Stochastic Simulation of Uplift Load-Displacement Behavior of Helical Anchors in Clays

by

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ABSTRACT: Recent studies have indicated that the uplift load-displacement behavior of helical anchors installed and tested within the same soil deposits and depths has exhibited a significant amount of variability. Appropriate estimates of displacement should consider the uncertainty associated with both the inherent soil variability and the uncertainty associated with the selected load-displacement model. This paper uses a load test database of uplift loading tests of helical anchors within fine-grained plastic soils to estimate and assess the uncertainty associated with inherent soil variability and model error for the purposes of simulating load-displacement behavior. Two load-displacement models are calibrated and their statistical performance quantified using normalized displacement and normalized load. Fitting of the model curves to the empirical data resulted in highly correlated model parameters. Thereafter, bivariate probability distribution functions were generated to allow adequate joint simulation of the load-displacement model parameters. Calibrated copula models were shown to provide superior estimates of load-displacement model parameters, and the resulting load-displacement curves suitably replicate the observed joint displacement uncertainty.

INTRODUCTION

The need to estimate displacement of foundation elements under service and strength limit states is increasing to meet the demand for improved design efficiency. Simultaneously, efforts are being focused on the calibration of probabilistic methods to estimate performance, such that serviceability and strength limit states can be tied to a particular acceptable probability of achieving the limit state. Central to these design movements is the characterization of the effect of natural or inherent soil variability and model error in the resulting uncertainty in displacement estimates of foundation elements.

Uncertainty in load-displacement behavior of helical anchors, despite similarities in geologic setting, loading protocols, and anchor geometry between tests has been well-documented. Clemence (1983) and Mooney et al. (1985) report the results of eight loading tests on multi-helix anchor constructed within marine clay. The helical anchors were installed to depths of 4 to 12 plate diameters. The anchor capacity varied about 20 percent, and the correlation with depth of embedment appeared weak. Lutenegger (2008) described the results of nine uplift load tests of single helix anchors in varved clay. The anchors had diameters that ranged from 203 to 406 mm and were installed to depths ranging from 7.5 to 23 plate diameters. Lutenegger (2008) showed that the load-displacement data appeared to follow a common curve when the displacements were normalized by plate diameter D and the anchor capacity was normalized by the uplift resistance at a displacement $0.1D$. Nonetheless, the resulting data exhibited variability on the order of 25 percent. Additionally, Stuedlein and Young (2013) describe the results of seven uplift loading tests of helical anchors in Beaumont clay, the results of which indicated significant uncertainty in load-displacement performance.

This paper assesses the uncertainty in the load-displacement behavior of helical anchors installed within cohesive soils and loaded in uplift by evaluating a load test database. Load-displacement models are fitted to the observed load-displacement data, and the performance of the models to represent load-displacement behavior is quantified. Following fitting of statistical distribution functions to the empirical load-displacement models, bivariate probability distributions are calibrated and evaluated for their ability to adequately replicate correlated load displacement model parameters for simulation purposes. This paper shows that the uncertainty in uplift load-displacement response of helical anchors can be appropriately simulated for the purposes of determining the upper and lower bounds of possible behavior as well as for calibration of future probabilistic serviceability limit state design procedures.

LOAD TEST DATABASE

A database of thirty-seven helical anchor uplift loading tests was compiled from seven sources, as indicated in Table 1. The tests were performed in soil profiles consisting of predominantly fine-grained plastic soils (i.e., anchors derived their support from cohesive soils). Criteria used to evaluate the suitability of the load test information included: (1) satisfactory characterization of soil profile, (2) satisfactory understanding of load-test procedures and anchor geometry, (3) relatively uniform conditions in proximity to anchor failure zone, (4) relatively rapid shearing of soil such that undrained conditions could be assumed, and (5) whether the development of sufficient displacement was achieved. Table 1 summarizes the geometry of the 18 single helix and 19 multi-helix anchors comprising the database. The anchor plate widths ranged from 76 to 406 mm in diameter, D, and the average plate diameter was 265 mm. The embedment ratio, H/D, describes the depth, H, to the shallowest helical anchor plate normalized by the plate diameter, and ranged from 0.12 to 48, with an average value of 14. In practice, H/D is generally limited to 5 or greater (Mooney et al. 1985, Seider 2012); however, lower values were included in this analyses conducted herein in order to account for the load-displacement behavior of possible shallow anchors. The undrained shear strength, s_u, ranged from 5 kPa to 239 kPa, and adequately spans the range in s_u possible for soil. The load test database, though limited to 37 tests, provides a suitable range in geometry and soil conditions typical in engineering applications of helical anchors.

