ECE 468 / CS 519: Digital Image Processing

Gradients, Harris Corners

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Image Gradient along X-axis

\[ I_x(x, y) = I(x + 1, y) - I(x, y) \]

\[ = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \star I(x, y) \]

\[ D_x(x, y) \]
Image Gradient along Y-axis

\[ I_y(x, y) = I(x, y + 1) - I(x, y) \]

\[
\begin{bmatrix}
0 & 0 & 0 \\
0 & -1 & 0 \\
0 & 1 & 0 \\
\end{bmatrix}
\]

\[ D_y(x,y) \]

\[ *I(x, y) \]
Filtering Image Gradient

\[ w(x, y; \sigma) \ast I_x(x, y) = w(x, y; \sigma) \ast D_x(x, y) \ast I(x, y) \]

\[ = [w(x, y; \sigma) \ast D_x(x, y)] \ast I(x, y) \]

convolution is associative
Filtering Image Gradient

\[ w(x, y; \sigma) * I_x(x, y) = w(x, y; \sigma) * D_x(x, y) * I(x, y) \]

\[ = [w(x, y; \sigma) * D_x(x, y)] * I(x, y) \]

\[ = [D_x(x, y) * w(x, y; \sigma)] * I(x, y) \]

convolution is commutative
Filtering Image Gradient

\[ w(x, y; \sigma) \ast I_x(x, y) = w(x, y; \sigma) \ast D_x(x, y) \ast I(x, y) \]

\[ = [w(x, y; \sigma) \ast D_x(x, y)] \ast I(x, y) \]

\[ = [D_x(x, y) \ast w(x, y; \sigma)] \ast I(x, y) \]

\[ = w_x(x, y; \sigma) \ast I(x, y) \]

derivative of the filter
Weighted Image Gradient

\[ w(x, y; \sigma) * I_x(x, y) = w_x(x, y; \sigma) * I(x, y) \]

\[ w(x, y; \sigma) * I_y(x, y) = w_y(x, y; \sigma) * I(x, y) \]

Image is discrete \( \Rightarrow \) Gradient is approximate

We always find the gradient of the filter!
Interest Points

• Harris corners
Properties of Interest Points

- Locality -- robust to occlusion, noise
- Saliency -- rich visual cue
- Stable under affine transforms
- Distinctiveness -- differ across distinct objects
- Efficiency -- easy to compute
Example of Detecting Harris Corners
Harris Corner Detector

scanning window

homogeneous region

\[ \downarrow \]

no change in all directions

edge

\[ \downarrow \]

no change along the edge

corner

\[ \downarrow \]

change in all directions

Source: Frolova, Simakov, Weizmann Institute
Harris Corner Detector

\[ E(x, y) = w(x, y) \ast [I(x + u, y + v) - I(x, y)]^2 \]

2D convolution

Source: Frolova, Simakov, Weizmann Institute
Harris Corner Detector

\[ E(x, y) = \sum_{m,n} w(m, n) [I(x + u + m, y + v + n) - I(x + m, y + n)]^2 \]

change  weighting window  shifted image  original image

Source: Frolova, Simakov, Weizmann Institute
Harris Detector Example

\[ I(x, y) \quad E(x, y) \]

input image \hspace{2cm} 2D map of changes
Harris Corner Detector

Taylor series expansion

For small shifts

\[
\begin{align*}
    u & \to 0 \\
    v & \to 0
\end{align*}
\]

\[
I(x + u, y + v) \approx I(x, y) + \frac{\partial I}{\partial x} u + \frac{\partial I}{\partial y} v
\]

\[
I(x + u, y + v) \approx I(x, y) + [I_x \quad I_y] \begin{bmatrix} u \\ v \end{bmatrix}
\]

image derivatives along x and y axes
Harris Corner Detector

\[ E(x, y) = w(x, y) \ast \left( I(x, y) + [I_x \ I_y] \begin{bmatrix} u \\ v \end{bmatrix} - I(x, y) \right)^2 \]

\[ = w(x, y) \ast \left( [I_x \ I_y] \begin{bmatrix} u \\ v \end{bmatrix} \right)^2 \]

\[ = w(x, y) \ast \left( [I_x \ I_y] \begin{bmatrix} u \\ v \end{bmatrix} \right)^T \left( [I_x \ I_y] \begin{bmatrix} u \\ v \end{bmatrix} \right) \]
Harris Corner Detector

