ECE468/CS519
Digital Image Processing

Feature Descriptors

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Outline

- Point descriptors -- SIFT, HOG

Affine Invariant Feature Detection

Affine Invariant Feature Detection

Given two images of the same scene
and related by an affine transform

$$I_2(x,y) = T[I_1(x,y)]$$

Features detected in both images should be the same

$$\begin{aligned} F\{I_1(x,y)\} &= F\{I_2(x,y)\} \\ &= F\{T[I_1(x,y)]\} \\ &= T'[F\{I_2(x,y)\}] \end{aligned}$$

Affine Invariant Feature Detection

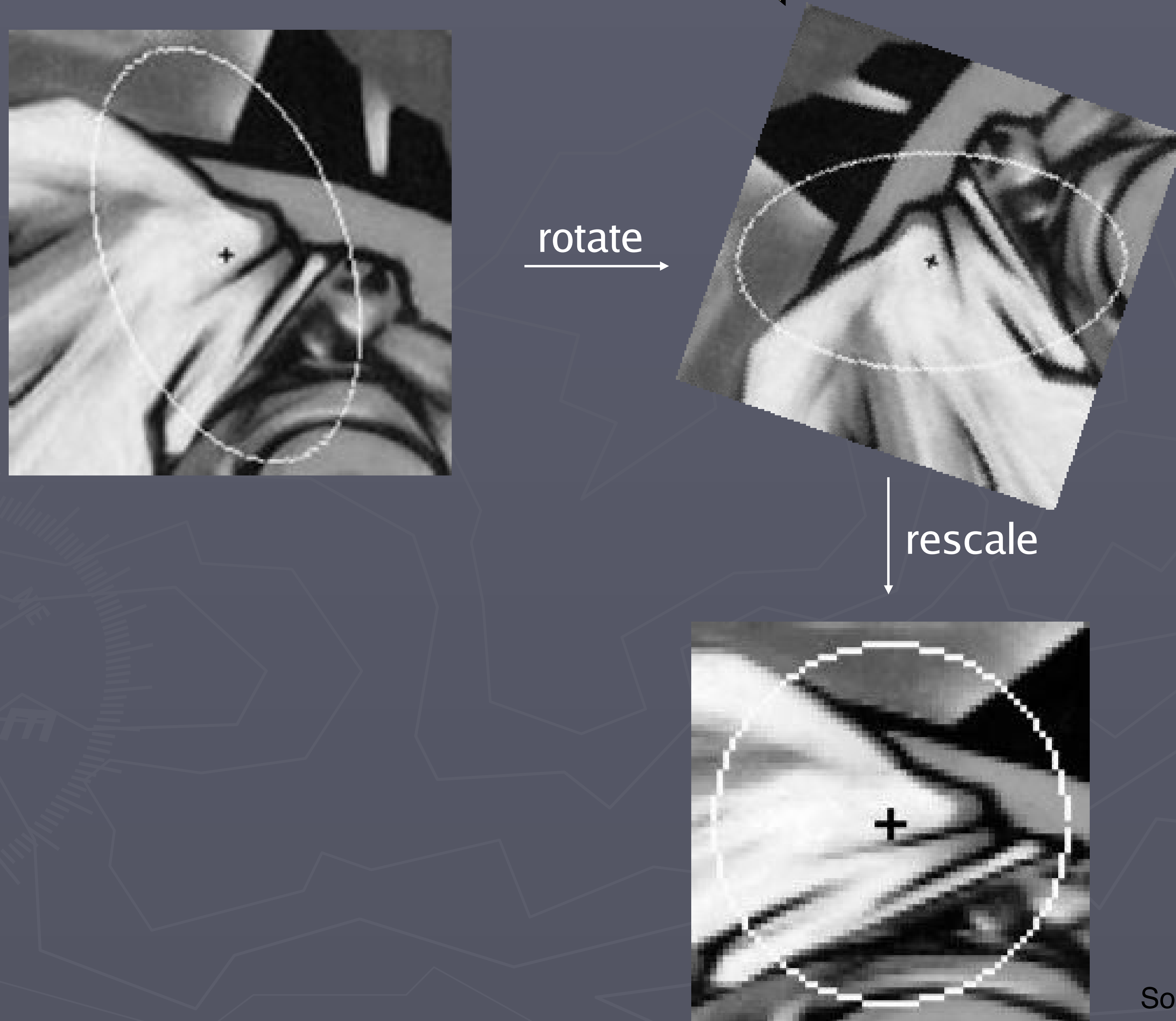
$$F\{I_1(x,y)\} = T'[F\{I_2(x,y)\}]$$

$$\Downarrow$$

$$T_1[F\{I_1(x,y)\}] = T_2[F\{I_2(x,y)\}]$$

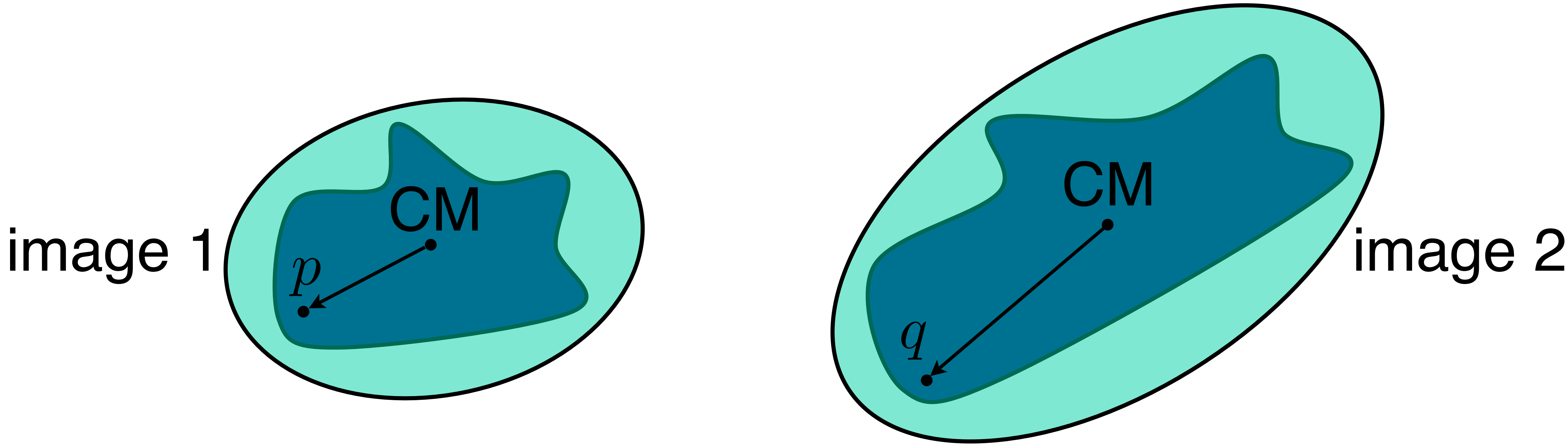
Each detected feature is normalized to a canonical view

Affine normalization (‘deskewing’)



Source: Tuytelaars

How to Normalize to a Canonical View?



If the points in two images are related by a transform their covariance matrices are also related by that transform

$$q = Tp$$

$$\Sigma_2 = T\Sigma_1 T^{\mathrm{T}}$$

Affine Normalization

Ellipse in image 1 $\Rightarrow$ Unit circle in image 2

$$p^T M^{-1} p = 1$$

the amount of intensity changes at detected interest point

$$q^T I^{-1} q = 1$$

where

$$q = Tp$$

$$T = ?$$

$$I = TMT^{\mathrm{T}}$$

$$I = T\Phi\Lambda\Phi^{\mathrm{T}}T^{\mathrm{T}}$$

$$I = T\Phi\Lambda^{\frac{1}{2}}(T\Phi\Lambda^{-\frac{1}{2}})^{\mathrm{T}}$$

$$T = \Phi^T\Lambda^{-\frac{1}{2}}$$

Descriptors

Point Descriptors

- Describe image properties in the neighborhood of a keypoint
- Descriptors = Vectors that are ideally affine invariant
- Popular descriptors:
  - Scale invariant feature transform (SIFT)
  - Steerable filters
  - Shape context and geometric blur
  - Gradient location and orientation histogram (GLOH)
  - Histogram of Oriented Gradients (HOGs)
  - DAISY