CONDITIONING OF LOAD-DISPLACMENT DATA

Serviceability limit state (SLS) design presents a desirable performance evaluation framework owing to the ability to predict displacement, s, directly. Phoon and Kulhawy (2008) and Lutenegger (2008) have determined that the dispersion in experimentally observed load-displacement, or Q-s, behavior can be decreased by considering normalized behavior, for example, Q divided by a reference capacity versus s divided by D. Lutenegger (2008) selected the capacity at 10 percent of the plate diameter, an admittedly arbitrary value, but appropriate in light of the lack of observed sensitivity of Q-s performance when normalized at other reference capacities. This study followed the work of Lutenegger (2008) by representing the displacement using the normalized anchor displacement, η =

Table 1. Helical anchor database used to calibrate generalized load-displacement data for uplift of helical anchors in clays.

Test Designation	H/D	S/D	D_{ave} (mm)	Ultimate Resistance, Q_{ult} (kN)	Slope Tangent Capacity, Q_{STC} (kN)	s_u (kPa)	k_1	k_2	Reference
A1	0.12	-	76	0.09	0.09	5	0.003	1.010	1
A2	0.33	-	76	0.15	0.12	5	0.018	0.735	1
A3	1.09	-	76	0.22	0.18	5	0.014	0.767	1
A4	3	-	76	0.29	0.21	5	0.023	0.675	1
A5	3	-	76	0.29	0.23	5	0.015	0.757	1
C1	4	3.76	248	53.2	42	24	0.016	0.723	2
C2	4	3.76	248	53.2	42.5	24	0.015	0.746	2
C3	8	3.76	248	49.4	45	24	0.007	0.883	2
C4	8	3.76	248	51.7	44	24	0.010	0.813	2
C5	10	3.76	248	49.3	44	24	0.006	0.869	2
C6	10	3.76	248	47.2	41.5	24	0.007	0.856	2
C7	12	3.76	248	49.6	47	24	0.005	0.926	2
C8	12	3.76	248	46.9	41.5	24	0.007	0.856	2
H1	10.5	3.6	254	134	128	79	0.006	0.887	3
LA-1	7.5	-	203	-	8.3	55	0.115	0.280	4
LA-2	5.0	-	305	-	23.3	55	0.099	0.334	4
LA-3	3.7	-	406	-	47	55	0.038	0.564	4
LA-4	15.0	-	203	-	21.9	191	0.050	0.486	4
LA-5	10.0	-	305	78	47	191	0.033	0.612	4
LA-6	7.5	-	406	-	49	191	0.084	0.341	4
LA-7	22.5	-	203	-	17	75	0.076	0.349	4
LA-8	15.0	-	305	-	39	75	0.048	0.498	4
LA-9	11.3	-	406	-	55	75	0.043	0.428	4
LB-1	27.8	0.75	200	-	13	31	0.041	0.513	5
LB-2	23.3	3	200	36.7	21	31	0.041	0.565	5
S3	18.97	3	330	364	295	97	0.021	0.704	6
S5	18.97	3	345	395	375	97	0.012	0.846	6
S6	18.97	3	345	310	270	86	0.046	0.579	6
S7	18.97	3	330	516	445	85	0.021	0.695	6
TC1	35.0		279	359	221	168	0.032	0.601	7
TC2	18.0		356	158	115	120	0.019	0.726	7
TC3	20.6		356	216	149	168	0.022	0.691	7
TC4	26.0		279	284	181	168	0.028	0.638	7
TC5	26.0		254	286	197	168	0.023	0.688	7
TC6	22.3		305	454	290	168	0.029	0.639	7
Lo1	48.0	-	203	277	175	239	0.024	0.592	7
Lo2	21.0	-	356	250	178	239	0.030	0.619	7

