ECE 468: Digital Image Processing

Lecture 24

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Outline

- 2D Wavelet Transform (Textbook 7.5)

2D Scaling and Wavelet Functions

$$\varphi(x, y) = \varphi(x)\varphi(y)$$

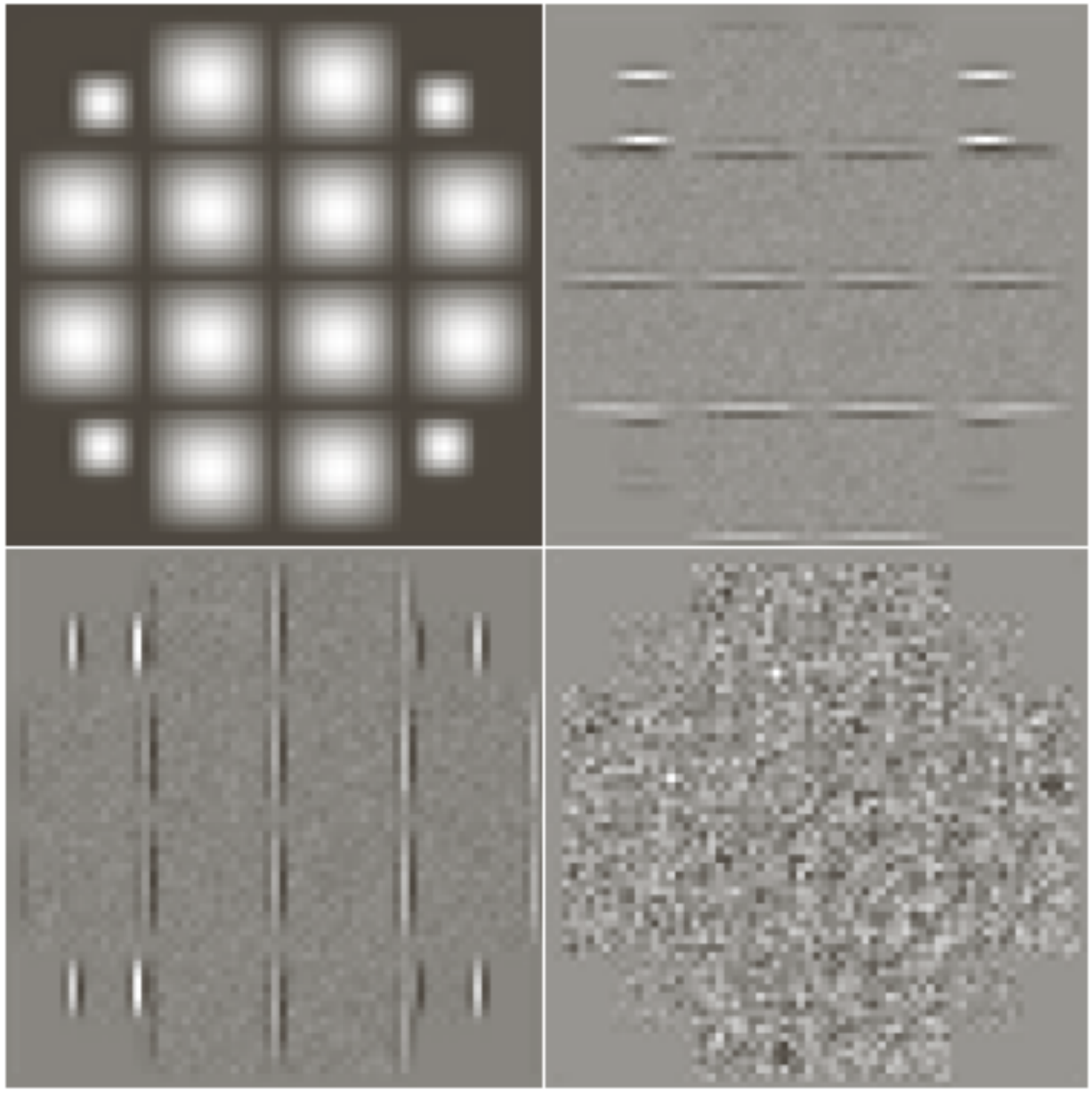
2D Scaling and Wavelet Functions

$$\psi^H(x,y) = \psi(x)\varphi(y)$$

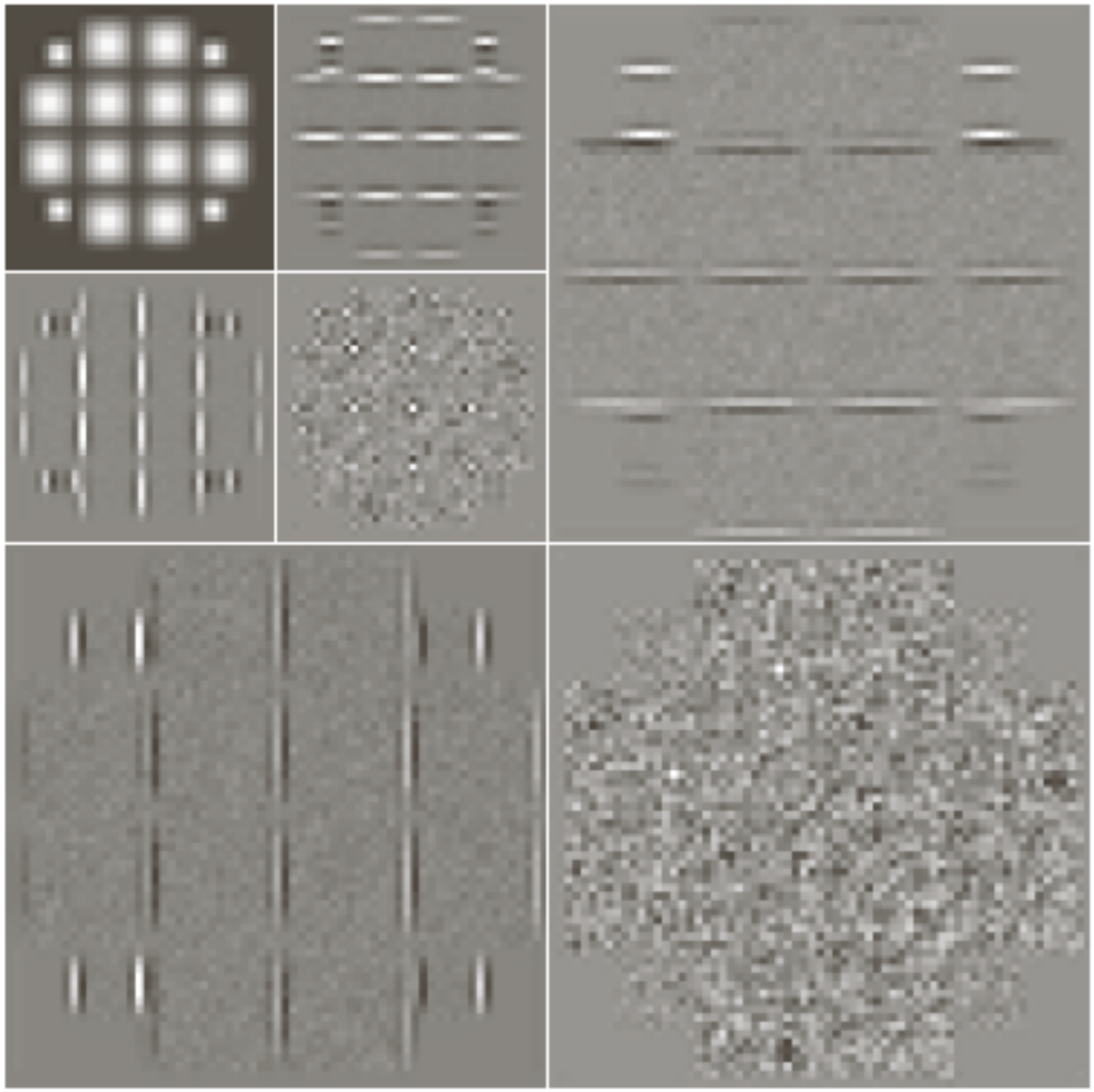
$$\psi^V(x,y) = \varphi(x)\psi(y)$$

$$\psi^D(x,y) = \psi(x)\psi(y)$$

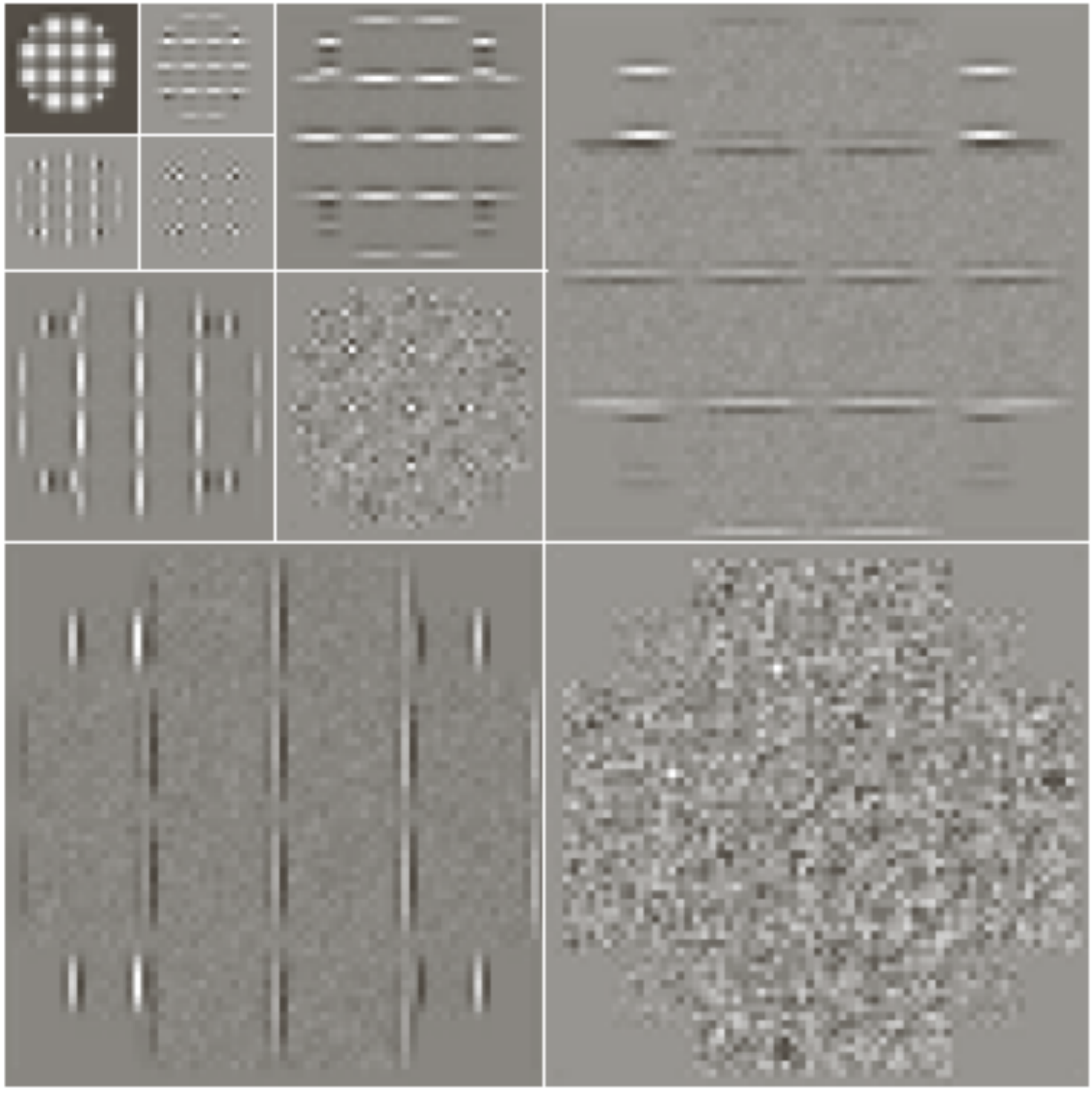
2D DWT



2D DWT



2D DWT



2D Scaling and Wavelet Functions

$$\varphi_{j,m,n}(x,y) = 2^{\frac{j}{2}}\varphi(2^j x - m, 2^j y - n)$$