SIFT and HOG

The key idea is that

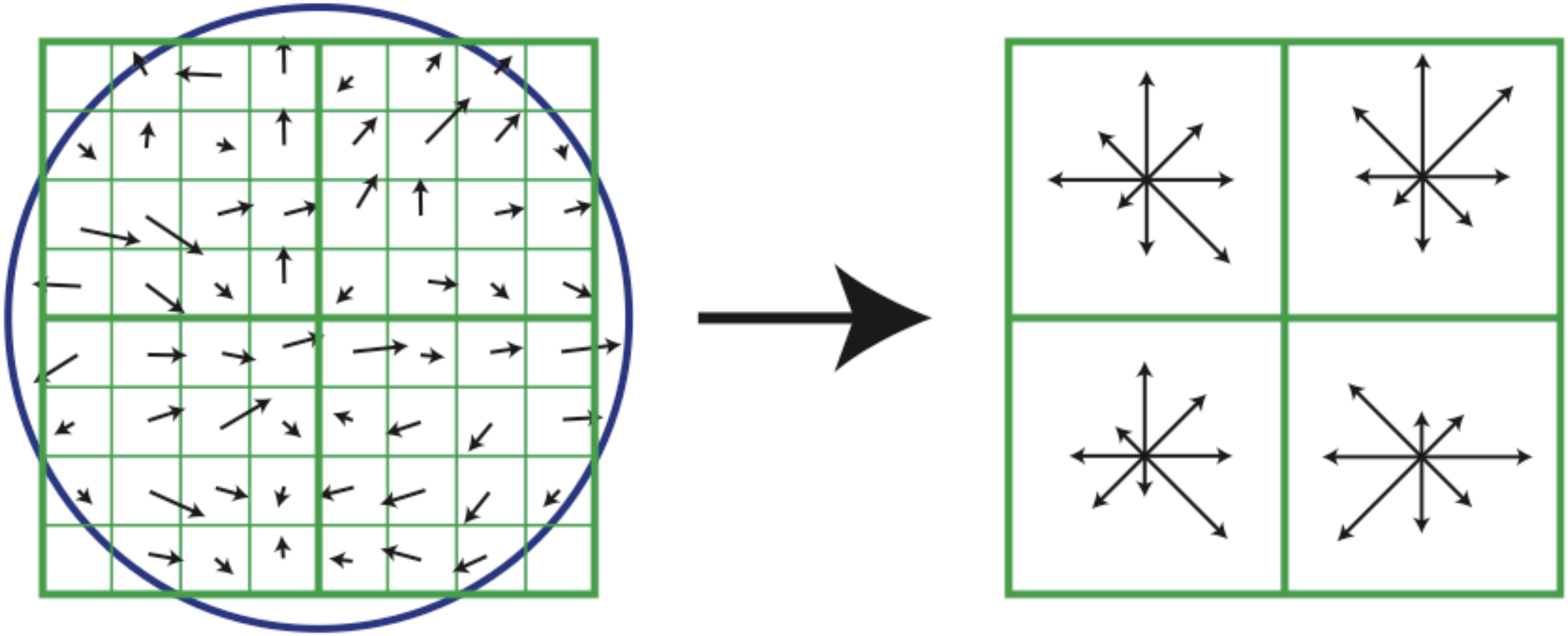
local object appearance and shape

can be described by the distribution of

intensity gradients or edge directions.

SIFT Descriptor

128-D vector = (4x4 blocks) x (8 bins of histogram)