[1] Ali (1969) [2] Clemence (1983) [3] Handojo (1997) [4] Lutenegger (2008)
[5] Lutenegger (2009) [6] Studlein and Young (2013) [7] Seider (2012)

s/D_{ave}, where D_{ave} equals the average plate diameter. The uplift capacity at each observed displacement value was normalized by the slope tangent capacity (Q_{STC}, Phoon and Kulhawy 2008) defined $\eta = s/D_{ave} = 0.05$ (Figure 1). The slope tangent capacity at 5 percent displacement was selected to represent levels of capacity and displacement commonly achieved in production anchors, and is not intended to represent an ultimate resistance, Q_{ULT}, associated with continuous pullout. However, Q_{ULT} is strongly correlated to Q_{STC}, as illustrated in Figure 2, indicating that if the ultimate resistance can be satisfactorily estimated, the entire load displacement curve can be estimated if a suitable load-displacement model is referenced.

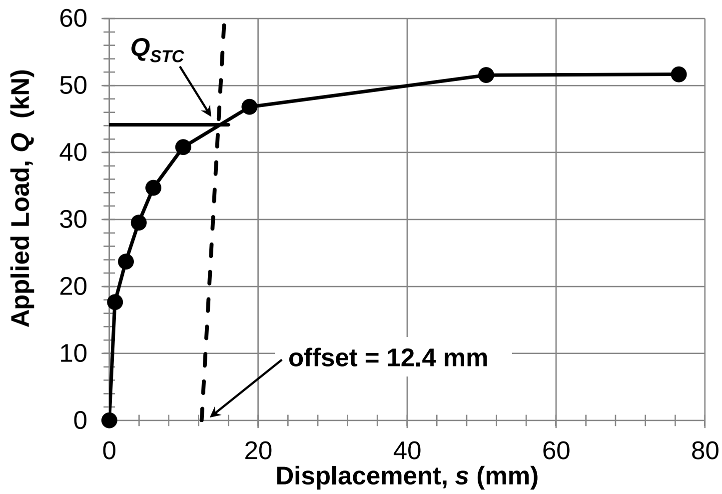

Figure 1. Definition of the slope tangent capacity, Q_{STC}; load test data for Anchor C4, with average plate diameter of 248 mm. Note: initial slope tangent determined through hyperbolic model fitting.