$$\psi^i_{j,m,n}(x,y) = 2^{\frac{j}{2}}\psi^i(2^j x - m, 2^j y - n), \quad i = H, V, D$$

$$\varphi(x,y) = \varphi(x)\varphi(y)$$

$$\psi^H(x,y) = \psi(x)\varphi(y)$$

$$\psi^V(x,y) = \varphi(x)\psi(y)$$

$$\psi^D(x,y) = \psi(x)\psi(y)$$

2D Discrete Wavelet Transform

$$W_\varphi(j,m,n) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\varphi_{j,m,n}(x,y)$$

$$W_\psi(j,m,n) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\psi_{j,m,n}(x,y)$$

$$\begin{aligned} f(x,y) &= \frac{1}{\sqrt{MN}} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} W_\varphi(j_0,m,n)\varphi_{j_0,m,n}(x,y) \\ &\quad + \frac{1}{\sqrt{MN}} \sum_{i=H,V,D} \sum_{j=j_0}^{\infty} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} W_\psi^i(j,m,n)\psi_{j,m,n}^i(x,y) \end{aligned}$$

2D Discrete Wavelet Transform

$$\varphi(x,y) = \varphi(x)\varphi(y)$$

$$\psi^H(x,y) = \psi(x)\varphi(y)$$
$$\psi^V(x,y) = \varphi(x)\psi(y)$$
$$\psi^D(x,y) = \psi(x)\psi(y)$$

$$\varphi_{j,m,n}(x,y) = 2^{\frac{j}{2}}\varphi(2^j x - m, 2^j y - n)$$
$$\psi^i_{j,m,n}(x,y) = 2^{\frac{j}{2}}\psi^i(2^j x - m, 2^j y - n), \quad i = H, V, D$$

$$W_\varphi(j,m,n) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\varphi_{j,m,n}(x,y)$$

