

ECE468/CS519: HOMEWORK 1

due 10/06/2017

Turn in a hard copy of your homework report in class

Problem 1 – 30 points

Image $f_1(x, y)$, $x = 1, \dots, M$, $y = 1, \dots, N$, is transformed to another image $f_2(x', y')$ using a 2D affine transform. After the transform, $f_2(x', y')$ is 2 times larger along rows, $x' = 1, \dots, 2M$, and 3 times larger along columns, $y' = 1, \dots, 3N$, than $f_1(x, y)$.

Problem 1.1 – 5 points

Compute the 3×3 matrix T_{12} of this 2D affine transform.

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = T_{12} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow T_{12} = ?$$

Problem 1.2 – 10 points

Compute the values of **all pixels** in the resulting image $f_2(x', y')$, $x' = 1, \dots, 2M$, $y' = 1, \dots, 3N$, in terms of the known pixel values of image $f_1(x, y)$, $x = 1, \dots, M$, $y = 1, \dots, N$, using bilinear interpolation. Note that some of pixels in $f_2(x', y')$ will be equal to their corresponding pixels in $f_1(x, y)$. Identify coordinates (x', y') of those pixels. Then, specify equations that bilinear interpolation requires for computing pixel values in $f_2(x', y')$ that are not present in $f_1(x, y)$.

Problem 1.3 – 15 points

Suppose that we rotate $f_1(x, y)$, $x = 1, \dots, M$, $y = 1, \dots, N$, to another image $f_3(x'', y'')$ by 30° degrees counter clockwise **around the image's center** $(x_c, y_c) = (\frac{N}{2}, \frac{M}{2})$. Compute the 3×3 affine transform matrix T_{13} that maps pixel coordinates of $f_1(x, y)$ to the corresponding pixel coordinates of $f_3(x'', y'')$:

$$\begin{bmatrix} x'' \\ y'' \\ 1 \end{bmatrix} = T_{13} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow T_{13} = ?$$

Problem 2 — 25 points

Pixels of an image $r = f_1(x, y)$ take integer values $r \in \{0, 1, \dots, 255\}$. Their histogram is estimated as:

$$h_1(r) = c_1 2^{-r}, \quad r \in \{0, 1, 2, \dots, 255\},$$

where c_1 is a normalizing constant such that $\sum_{r=0}^{255} h_1(r) = 1$. We would like to process image $f_1(x, y)$ to another image $f_2(x, y)$ such that the histogram of pixel values $s = f_2(x, y)$ is specified as:

$$h_2(s) = c_2 e^{-s}, \quad s \in \{0, 1, 2, \dots, 255\},$$

where c_2 is a normalizing constant such that $\sum_{s=0}^{255} h_2(s) = 1$. Find the transformation $s = T(r) = ?$.

Hint: Finite geometric series $S_n = \sum_{k=0}^n \alpha^k = \frac{1-\alpha^{n+1}}{1-\alpha}$, **only for** $|\alpha| < 1$

Problem 3 — 45 points: Affine Transformation of Images

Write a MATLAB code for the following image processing steps:

- 1) Create the checkerboard image, f , where the side of every square is 50 pixels in length.
- 2) Compute the affine transformation matrix T that rotates an image by angle $\theta = \pi/6$ counter-clockwise, and then scales the rotated result with the scaling parameters $s_x = 0.75$ and $s_y = 1.25$.
- 3) Apply T to f . The resulting image is $f1$.
- 4) Apply the inverse transform T^{-1} to $f1$. The resulting image is $f2$.
- 5) Compute the difference image $f3 = f - f2$ (hint: apply zero padding to f or $f2$ if the two images have different sizes.)
- 6) Display f , $f1$, and $f3$.

Your report should include:

- 1) (10 points) A print-out of the above specified MATLAB code (please use comments in your code).
- 2) (30 points) Figure 1 showing the three images f , $f1$, and $f3$.
- 3) (5 points) In the caption of Figure 1, comment on the meaning of bright and dark pixels of $f3$.

(Hint: Use the following MATLAB commands: 'maketform', 'imtransform')