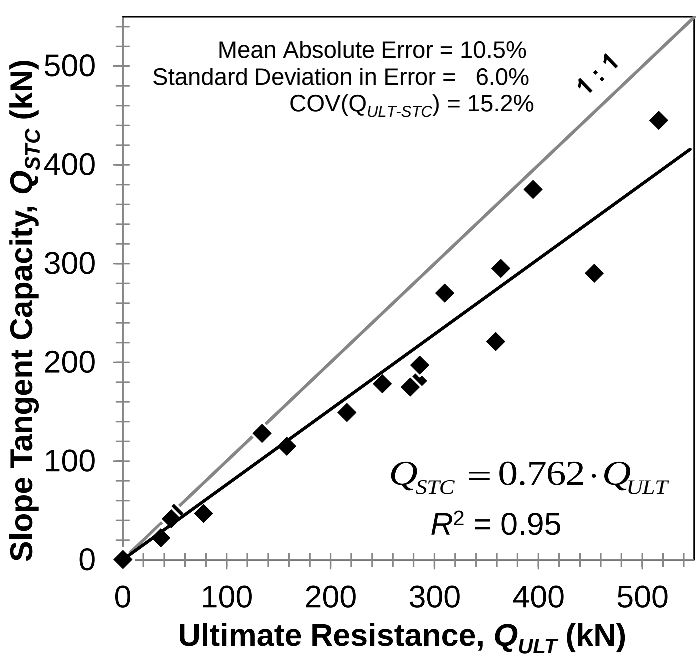

Figure 2. Correlation of ultimate resistance, Q_{ULT}, and slope tangent capacity, Q_{STC}. Only those load tests that exhibited a true ultimate resistance, or could be reliably extrapolated to an ultimate resistance, were used to generate this relationship (see Table 1).

LOAD-DISPLACMENT MODEL SELECTION

Uncertainty generally arises from aleatory and epistemic sources. Aleatory uncertainty stems from the natural or inherent variability of soil properties, in the present case, undrained shear strength. Epistemic sources of uncertainty range from variability in material fabrication (e.g., stiffness of the steel anchor), testing and measurement errors, model errors and transformation errors in model parameters. The possibility of reducing the epistemic uncertainty is non-zero upon improvement of the general knowledge of testing and measurement processes, correlations, and models. In the assessment of the load-displacement model selected for representing the uplift helical anchor data, aleatory uncertainty, transformation error, and model error are considered in a lumped sense; however, measurement uncertainty is not directly treated. Two load-displacement models were considered for this study: the hyperbolic model and power law model, based largely on their performance in previous studies on footings (e.g., Akbas and Kulhawy 2009; Uzielli and Mayne 2011a; 2011b), augered cast-in-place piles (e.g., Phoon and Kulhawy 2008), and other geotechnical elements. The hyperbolic model is given by:

$$\frac{Q}{Q_{STC}} = \frac{\eta}{k_1 + \eta \cdot k_2} \qquad [1]$$

where the Q = the applied load, Q_{STC} = the slope tangent capacity (defined below), η = the normalized displacement, defined as the displacement divided by the average plate diameter, and k_1 and k_2 are hyperbolic model parameters fitted using least squares regression. The power law model is given by:

$$\frac{Q}{Q_{STC}} = k_3 \cdot \eta^{\cdot k_4} \qquad [2]$$

where k_3 and k_4 equal an empirical fitting coefficient and exponent, respectively. Each of the uplift load-displacement curves were fit to the models represented by Eq. (1) and (2) using generalized least squares regression over the entire range in Q-s performance, resulting in the fitting parameters plotted in Figure 3. The model parameters cluster tightly and exhibit a distinct correlation. The Pearson product-moment correlation coefficient, ρ, was calculated for each model, and resulted in a value of -0.92 and 0.94 for the hyperbolic and power law models, respectively. These large correlation coefficient values indicate the existence of statistical dependence between model parameters. Operationally, this suggests that stochastic simulation of the load-displacement data should incorporate a bivariate model parameter distribution with dependent marginals in order to adequately capture the range in possible Q-s performance.

The model performance was assessed quantitatively using the average bias, λ, defined as the ratio of measured to calculated load ratio (i.e., Q/Q_{STC}), for each of the Q-s curves. Additional performance metrics included the root mean squared error ($RMSE$), the coefficient of determination (R^2), and the coefficient of variation (COV_λ), defined as the ratio of the standard deviation in bias to the mean bias. The goodness-of-fit parameters for each of the models are shown in Table 2. Based on the goodness-of-fit parameters, both the hyperbolic model and the power law model appeared to satisfactorily capture the Q-s data. The power law model parameters provide the mean bias values closest to unity, indicating an overall accurate fit, and low dispersions around the mean values; this is largely due to better fits near the initial portion of each load-displacement curve as compared to the hyperbolic model. However, the hyperbolic model best approximates the curvature of the load displacement curves through the higher coefficients of determination and lower overall average error, indicating that the hyperbolic model may better represent the entire load-displacement behavior. Although it is likely the power law model could provide a good representation of the *Q*-*s* data, the hyperbolic model was selected for further probabilistic modeling.