gradients of a 16x16 patch centered at the point

histogram of gradients at certain angles of a 4x4 subpatch

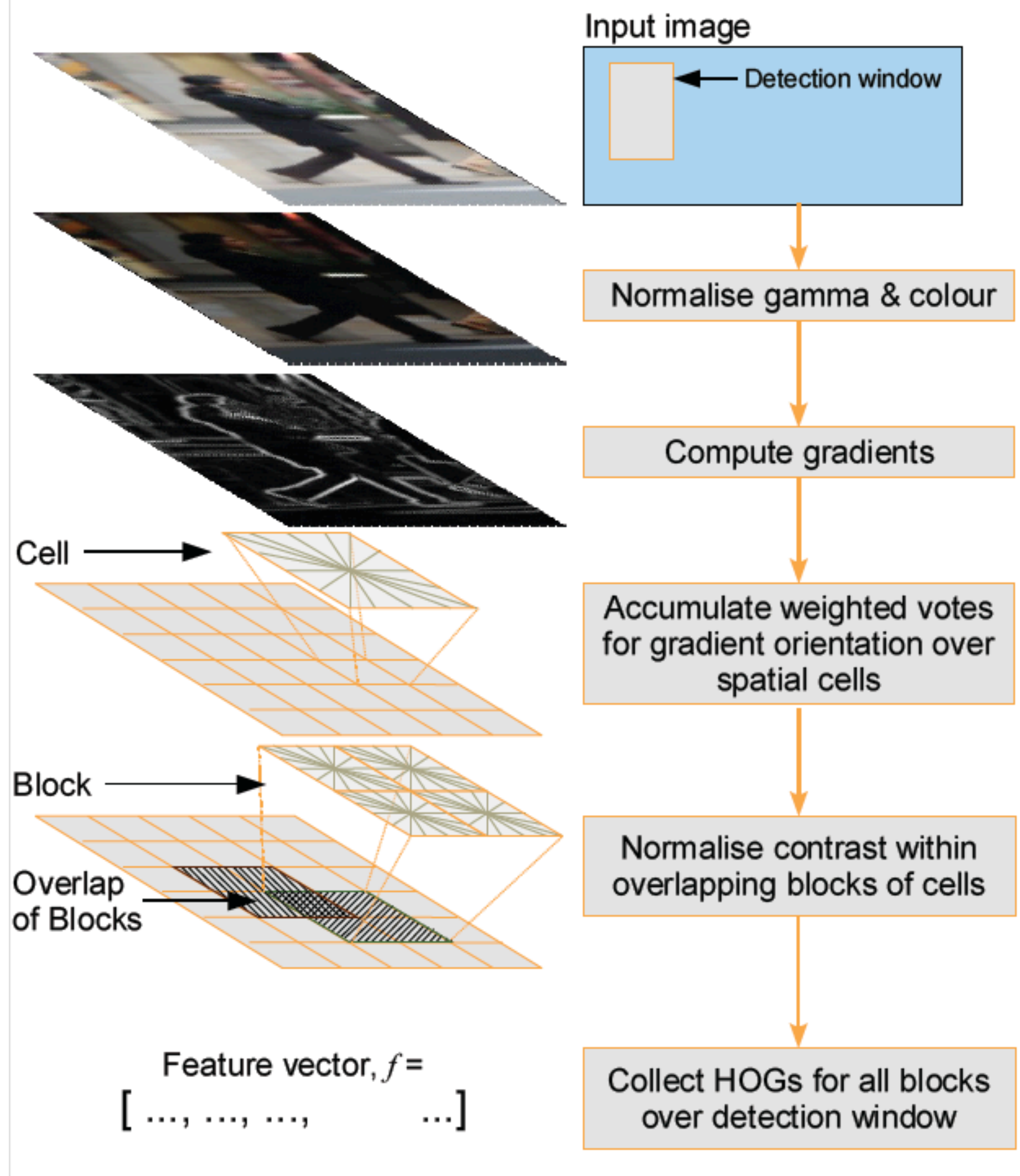
The figure illustrates only 8x8 pixel neighborhood that is transformed into 2x2 blocks, for visibility

PCA-SIFT

- Instead of using 8 fixed bins for the histogram of gradients
- Learn the principal axes of all gradients observed in training images
- For a given interest point
    - Compute SIFT with
    - Gradients in the vicinity of the point projected onto the principal axes

Histogram of Oriented Gradients

HOG is a histogram of orientations of the image gradients within a patch



Evaluation of Feature Detectors and Descriptors

Evaluation of Detectors

- Find corresponding points in two images showing the same scene.
- After projecting image 2 to image 1:
- Corresponding features have an overlap defined by the ratio of intersection and union of their associated ellipses:
- Similarity = intersection / union