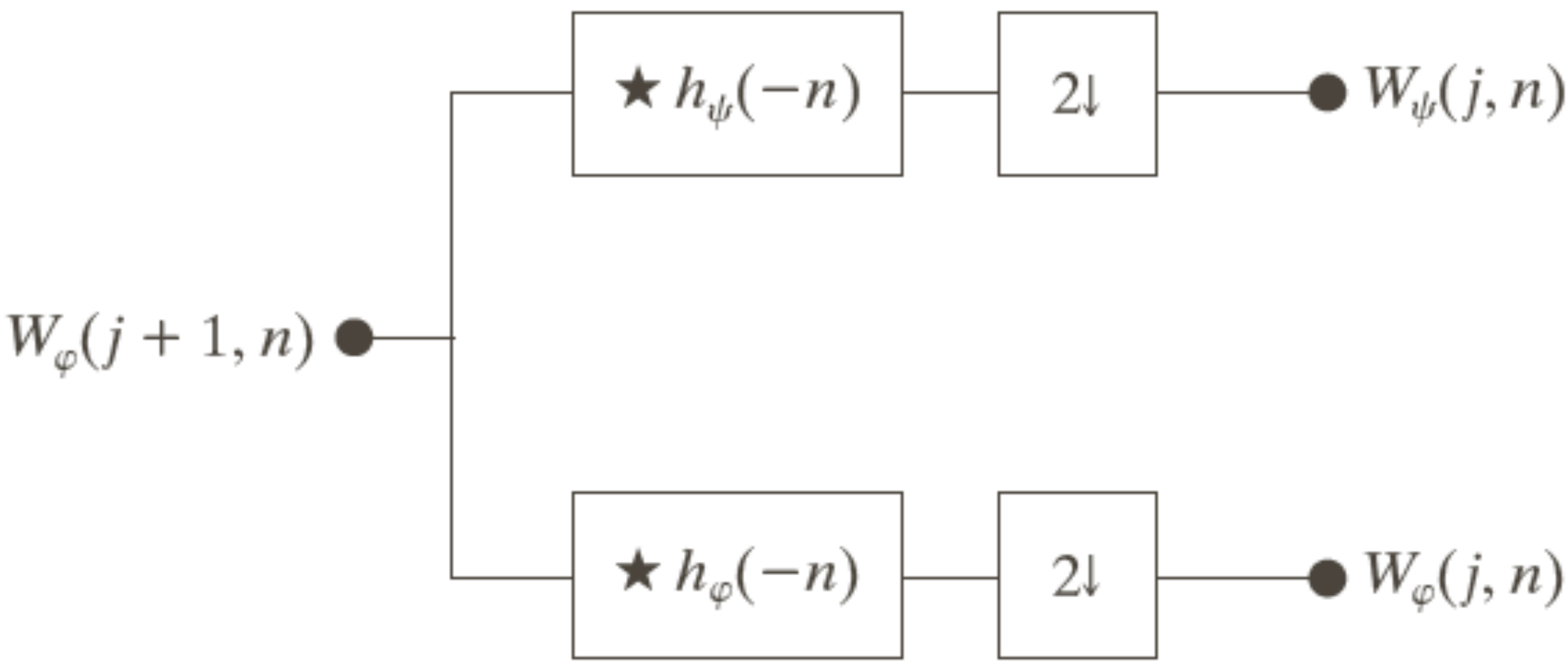
$$W_\psi(j,m,n) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\psi_{j,m,n}(x,y)$$

$$\begin{aligned} f(x,y) &= \frac{1}{\sqrt{MN}} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} W_\varphi(j_0,m,n)\varphi_{j_0,m,n}(x,y) \\ &\quad + \frac{1}{\sqrt{MN}} \sum_{i=H,V,D} \sum_{j=j_0}^{\infty} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} W^i_\psi(j,m,n)\psi^i_{j,m,n}(x,y) \end{aligned}$$

Fast 1D Wavelet Transform

Recall:

$$W_\varphi(j,n) = \sum_k h_\varphi(k-2n) W_\varphi(j+1,k)$$



Fast 2D Wavelet Transform

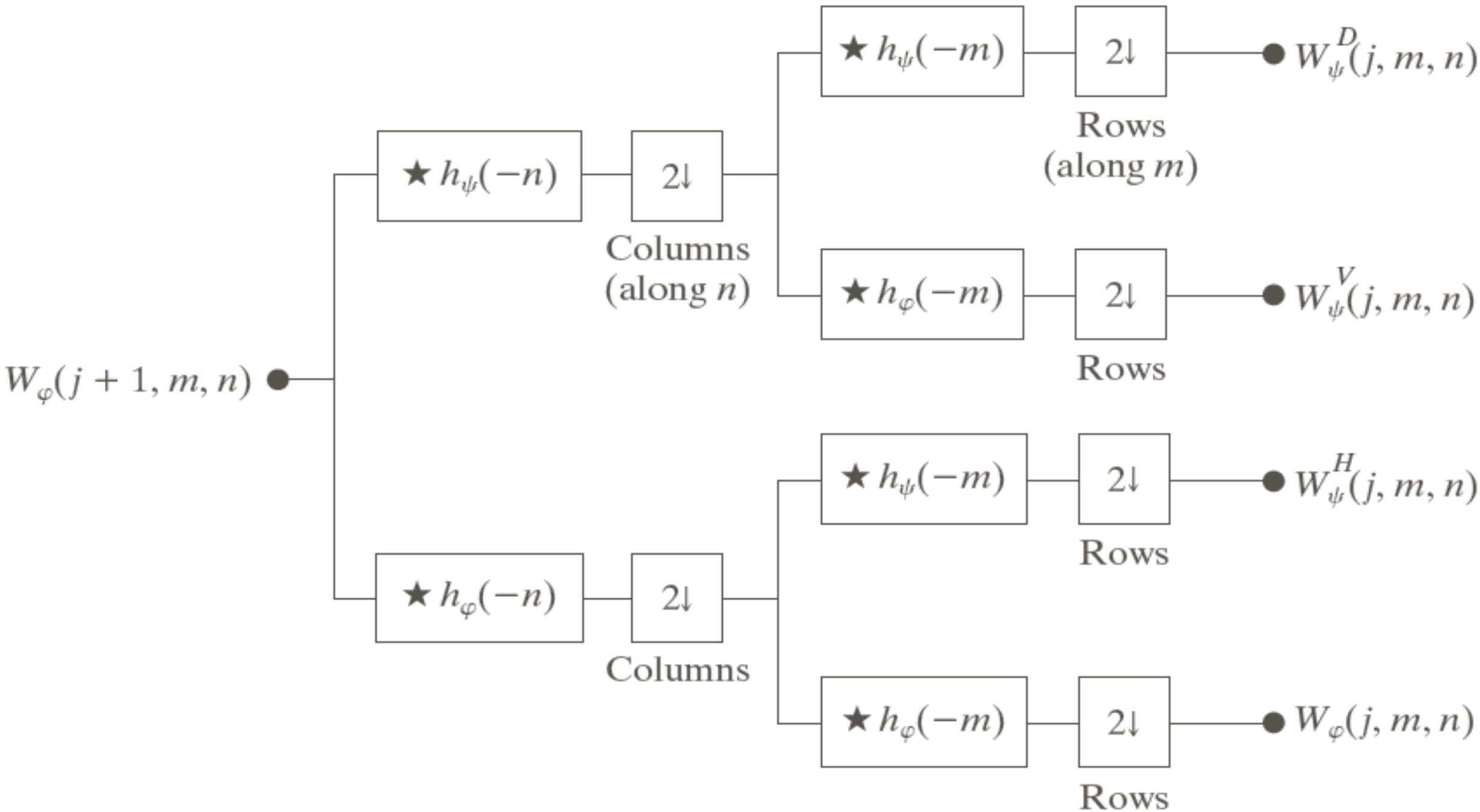
1D case:

