ECE468/CS519: EXAM 1 – Fall 2016

NAME: _____

INSTRUCTIONS

This is a 50 minute, **closed-book** exam with **THREE** problems. Cheating during the exam will result in a failing grade for the entire course.

Problem	Estimated minutes	Max points	Earned points
1.1	15 minutes	20 points	
1.2	5 minutes	10 points	
1.3	10 minutes	20 points	
1.4	10 minutes	20 points	
2	5 minutes	15 points	
3	5 minutes	15 points	
TOTAL	50 minutes	100 points	

Problem 1 — 50 points; 40 minutes

Problem 1.1 — 20 points; 15 minutes

Let $h(x, y)$ and $H(u, v)$ denote a 2D spatial function and its 2D DFT. Compute $H(u, v) =?$ when $h(x, y)$ is defined as:

$$h(x, y) = \begin{bmatrix} -1 & -1 & -1 \\ -1 & 9 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

Problem 1.2 — 10 points; 5 minutes

Suppose that the filter $h(x, y)$ defined in Prob. 1.1 is applied to an image $f(x, y)$, resulting in $g(x, y) = h(x, y) * f(x, y)$. Is $g(x, y)$ more blurry or sharper than $f(x, y)$, or is $g(x, y)$ the response of edge or corner detections in $f(x, y)$? Explain your reasoning.

Problem 1.3 — 20 points; 10 minutes

Suppose that image $f(x, y)$ and degradation filter $h(x, y)$, defined in Prob. 1.1, are part of the image system depicted in the figure above. As can be seen from the figure, $f(x, y)$ represents a noise-corrupted image, $f(x, y) = f_0(x, y) + \eta(x, y)$, where $f_0(x, y)$ denotes the original image without noise, and $\eta(x, y)$ denotes stationary additive Gaussian noise with mean $\mu = 0$ and variance $\sigma^2 \neq 0$.

Compute variance Var$\{h(x, y) * f(x, y)\}$=?

(Hint: $h(x, y) * f(x, y)$ does NOT reduce the amount of additive noise in $f(x, y)$)

Problem 1.4 — 20 points; 10 minutes

Given the image degradation system depicted in the figure above, with all elements defined in the previous problems Prob. 1.1 and Prob. 1.3, specify the Wiener filter $W(u, v)$ in the 2D DFT domain for restoring the original uncorrupted image $f_0(x, y)$ whose power is known to be constant (i.e., energy per unit spatial area) $|F_0(u, v)|^2 = 1$.

$$f_0'(x, y) = [(f_0(x, y) + \eta(x, y)) * h(x, y)] * w(x, y).$$

(Hint: Note that the degradation filter $h(x, y)$ is applied after the noise addition. This is different from the image degradation system that we covered in class. How does this affect the specification of the Wiener filter? Use the result from Prob. 1.3)

Problem 2 – 15 points; 5 minutes

The figure above shows two images f1 and f2. Both images have only four non-zero pixels with values $\{1, 2, 3, 4\}$, and the rest of pixels are equal to zero. Specify the 3×3 affine transform matrix that maps pixels of f1 to pixels of f2, for the coordinate system centered in the middle of the image, as marked in the figure.

Problem 3 — 15 points; 5 minutes

Consider a very large image $f(x, y)$, encoded by 8 bits, whose pixels are just random noise,

$$f(x, y) = \text{round}(7 * \text{rand}), \quad \text{for all } x = 1, ..., M, \quad \text{and} \quad y = 1, ..., N,$$

where $M \to \infty$ and $N \to \infty$, and "rand" is a random generator of real numbers from the uniform distribution in the interval [0,1]. Specify the histogram equalization of $f(x, y)$.

$\sin(x) = \dfrac{e^{jx} - e^{-jx}}{2j}, \quad \cos(x) = \dfrac{e^{jx} + e^{-jx}}{2}, \quad e^{jx} = \cos(x) + j\sin(x)$
Rectangular Pulse: $f(x,y) = A, -X/2 \le x \le X/2,$ and $-Y/2 \le y \le Y/2$ and $f(x,y) = 0$, otherwise $F(\mu,\nu) = CFT\{f(x,y)\} = AXY \frac{\sin(\pi X\mu)}{\pi X\mu} \frac{\sin(\pi Y\nu)}{\pi Y\nu}$
$f(x,y) = \dfrac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v) e^{j2\pi(\frac{xu}{M} + \frac{yv}{N})}, \quad x = 0, ..., M-1, \quad y = 0, ..., N-1$
$F(u,v) = DFT\{f(x,y)\} = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(\frac{xu}{M} + \frac{yv}{N})}, \quad u = 0, ..., M-1, \quad v = 0, ..., N-1$
Translation in space: $DFT\{f(x+x_0, y+y_0)\} = F(u,v) e^{j2\pi(\frac{x_0 u}{M} + \frac{y_0 v}{N})}$
Translation in frequency: $DFT^{-1}\{F(u+u_0, v+v_0)\} = f(x,y) e^{-j2\pi(\frac{x u_0}{M} + \frac{y v_0}{N})}$
Example: $DFT\{\delta(x,y)\} = 1, \quad DFT\{\delta(x+x_0, y+y_0)\} = e^{j2\pi(\frac{x_0 u}{M} + \frac{y_0 v}{N})}$
Rotation in space: $DFT\left\{ f(x,y) e^{-j2\pi(\frac{x u_0}{M} + \frac{y v_0}{N})} \right\} = F(u+u_0, v+v_0)$
Rotation in frequency: $DFT^{-1}\left\{ F(u,v) e^{j2\pi(\frac{x_0 u}{M} + \frac{y_0 v}{N})} \right\} = f(x+x_0, y+y_0)$
Image sharpening: $g(x,y) = f(x,y) \pm c\nabla^2 f(x,y),$ where Laplacian $\nabla^2 f(x,y) = \dfrac{\partial^2 f(x,y)}{\partial x^2} + \dfrac{\partial^2 f(x,y)}{\partial y^2}$
Derivative approximation: $\dfrac{\partial f(x,y)}{\partial x} \approx \dfrac{f(x+1,y) - f(x-1,y)}{2}, \quad \dfrac{\partial f(x,y)}{\partial y} \approx \dfrac{f(x,y+1) - f(x,y-1)}{2}$
Laplacian approximation: $\nabla^2 f(x,y) \approx f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$
Unsharp masking: $g(x,y) = f(x,y) + c(f(x,y) - \overline{f(x,y)}),$ where $\overline{f(x,y)}$ is a smoothed $f(x,y)$
Affine transformation matrix: $\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x \cos\theta & \mp s_x \sin\theta & t_x \\ \pm s_y \sin\theta & s_y \cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$
Bilinear interpolation: $f(x,y) = ax + by + cxy + d$, where coefficients a, b, c, d can be obtained from 4 known pixel values $f(x_1,y_1)$, $f(x_2,y_2)$, $f(x_3,y_3)$, $f(x_4,y_4)$ at locations $(x_i,y_i), i = 1,2,3,4$ around the unknown pixel at (x,y).
Histogram equalization: $s = \sum_{i=0}^{r} h(i)$, where h is the histogram of input pixels r, and s is output
Histogram specification: $z = T_2(s)$ and $z = T_1(r)$, then $s = T_2^{-1}(T_1(r))$ where input and output pixels are r and s
The Harris corner detector computes a 2D map $E = [u \;\; v] \begin{bmatrix} (w_x * f)^2 & (w_x * f)(w_y * f) \\ (w_x * f)(w_y * f) & (w_y * f)^2 \end{bmatrix} [u \;\; v]^\top$