\[ E(x, y) = w(x, y) \ast \begin{bmatrix} u \\ v \end{bmatrix} \begin{bmatrix} I_x \\ I_y \end{bmatrix} \begin{bmatrix} I_x & I_y \\ I_x & I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \]

\[ = [u \ v] \left( w(x, y) \ast \begin{bmatrix} I_x^2(x, y) \\ I_x(x, y)I_y(x, y) \\ I_x(x, y)I_y(x, y) \\ I_y^2(x, y) \end{bmatrix} \right) \begin{bmatrix} u \\ v \end{bmatrix} \]

\[ = [u \ v] \left( M(x, y) \right) \]
Filtering Image Gradient

\[ w(x, y; \sigma) \ast I_x(x, y) = w(x, y; \sigma) \ast D_x(x, y) \ast I(x, y) \]

\[ = [w(x, y; \sigma) \ast D_x(x, y)] \ast I(x, y) \]

\[ = [D_x(x, y) \ast w(x, y; \sigma)] \ast I(x, y) \]

\[ = w_x(x, y; \sigma) \ast I(x, y) \]

derivative of the filter
Harris Corner Detector

\[ E(x, y) = \begin{bmatrix} u & v \end{bmatrix} M(x, y) \begin{bmatrix} u \\ v \end{bmatrix} \]

\[ M(x, y) = \begin{bmatrix} (w_x \ast I)^2 & (w_x \ast I)(w_y \ast I) \\ (w_x \ast I)(w_y \ast I) & (w_y \ast I)^2 \end{bmatrix}_{(x, y)} \]
Eigenvalues and Eigenvectors of $M$

Eigenvalues of $M$ -- the amount of change along eigenvectors

Eigenvectors of $M$ -- directions of the largest change of $E(x,y)$

$E(x, y) = [u \ v] M(x, y) \begin{bmatrix} u \\ v \end{bmatrix}$
Detection of Harris Corners using the Eigenvalues of $M$

- "Corner": $\lambda_1$ and $\lambda_2$ are large, $\lambda_1 \sim \lambda_2$; $E$ increases in all directions.
- "Edge": $\lambda_1 \gg \lambda_2$.
- "Flat" region.
Harris Detector

\[ f(\sigma) = \frac{\lambda_1(\sigma)\lambda_2(\sigma)}{\lambda_1(\sigma) + \lambda_2(\sigma)} = \frac{\det(M(\sigma))}{\text{trace}(M(\sigma))} \]

objective function
Harris Detector Example

input image
Harris Detector Example

f values color coded
red = high values
blue = low values
Harris Detector Example

f values thresholded
Harris Detector Example

Harris features = Spatial maxima of f values
Example of Detecting Harris Corners
Properties of Harris Corners

- Invariance to variations of imaging parameters:
  - Illumination?
  - Camera distance, i.e., scale?
  - Camera viewpoint, i.e., affine transformation?
Harris/Hessian Detector is NOT Scale Invariant

Source: L. Fei-Fei
Automatic scale selection

Function responses for increasing scale
Scale trace (signature)

Source: Tuytelaars
Automatic scale selection

Function responses for increasing scale
Scale trace (signature)

\[ f(I_{i_1, \ldots, i_m}(x, \sigma)) \]

Source: Tuytelaars
Automatic scale selection

Function responses for increasing scale
Scale trace (signature)

\[ f(I_{i_1...i_m}(x, \sigma)) \]

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Function responses for increasing scale
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$f(I_{i_{1..i_m}}(x, \sigma))$

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Automatic scale selection

Function responses for increasing scale
Scale trace (signature)

$f(I_{i_1, ..., i_m}(x, \sigma))$

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Function responses for increasing scale
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Function responses for increasing scale
Scale trace (signature)

$f(I_{i_1...i_m}(x, \sigma))$

$f(I_{i_1...i_m}(x', \sigma))$

Source: Tuytelaars
Automatic scale selection

Function responses for increasing scale
Scale trace (signature)

$f(I_{i_1...i_m}(x,\sigma))$

$f(I_{i_1...i_m}(x',\sigma))$

Source: Tuytelaars
Automatic scale selection

Function responses for increasing scale
Scale trace (signature)

Source: Tuytelaars