$$W_\varphi(j,m) = \sum_k h_\varphi(k-2m) W_\varphi(j+1,k)$$

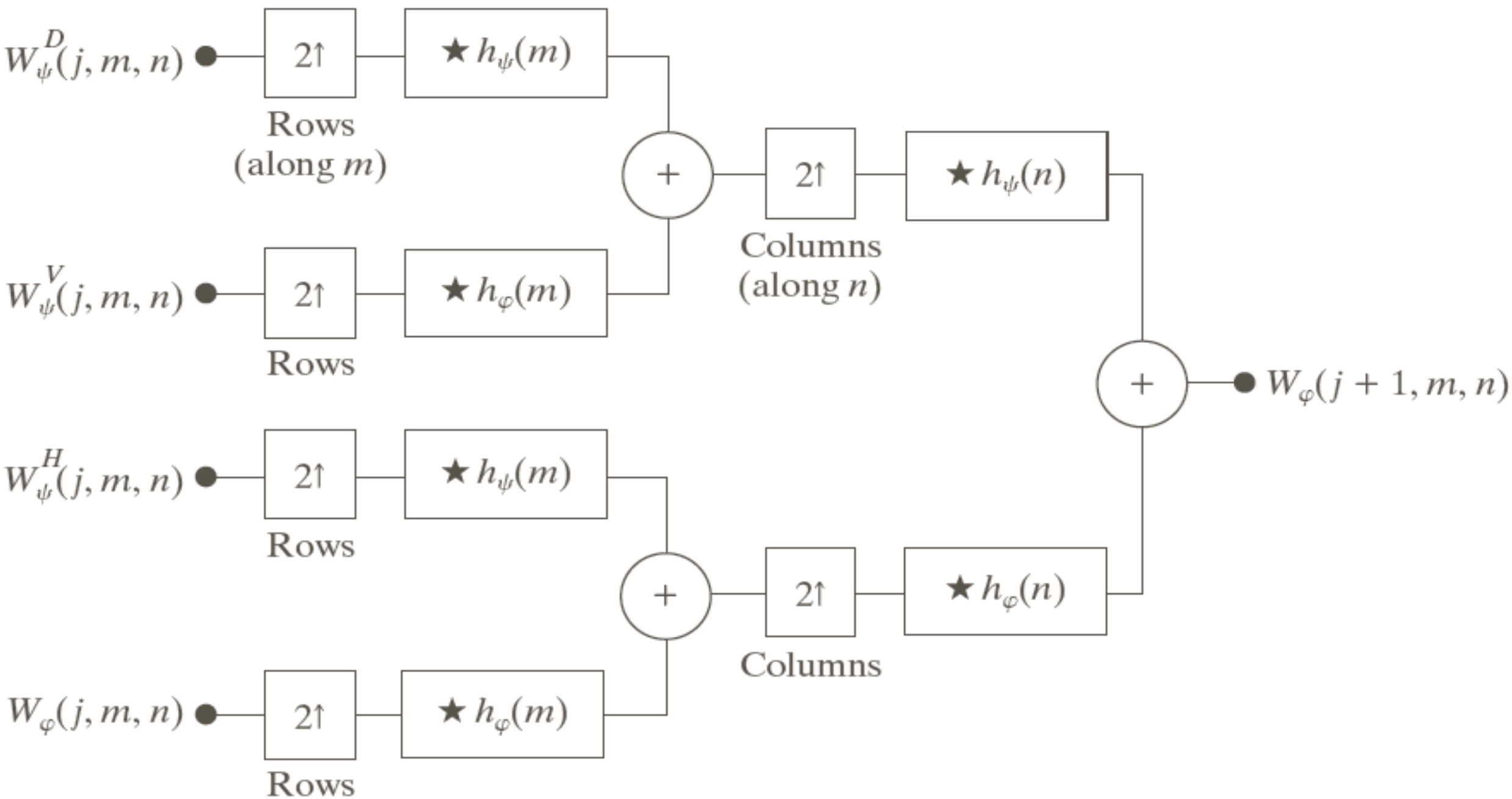
2D case:

