ECE468: EXAM 2

NAME: ____

INSTRUCTIONS

- This is a 60 minute exam containing **FOUR** problems: 1.1–1.3; 2; 3; 4
- For the exam, you may use the textbook, one letter-size crib sheet, calculator, and pens/pencils
- Cheating during the exam will result in a failing grade for the entire course

Problem	Max points	Earned points
1.1	30pts	
1.2	10pts	
1.3	20pts	
2	40pts	
3	40pts	
4	40pts	
TOTAL	180pts	

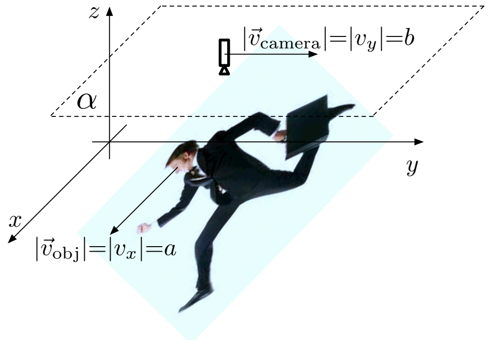


Fig. 1. The image acquisition system moves along y-axis, while the object moves along x-axis. If the imaging process takes a relatively long period of time, the obtained image will be corrupted by motion blur.

Problem 1 – (60pts)

Consider a video capturing system shown in Fig. 1. The object moves in the x-y plane, along x-axis, at a constant speed of a units per second. The camera moves in plane α, which is parallel to the x-y plane, along y-axis, at a constant speed of b units per seconds. Assume the video capturing process continues for a period of T seconds, and the lens opening and closing are both instantaneous.

1.1. (30pts)

Derive the discrete, $M \times N$, frequency-domain filter, $H(u, v)$, corresponding to the motion blur caused by the imaging process.

1.2. (10pts)

Explain formally whether $H(u, v)$ is a lowpas or highpass filter?

1.3. (20pts)

Suppose our goal is to eliminate the motion blur present in the image that is captured by the imaging system show in Fig. 1. This would entail developing an image restoration system shown in Fig. 2. In this system, $H(u, v)$ is the motion-blur degradation function in the frequency domain that you computed in 1.1; $\eta(x, y)$ is an additive noise; and $H_R(u, v)$ is the restoration filter, aimed at restoring the original, uncorrupted visual input. Let the signal-to-noise ratio be constant $K \neq 0$.

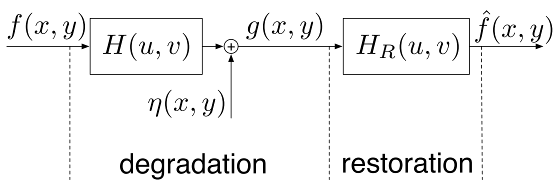


Fig. 2. The image acquisition system shown in Fig. 2 can be represented as a degradation/restoration system. $H(u, v)$ models the effect of motion blur in the frequency domain on visual input $f(x, y)$; $\eta(x, y)$ is an additive noise; and $H_R(u, v)$ is the restoration filter, aimed at restoring the original, uncorrupted visual input.

- (10pts) What is the expression for $H_R(u, v)$, if this filter is defined as inverse filter? Find the Fourier transform of $\hat{f}(x, y)$, denoted as $\hat{F}(u, v)$, when $H_R(u, v)$ is the inverse filter. Is the inverse filter suitable for restoration in this case? Explain your reasoning.

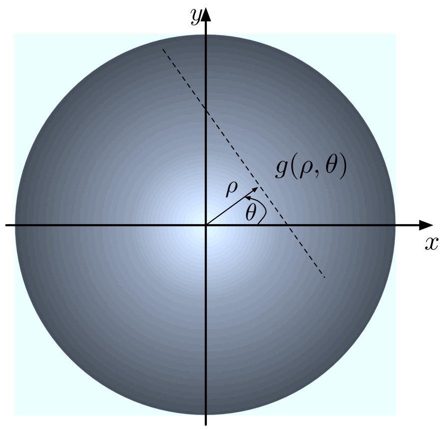
- (10pts) What is the expression for $H_R(u, v)$, if this filter is defined as Wiener filter? Is the Wiener filter suitable for restoration in this case? Explain your reasoning.

Problem 2 – (40pts)

Consider the object $f(x, y)$ shown in Fig. 3, which has a non-zero attenuation profile modeled as

$$f(x,y) = \exp\left[-\frac{x^2+y^2}{4}\right] + \exp\left[-\frac{x^2+y^2}{2}\right]$$

along x and y axes. Find the Radon Transform, $g(\rho, \theta)$, of $f(x, y)$.



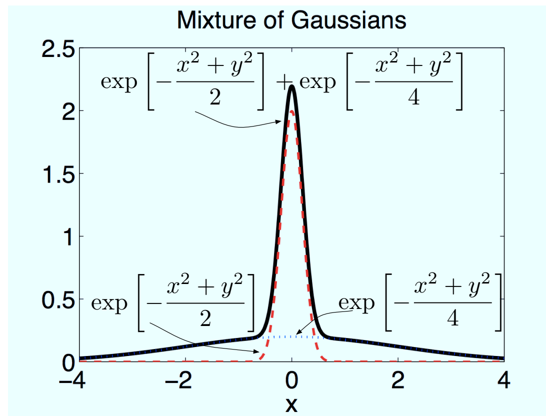


Fig. 3. (a) The object's attenuation profile along x and y axes; (b) Cross section of the object's attenuation profile (solid black line) at y=0. The dashed lines show two components $\exp(-\frac{x^2+y^2}{4})$ and $\exp(-\frac{x^2+y^2}{2})$ of the object's attenuation function $f(x, y)$.

Problem 3 – (40pts)

Find the original image, $f(x, y)$, if its projection is $g(\rho, \theta) = \delta(\rho - 3\cos\theta + 2\sin\theta)$.

Problem 4 – (40pts)

Find the original image, $f(x, y)$, if the 1D Fourier Transform of its projection is $G(\omega, \theta) = \exp(-\frac{\omega^2}{2})$.

(Hint: Use the Continuous Fourier Transform pair: $A2\pi\sigma^2 \exp(-2\pi^2\sigma^2(x^2 + y^2)) \leftrightarrow A \exp(-\frac{\mu^2+\nu^2}{2\sigma^2})$, where x, y are coordinates in the continuous spatial domain, and μ, ν are coordinates in the continuous frequency domain.)