Table 2. Summary of statistical goodness-of-fit parameters for hyperbolic (k_1, k_2) and power models (k_3, k_4) fit to the helical anchor load-displacement curves.

			Fitted Model Parameter		Goodness-of-Fit Parameters	
	k_1	k_2	Mean λ	COV$_\lambda$	R^2	RMSE
Mean	0.032	0.647	1.083	0.207	0.972	0.053
COV	0.809	0.264	0.214	1.318	0.033	0.500
	k_3	k_4				
Mean	2.729	0.408	0.984	0.165	0.923	0.084
COV	0.388	0.446	0.084	0.849	0.086	0.500

The uplift load-displacement behavior of helical anchors in clays can be simulated through randomly sampling hyperbolic model parameters k_1 and k_2 from marginal distributions that suitably replicate sample data. Beta-type distributions were selected to model k_1 and k_2 because they allow the specification of upper and lower bounds, which is critical in light of physical soil behavior (e.g., Najjar and Gilbert 2009). The probability density function of the Beta distribution for a continuous random variable θ is given by

$$f(\theta) = \frac{1}{B(\alpha_1, \alpha_2)} \frac{(\theta - \theta_l)^{\alpha_1 - 1} (\theta_u - \theta)^{\alpha_2 - 1}}{R(\theta)^{\alpha_1 + \alpha_2 - 1}} \quad [3]$$

where $B(\alpha_1,\alpha_2)$ equals the beta function with parameters α_1 and α_2, $R(\theta) = \theta_u - \theta_l$ is the difference between the upper-bound value, θ_u, and the lower-bound value, θ_l, of the continuous random variable.

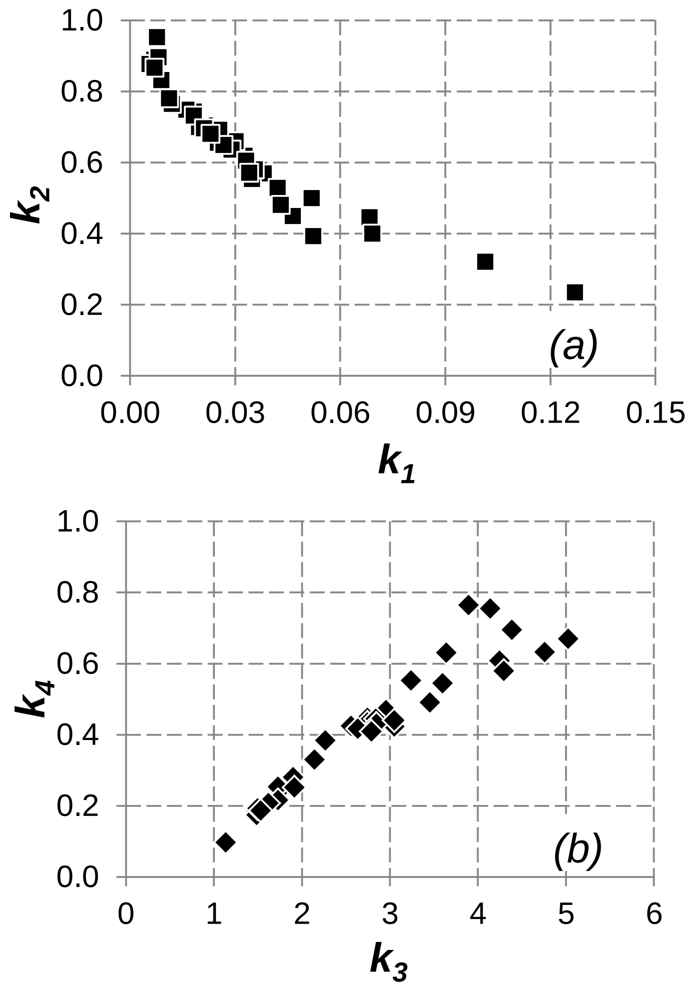

Figure 3. Scatter plots of fitted model parameters for (a) the hyperbolic model, and (b) the power model.