$$W_\varphi(j,m,n) = \sum_l h_\varphi(l-2n) \left[\sum_k h_\varphi(k-2m) W_\varphi(j+1,k,l) \right]$$

2D DWT


$W_\varphi(j+1, m, n)$
$\star\, h_\psi(-n)$
2↓
Columns (along n)
$\star\, h_\psi(-m)$
2↓
Rows (along m)
$W_\psi^D(j, m, n)$
$\star\, h_\varphi(-m)$
2↓
Rows
$W_\psi^V(j, m, n)$
$\star\, h_\varphi(-n)$
2↓
Columns
$\star\, h_\psi(-m)$
2↓
Rows
$W_\psi^H(j, m, n)$
$\star\, h_\varphi(-m)$
2↓
Rows
$W_\varphi(j, m, n)$

Inverse 2D DWT



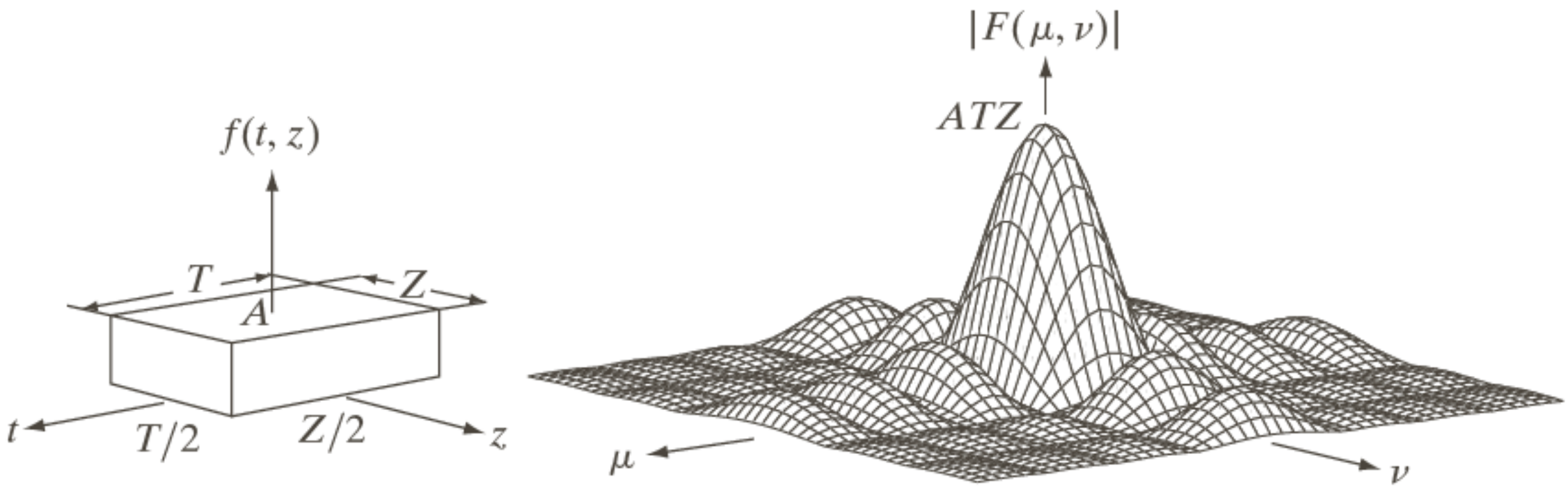
Uncertainty Principle

Energy spread of a function and its Fourier transform

CANNOT BE

simultaneously arbitrarily small.

Example: Uncertainty Principle



a b

FIGURE 4.13 (a) A 2-D function, and (b) a section of its spectrum (not to scale). The block is longer along the t-axis, so the spectrum is more "contracted" along the μ-axis. Compare with Fig. 4.4.

