, 7$, and label the axes and main points of the plot.

2.2. (20pts)

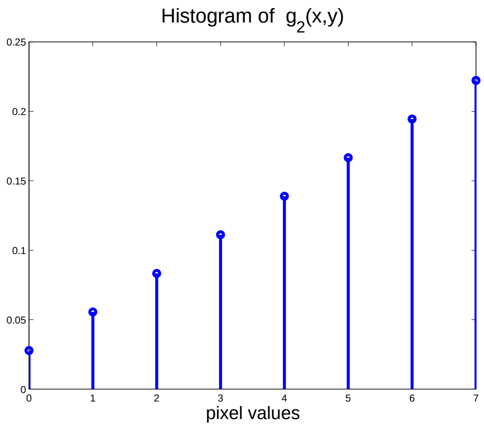

Fig. 2. Histogram of pixel values of $g_2(x, y)$.

Find transformation $g_2(x, y) = T_2(f(x, y))$ so that the pixel histogram of output image $g_2(x, y)$, denoted as h_{g_2}, is a linear function shown in Fig. 2, and defined as $h_{g_2}(i)=\dfrac{i+1}{36}, \quad i=0, 1, \ldots, 7$. Plot $T_2(i)$, $i = 0, 1, \ldots, 7$, and label the axes and main points of the plot.

2.3. (5pts)

Explain which of the two images, $g_1(x, y)$ or $g_2(x, y)$, will be brighter, without computing their pixel values. Simple answers, like "$g_1(x, y)$ is brighter" or "$g_2(x, y)$ is brighter", without any additional explanation, will be interpreted as random guessing, and will NOT earn you points.

Problem 3 – (25pts)

Suppose an input image is filtered using filter $H(u,v)$ in the frequency domain. Explain the effect of this filtering in the output image when the filter is defined as:

3.1. (10pts)

- $H(u,v) = -4\pi^2(u^2+v^2)$

- $H(u,v) = [1 + 4\pi^2(u^2+v^2)]$

3.2. (15pts)

- $H(u,v) = \exp\left(-\dfrac{u^2+v^2}{2D_0}\right)$, $D_0 = 1$

- $H(u,v) = \exp\left(-\dfrac{u^2+v^2}{2D_0}\right)$, $D_0 \to \infty$

- $H(u,v) = \exp\left(-\dfrac{u^2+v^2}{2D_0}\right)$, $D_0 \to 0$