ECE468: EXAM 1

NAME: ____

INSTRUCTIONS

- This is a 45 minute exam containing **THREE** problems: 1.1–1.5; 2; and 3.1–3.2
- For the exam, you may use the textbook, one letter-size crib sheet, calculator, and pens/pencils
- Cheating during the exam will result in a failing grade for the entire course

Problem	Max points	Earned points
1.1	30pts	
1.2	5pts	
1.3	10pts	
1.4	20pts	
1.5	5pts	
2	20pts	
3.1	20pts	
3.2	10pts	
TOTAL	120pts	

$$f(x,y) \longrightarrow \boxed{H(u,v)} \longrightarrow g(x,y)$$

Fig. 1. A discrete-space system.

Problem 1 – (65pts)

Consider a discrete-space system, shown in Fig. 1, where $f(x,y)$ is input, $g(x,y)$ is output, and $H(u,v)$ is the filter transfer function in the frequency domain, defined as

$$H(u,v) = \frac{j}{2}\left[\sin(2\pi u/M) - \sin(2\pi v/N)\right], \quad u = 0, 1, \ldots, M-1, \quad v = 0, 1, \ldots, N-1 \ .$$

The input image is corrupted by stationary, additive, Gaussian noise, η, as $f(x,y) = f_0(x,y) + \eta$, where $f_0(x,y)$ is the original image. It is known that noise η has the Gaussian distribution with mean μ and variance σ^2, and that it is stationary.

1.1. (30pts)

Compute the nine elements of 3×3 spatial filter $h(x,y)$ that represents the spatial-domain equivalent of $H(u,v)$.

1.2. (5pts)

Can we use $h(x, y)$ for image smoothing? Why?

1.3. (10pts)

Compute the expected value and variance of input image $f(x, y)$, where the original image, $f_0(x, y)$, is assumed deterministic. Is $f(x, y)$ stationary?

1.4. (20pts)

Compute the variance of the output image $g(x, y)$.

1.5. (5pts)

Does filter $h(x, y)$ reduce the noise? Why?

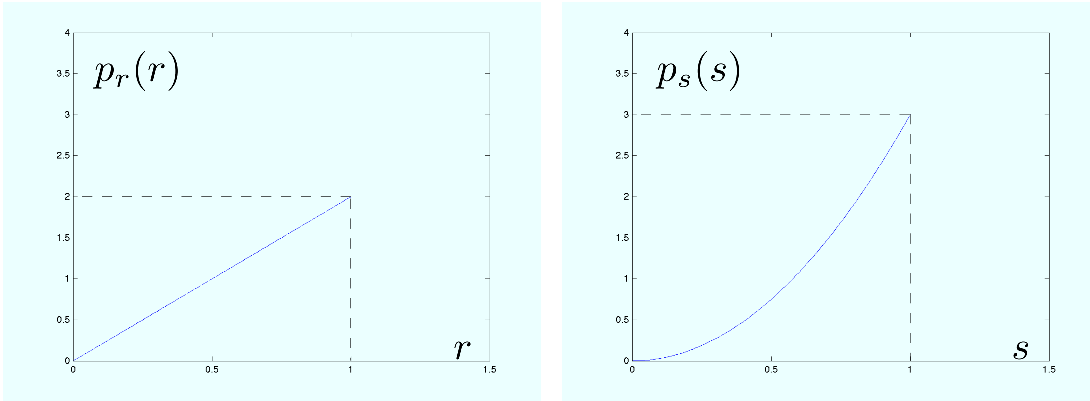

Fig. 2. Probability density functions of pixel values: (left) $p_r(r)$ is for the input image; (right) $p_s(s)$ is for the output image.

Problem 2 – (20pts)

Suppose we are given an image with pixel intensities in the interval $[0, 1]$. The values of pixels in the image, r, are characterized by a probability density function (pdf), $p_r(r) = 2r$ for $r \in [0, 1]$, and $p_r(r) = 0$, otherwise. Our task is to transform the intensity levels of this image, so the pixel values in the new image, s, are characterized by another pdf, $p_s(s) = 3s^2$ for $s \in [0, 1]$, and $p_s(s) = 0$, otherwise. The plots of $p_r(r)$ and $p_s(s)$ are shown in Figure 2. Assuming that pixel values can take continuous real values in $[0, 1]$, find this transformation in terms of input r and output s values.

Problem 3 – (20pts)

Let $F_1(u, v)$ and $F_2(u, v)$ denote the DFT of images $f_1(x, y)$ and $f_2(x, y)$. The relationship between these two images is given in the frequency domain as

$$F_1(u, v) = F_2(u, v) \left(2 - \frac{\sin(3\pi u)}{3\pi u} \frac{\sin(3\pi v)}{3\pi v}\right) .$$

3.1 (20pts)

Compute the relationship between $f_1(x, y)$ and $f_2(x, y)$ in the spatial domain.

3.1 (10pts)

- Which image, $f_1(x,y)$ or $f_2(x,y)$, will have sharper edges? Why?

- What is the name of filtering that transforms $f_2(x,y)$ into $f_1(x,y)$?

- Which image, $f_1(x,y)$ or $f_2(x,y)$, will have sharper edges if their relationship in the frequency domain has changed to

$$F_1(u,v) = F_2(u,v)\left(1 - \frac{\sin(3\pi u)}{3\pi u}\frac{\sin(3\pi v)}{3\pi v}\right) \ .$$