Beta distribution parameters were obtained by maximum likelihood estimation based on the empirical sample distribution, resulting in α_1 = 1.14, α_2 = 4.86, θ_l = 0.004 and θ_u = 0.145 for k_1, and α_1 = 3.33, α_2 = 2.67, θ_l = 0.18 and θ_u = 1.02 for k_2. Figure 4 shows the cumulative distribution functions (CDFs) of the fitted beta distributions with the empirical CDFs of k_1 and k_2. The continuous beta distributions, set within an appropriate bivariate distribution model, can be used to simulate random estimates of hyperbolic model parameters in light of their observed correlation. Note that the empirical CDFs of k_1 and k_2 could be represented by lognormal and normal distributions, respectively; however, these distribution types could provide unrealistically small values of the hyperbolic model parameters, and could therefore result in very large estimates of normalized load due to the location of the model parameters in the denominator of Eq. (1). Therefore, constrained Beta distributions are preferred for modeling the load-displacement model parameters.

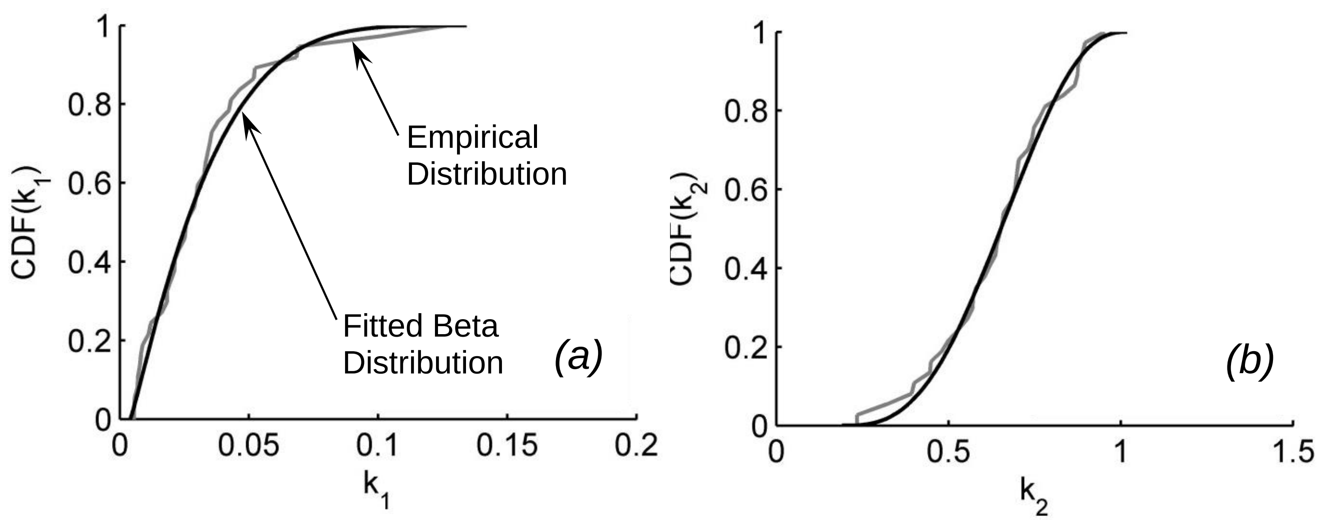

Figure 4. Empirical cumulative distribution functions of the sample hyperbolic model parameters and fitted CDFs for (a) k_1 and (b) k_2.