Repeatability of Detectors

image 1

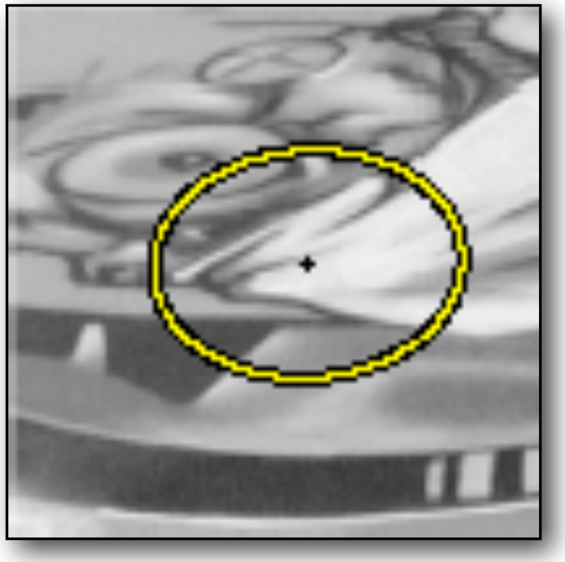
image 2

Evaluation of Detectors

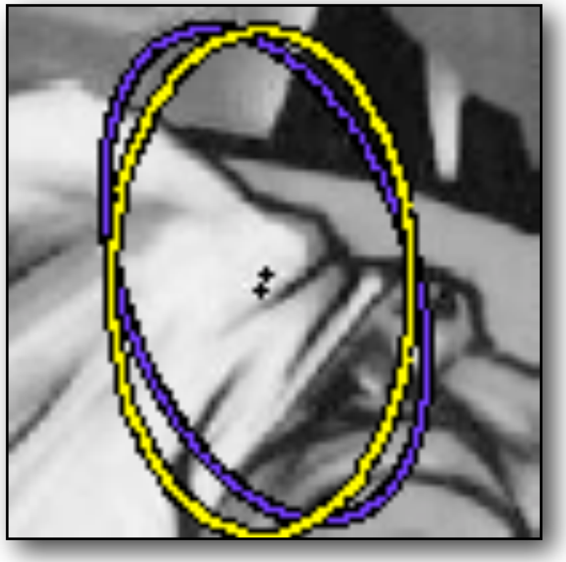
image 1

image 2

point detected in image 1



point detected in image 2



projection of image 2 to image 1

Evaluation of Descriptors -- Task Driven

- Match points in two images showing the same scene
- Matches are nearest neighbors in the descriptor space
- Precision: percentage of correctly matched feature points of the total number of matches

Matching Cost of Two Descriptors

- Euclidean distance: $\psi(\mathbf{d}_1, \mathbf{d}_2) = \|\mathbf{d}_1 - \mathbf{d}_2\|^2$

- Chi-squared distance: $\psi(\mathbf{d}_1, \mathbf{d}_2) = \sum_i \frac{(\mathbf{d}_{1i} - \mathbf{d}_{2i})^2}{\mathbf{d}_{1i} + \mathbf{d}_{2i}}$

Matching Formulation

Given two sets of descriptors to be matched

$$V = \{\mathbf{d}_1, \mathbf{d}_2, \ldots, \mathbf{d}_N\}, \text{ and } V' = \{\mathbf{d}'_1, \mathbf{d}'_2, \ldots, \mathbf{d}'_M\}$$

Find the legal mapping $f \in \mathcal{F}$

$$f := \{(\mathbf{d}, \mathbf{d}') : \ \mathbf{d} \in V, \ \ \mathbf{d}' \in V'\}$$

Which minimizes the total cost of matching

$$\hat{f} = \min_{f \in \mathcal{F}} \sum_{(\mathbf{d}, \mathbf{d}') \in f} \psi(\mathbf{d}, \mathbf{d}'), \ \ \psi(\mathbf{d}, \mathbf{d}') \geq \mathbf{0}$$