Example: Uncertainty Principle

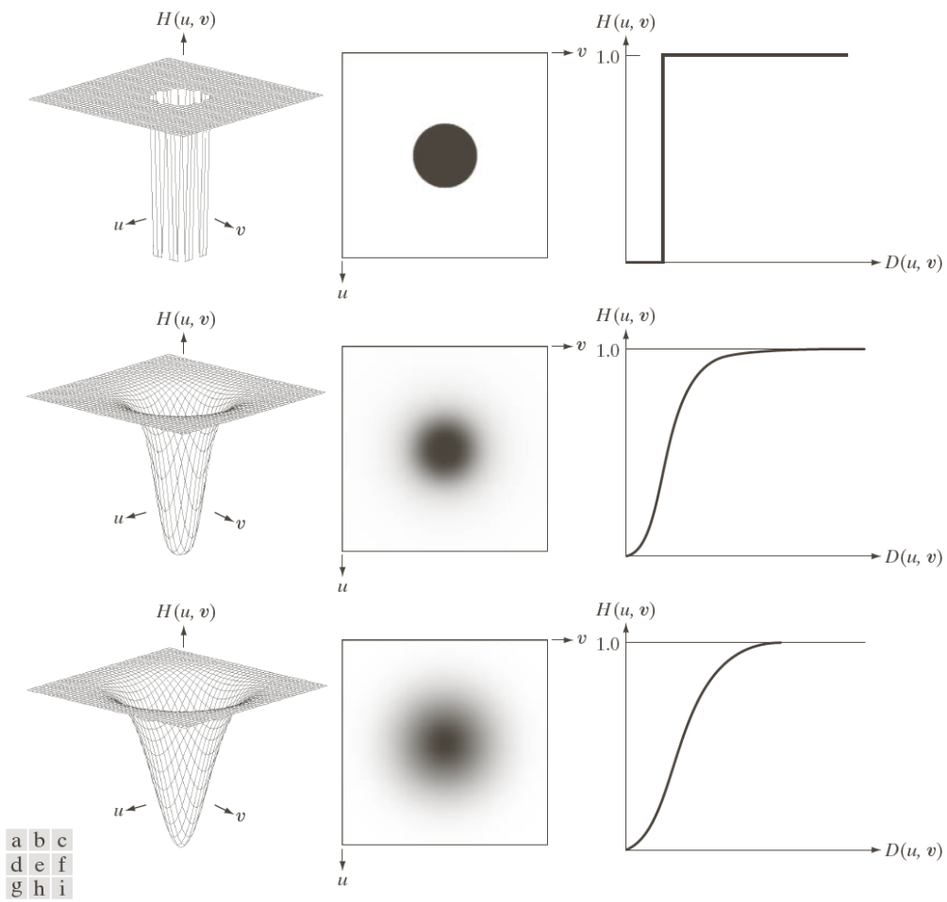


FIGURE 4.52 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

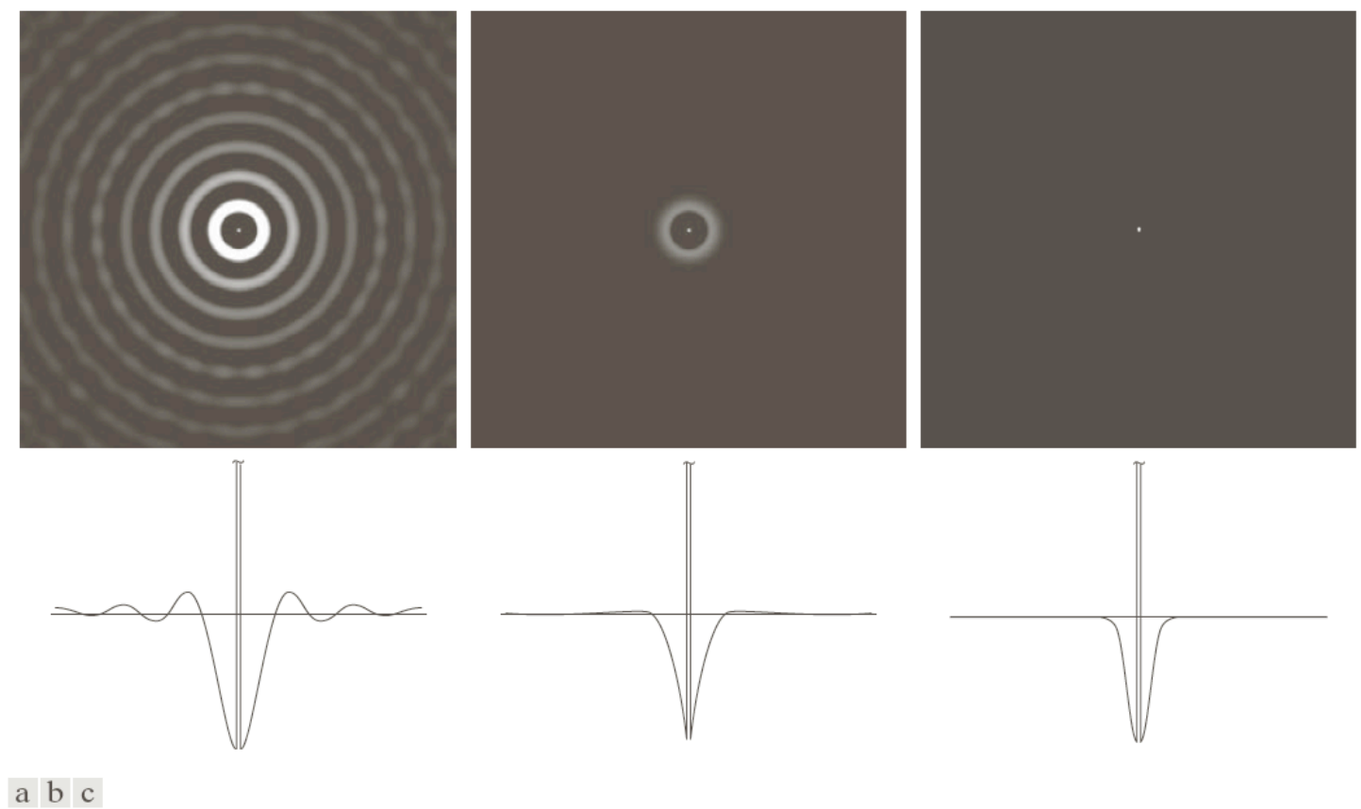


FIGURE 4.53 Spatial representation of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding intensity profiles through their centers.