Total Cost of Matching

$$A = \begin{bmatrix} \psi_{11'} & \psi_{12'} & \psi_{13'} & \dots & \psi_{1M} \\ \psi_{21'} & \psi_{22'} & \psi_{23'} & \dots & \psi_{2M} \\ & & \dots & & \end{bmatrix}_{N \times M}$$

cost matrix

$$\sum_{(\mathbf{d},\mathbf{d}')} \psi(\mathbf{d}, \mathbf{d}') = \mathrm{tr}(A^T I)$$

identity matrix

Total Cost of Matching

$$A = \begin{bmatrix} \psi_{11'} & \psi_{12'} & \psi_{13'} & \cdots & \psi_{1M} \\ \psi_{21'} & \psi_{22'} & \psi_{23'} & \cdots & \psi_{2M} \\ & & \cdots & & \end{bmatrix}_{N \times M}$$

cost matrix

$$\sum_{(\mathbf{d},\mathbf{d}')} \psi(\mathbf{d}, \mathbf{d}') = \mathrm{tr}(A^T I)$$

identity matrix

$$\sum_{(\mathbf{d},\mathbf{d}') \in f} \psi(\mathbf{d}, \mathbf{d}') = ?$$

Linearization

Linearization by introducing an indicator matrix

$$X = \begin{bmatrix} 0 & 0 & 1 & ... & 0 & 0 \\ 0 & 1 & 0 & ... & 0 & 0 \\ & & ... & & & \end{bmatrix}_{N \times M}$$

$$x(\mathbf{d}, \mathbf{d}') = 1, \;\; \text{if} \;\; (\mathbf{d}, \mathbf{d}') \in f \;\; \text{matched pair}$$

$$x(\mathbf{d}, \mathbf{d}') = 0, \;\; \text{if} \;\; (\mathbf{d}, \mathbf{d}') \notin f \;\; \text{unmatched pair}$$

Linearization

$$A = \begin{bmatrix} \psi_{11'} & \psi_{12'} & \psi_{13'} & \dots & \psi_{1M} \\ \psi_{21'} & \psi_{22'} & \psi_{23'} & \dots & \psi_{2M} \\ & & \dots & & \end{bmatrix}_{N \times M}$$

$$X = \begin{bmatrix} 0 & 0 & 1 & \dots & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 & 0 \\ & & \dots & & & \end{bmatrix}_{N \times M}$$

$$\Downarrow$$

$$\sum_{(\mathbf{d},\mathbf{d}') \in f} \psi(\mathbf{d}, \mathbf{d}') = \sum_{(\mathbf{d},\mathbf{d}')} \psi_{\mathbf{d},\mathbf{d}'} x_{\mathbf{d},\mathbf{d}'} = \mathrm{tr}(A^T X)$$

Matching Formulation

$$\hat{X} = \min_{X} \ \mathrm{tr}(A^T X)$$

$$\Downarrow$$

$$\hat{X} = \mathbf{0}$$ trivial solution

$$\Downarrow$$

we need to constrain the formulation

Matching Formulation

$$\min_{X} \ \mathrm{tr}(A^T X)$$

subject to:

$$\forall \mathbf{d} \in V, \ \forall \mathbf{d}' \in V', \ x_{\mathbf{d}\mathbf{d}'} \in \{0, 1\}$$

$$\forall \mathbf{d}, \quad \sum_{\mathbf{d}'} x_{\mathbf{d}\mathbf{d}'} = 1$$

$$\forall \mathbf{d}', \quad \sum_{\mathbf{d}} x_{\mathbf{d}\mathbf{d}'} = 1$$

what is the meaning of this constraint?

Matching Formulation

$$\min_{X} \; \mathrm{tr}(A^T X)$$

subject to:

$$\forall \mathbf{d} \in V,\; \forall \mathbf{d}' \in V',\; x_{\mathbf{d}\mathbf{d}'} \in \{0, 1\}$$

$$\forall \mathbf{d}, \quad \sum_{\mathbf{d}'} x_{\mathbf{d}\mathbf{d}'} = 1$$

one-to-one matching

$$\forall \mathbf{d}', \quad \sum_{\mathbf{d}} x_{\mathbf{d}\mathbf{d}'} = 1$$

Relaxation

$$\min_X \ \mathrm{tr}(A^T X)$$

subject to:

$$\forall \mathbf{d} \in V, \ \forall \mathbf{d}' \in V', \ x_{\mathbf{dd}'} \in [0, 1]$$

$$\forall \mathbf{d}, \quad \sum_{\mathbf{d}'} x_{\mathbf{dd}'} = 1$$

one-to-one matching

$$\forall \mathbf{d}', \quad \sum_{\mathbf{d}} x_{\mathbf{dd}'} = 1$$

Linear Assignment Problem

$$\min_X \ \mathrm{tr}(A^T X)$$

subject to:

$$\forall \mathbf{d} \in V,\ \forall \mathbf{d}' \in V',\ x_{\mathbf{dd}'} \in [0,1]$$

$$\forall \mathbf{d},\quad \sum_{\mathbf{d}'} x_{\mathbf{dd}'} = 1$$

one-to-one matching

$$\forall \mathbf{d}',\quad \sum_{\mathbf{d}} x_{\mathbf{dd}'} = 1$$

Hungarian algorithm for the balanced problem |V| = |V'|

Hungarian Algorithm

The Hungarian Algorithm

1. Find the min element along each row (column) of *A*, and subtract it from all elements in the respective rows (columns). Replace *A* with the resulting matrix.

2. Cross out the minimum number of rows and columns in *A* to cover all zero elements of *A*

3. If min(*N*,*M)* rows and columns of *A* are crossed out, then go to step 5.

4. Otherwise, find the minimal entry of *A* that is not crossed out. Add this entry to all elements that are doubly crossed out (by both a horizontal and vertical line), and subtract it from all entries of *A* that are not crossed out. Return to step 2 with the new matrix.

5. Solutions are zero elements of A. Go first for the zero element which is unique in its row and column. Then, delete that row and column from A. Repeat until you delete all rows or columns from A.

Example -- The Hungarian Algorithm

given a
cost matrix

$$A = \begin{bmatrix} 14 & 5 & 8 & 7 \\ 2 & 12 & 6 & 5 \\ 7 & 8 & 3 & 9 \\ 2 & 4 & 6 & 10 \end{bmatrix}$$

$\Downarrow$ step 1: find minimums in each row and subtract

$$A = \begin{bmatrix} 9 & 0 & 3 & 2 \\ 0 & 10 & 4 & 3 \\ 4 & 5 & 0 & 6 \\ 0 & 2 & 4 & 8 \end{bmatrix}$$

Example -- The Hungarian Algorithm -- Solution

go for the unique solution first

step 5:

$$A = \begin{array}{cc} \begin{matrix} \\ \\ \rightarrow \\ \\ \end{matrix} & \begin{bmatrix} 10 & 0 & 3 & 0 \\ 0 & 9 & 3 & 0 \\ 5 & 5 & \boxed{0} & 4 \\ 0 & 1 & 3 & 5 \end{bmatrix} \\ & \begin{matrix} & & \uparrow & \end{matrix} \end{array}$$

$$f = \{(\mathbf{d}_3, \mathbf{d}'_3)\}$$

Example -- The Hungarian Algorithm -- Solution

go for the unique solution first

step 5:

$$A = \begin{bmatrix} & 10 & 0 & 3 & 0 \\ & 0 & 9 & 3 & 0 \\ & 5 & 5 & \boxed{0} & 4 \\ \rightarrow & \boxed{0} & 1 & 3 & 5 \\ & \uparrow & & & \end{bmatrix}$$

$$f = \{(\mathbf{d}_3, \mathbf{d}'_3),\ (\mathbf{d}_4, \mathbf{d}'_1)\}$$

Example -- The Hungarian Algorithm -- Solution

go for the unique solution first

step 5:

$$A = \begin{bmatrix} \rightarrow & 10 & \boxed{0} & 3 & 0 \\ & 0 & 9 & 3 & 0 \\ & 5 & 5 & \boxed{0} & 4 \\ & \boxed{0} & 1 & 3 & 5 \\ & & \uparrow & & \end{bmatrix}$$

$$f = \{(\mathbf{d}_3, \mathbf{d}'_3),\ (\mathbf{d}_4, \mathbf{d}'_1),\ (\mathbf{d}_1, \mathbf{d}'_2)\}$$

Example -- The Hungarian Algorithm -- Solution

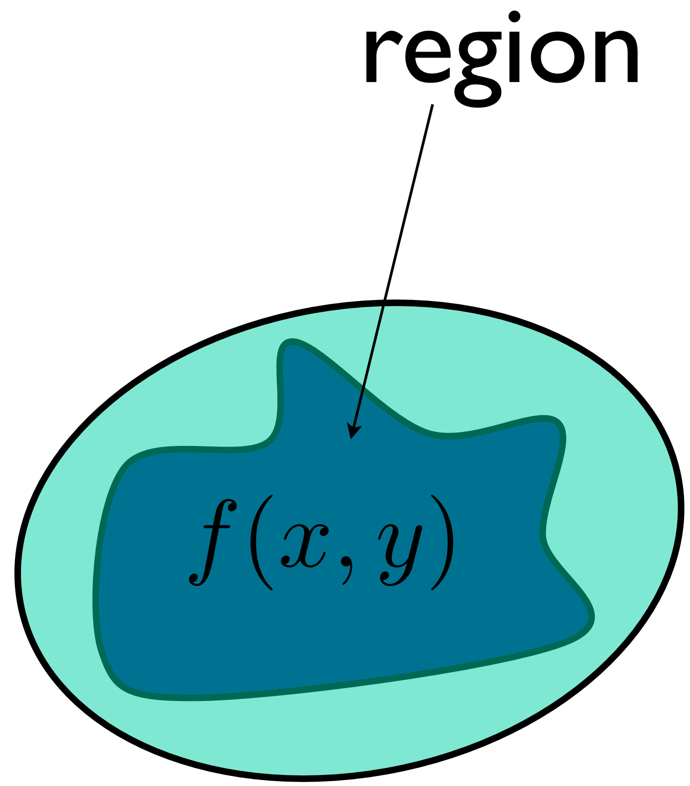
go for the unique solution first

step 5:

$$A = \begin{array}{c} \\ \rightarrow \\ \\ \\ \\ \end{array} \begin{array}{c} \left[\begin{array}{cccc} 10 & \boxed{0} & 3 & 0 \\ 0 & 9 & 3 & \boxed{0} \\ 5 & 5 & \boxed{0} & 4 \\ \boxed{0} & 1 & 3 & 5 \end{array}\right] \\ \begin{array}{cccc} & & & \uparrow \end{array} \end{array}$$

$$f = \{(\mathbf{d}_3, \mathbf{d}'_3),\ (\mathbf{d}_4, \mathbf{d}'_1),\ (\mathbf{d}_1, \mathbf{d}'_2),\ (\mathbf{d}_2, \mathbf{d}'_4)\}$$

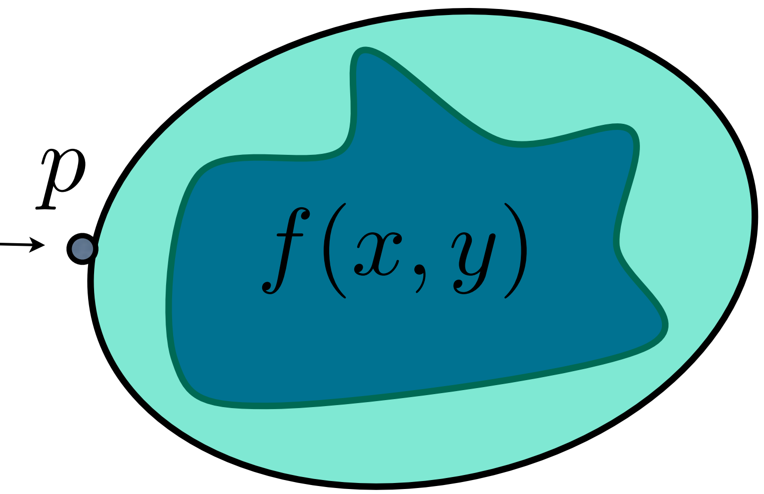
How to Fit an Ellipse to a Region



$$f(x,y) = \left\{ \begin{array}{ll} 1 & \text{inside the region} \\ 0 & \text{otherwise} \end{array} \right]$$

Ellipse

$$p^T\Sigma^{-1}p = 1$$



$$\Sigma = \begin{bmatrix} m_{20} & m_{11} \\ m_{11} & m_{02} \end{bmatrix}$$

CM = center of mass

$$CM_x = \int_{\mathbb{R}^2} xf(x,y)dx \qquad CM_y = \int_{\mathbb{R}^2} yf(x,y)dx$$

$$m_{pq} = \int_{\mathbb{R}^2} (x - CM_x)^p(y - CM_y)^q f(x,y)dxdy$$

Next Class

- Shape Descriptors
- Image segmentation