1D Case: Gabor -- Ville -- Wigner

- Decompose a signal
- over elementary waveforms
- that have a minimal spread in the time-frequency plane

1D Case: Time-Frequency Plane

A Wavelet Tour of Signal Processing
Stéphane Mallat, Academic Press 1999 (2nd edition)

u,v = offsets in time
ξ, γ = offsets in frequency

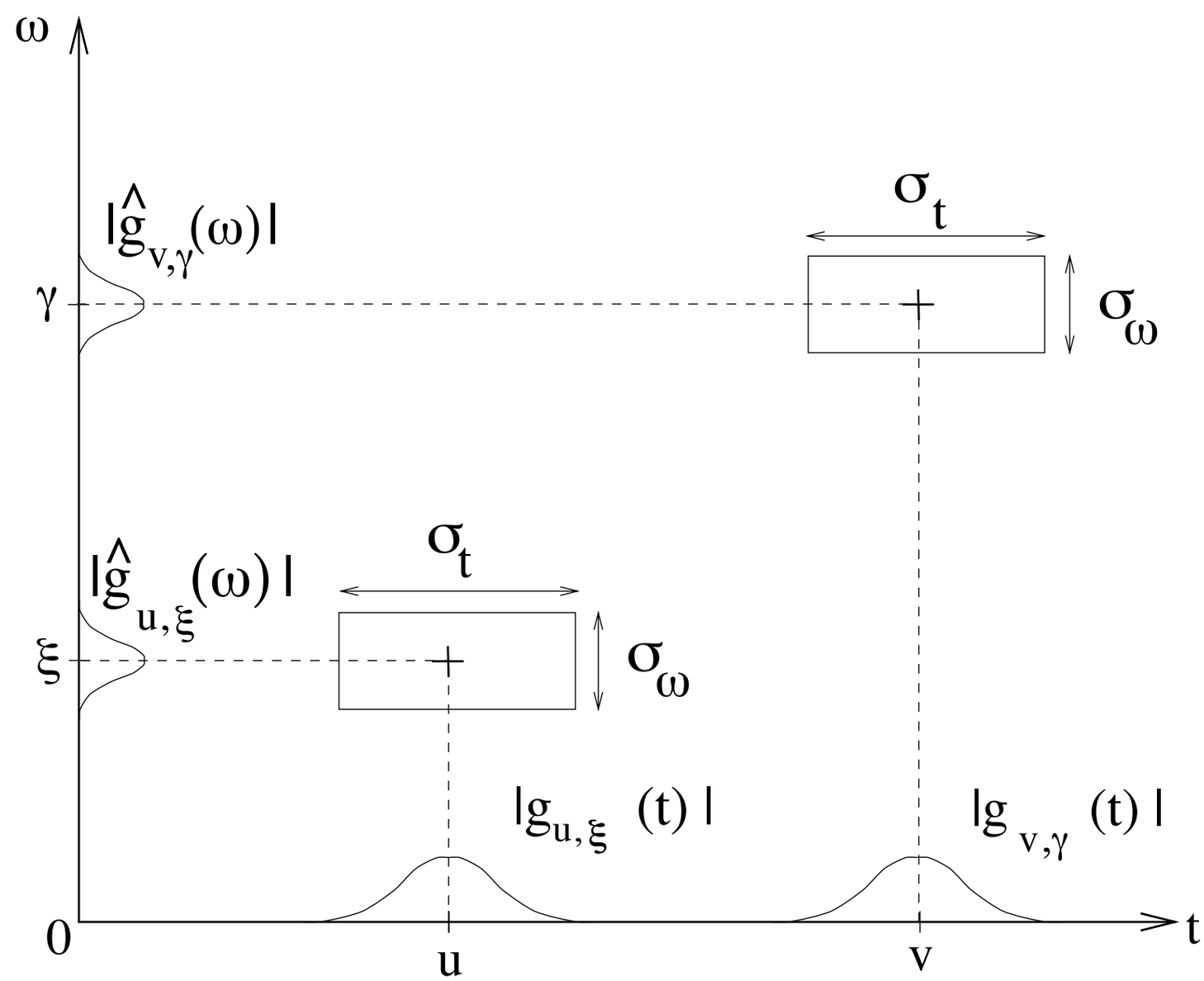


Figure 1.1: Time-frequency boxes ("Heisenberg rectangles") representing the energy spread of two Gabor atoms

Gabor Wavelets

Gabor generator

shift

rotation

$$g_{j,k,u}(x) = 2^{\frac{j}{2}} g(2^j x - k) e^{jux}$$

support

frequency