SELECTION OF BIVARIATE PROBABILITY DISTRIBUTION FUNCTIONS

The two-parameter load-displacement models clearly exhibit statistical correlation, indicating that any subsequent simulation should incorporate the correlation to adequately capture the possible range load-displacement behavior. Pairs of k_1 and k_2 can be represented by a calibrated bivariate probability distribution function, provided independence or randomness in marignals can be confirmed. Typically, randomness can be assessed through the comparison of model parameters k_1 and k_2 against geometrical factors, like the average plate diameter, or soil parameters, for instance the undrained shear strength. Non-parametric dependence statistics, such as Kendall's tau (Daniel 1990), are suited for the evaluation of model parameter randomness. Kendall's tau was calculated as $\tau_{D,k1}$ = 0.13, $\tau_{D,k2}$ = -0.12 for the correlation to average diameter, and $\tau_{su,k1}$ = 0.20, and $\tau_{su,k2}$ = -0.21 for correlation to undrained shear strength, and produced *p*-values of 0.32, 0.36, 0.14, and 0.13, respectively. These p-values indicate that no correlation exists for k_1 and k_2 and the average diameter and undrained shear strength at the 0.05 significance level. Several approaches are available to simulate the bivariate distribution of the load-displacement model parameters, including the translation model (e.g., Phoon and Kulhawy 2008) and the more powerful copula model (e.g., Nelsen 1999), each of which can accommodate various distribution types (e.g., normal, lognormal, etc.) with varying degrees of sophistication. The performance of these models is assessed herein to demonstrate the importance of satisfactorily modeling highly correlated bivariate uplift load-displacement data for helical anchors in uplift.

Simulation of Hyperbolic Model Parameters using the Translation Model

Model parameters k_1 and k_2 can be simulated using a bivariate probability distribution model that translates randomly generated uncorrelated standard normal random variables Z_1 and Z_2 with mean = 0 and standard deviation = 1.0 to correlated random variables X_1 and X_2:

$$X_1 = Z_1 \tag{4}$$

$$X_2 = Z_1 \cdot \rho_{\ln} + Z_2\sqrt{1-\rho_{\ln}^2} \tag{5}$$

where ρ_{ln} represents the equivalent-normal correlation coefficient for lognormally distributed parameters (Phoon and Kulhway 2008):

$$\rho_{\ln} = \frac{\ln\left[\rho \cdot \sqrt{\left(e^{\zeta_1^2}-1\right)\left(e^{\zeta_2^2}-1\right)}+1\right]}{\zeta_1 \cdot \zeta_2} \tag{6}$$

and where λ_1, ζ_1 and λ_2, ζ_2 are the approximate lognormal mean and standard deviation of k_1 and k_2, respectively, both of which were calculated from the sample mean, $\bar{k}_i$, and standard deviation, σ_i. For the lognormal case, the lognormal mean and standard deviation is calculated as:

$$\lambda_1 = \ln(\bar{k}_1) - 0.5 \cdot \zeta_1^2 \tag{7}$$

$$\zeta_1 = \sqrt{\ln(1+\sigma_1^2/\bar{k}_1^2)} \tag{8}$$

For the present case, k_1 may represented by the lognormal distribution, but k_2 is more appropriately modeled using a normal distribution. The corresponding equivalent-normal correlation coefficient for the mixed normal-lognormal distributions can be computed using:

$$\rho_{\ln} = \frac{\rho \cdot \sqrt{e^{\zeta_1^2}-1}}{\zeta_1} \tag{9}$$

Assuming model parameters k_1 and k_2 can be modeled using lognormal and normal marginal distributions, the model parameters can be simulated using:

$$k_1 = e^{(\zeta_1 \cdot X_1 + \lambda_1)} \tag{10a}$$

$$k_2 = \mu_2 + X_2 \cdot \sigma_2 \tag{10b}$$

where μ_2 and σ_2 are the normal mean and standard deviation of k_2, respectively, as long as the bounds of the equivalent-normal correlation coefficient (i.e., -1 to 1) are not encountered prior to achieving the empirical normal correlation coefficient, ρ, for the vector (k_1, k_2). Figure 5a shows the relationship of ρ to ρ_{ln} for the lognormal statistics of k_1 and k_2, and that the effective maximum value of ρ that can be modeled is -0.877. Because the actual correlation between k_1 and k_2 equals -0.92, the translation model is an inappropriate tool for modeling the empirical bivariate data representing the uplift load-displacement curves of helical anchors in clay. The inability to model the empirical k_1 and k_2 is evident in Figure 5b, which shows pairs of (k_1, k_2) resulting from 500 simulations assuming ρ_{ln} equal to -0.9 and -1.0. For the case of ρ_{ln} equal to -0.9, the scatter modeled near the middle of the data cloud is too dispersed. When ρ_{ln} is set equal to -1.0, a lognormal function results, exhibiting no scatter, and is entirely unacceptable. Thus, another approach for modeling the highly correlated bivariate hyperbolic model parameters must be evaluated.

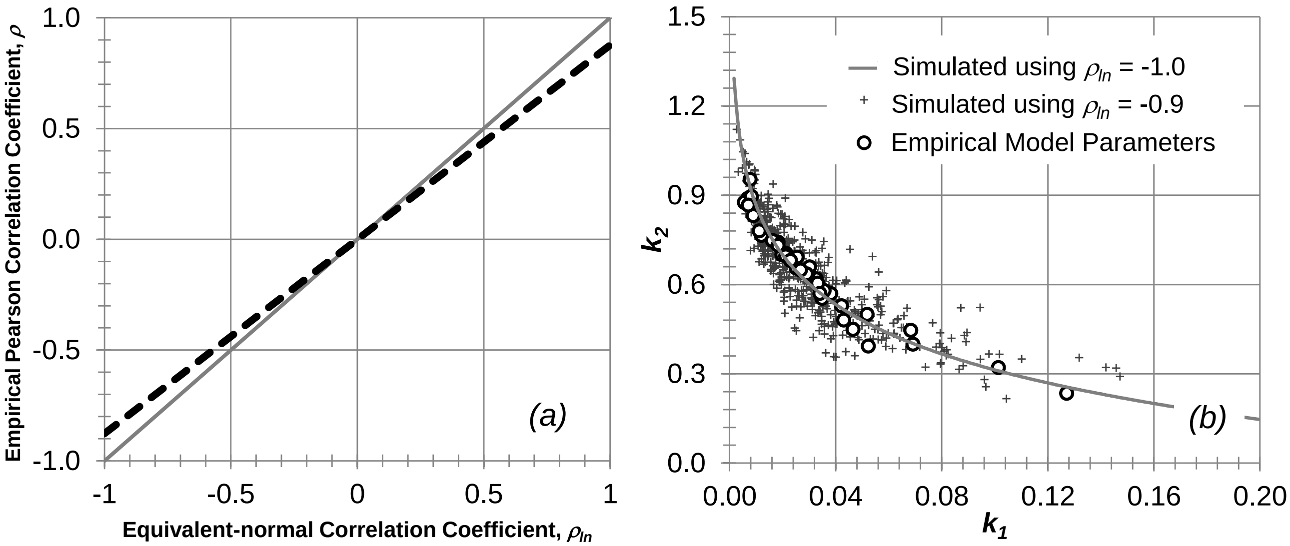

Figure 5. Performance of translation model to simulate empirical bivariate hyperbolic model parameters: (a) relationship between equivalent-normal correlation coefficient and empirical Pearson correlation coefficient, and (b) comparison of simulated hyperbolic model parameters to empirical model parameters for various equivalent normal correlation coefficients.

Simulation of Hyperbolic Model Parameters using the Copula Model

Copulas are becoming increasingly popular for modeling multivariate data, and specifically for producing the joint distribution of load-displacement data and correlated load-displacement model parameters. Li et al. (2011) employed copula theory for modeling the displacement of pile foundations. Uzielli and Mayne (2011a; 2011b) used copula theory for simulating the load-displacement performance of footings on sands. Nelson (1999) provides a general overview of copula theory; practical geotechnical application of copula theory, including fitting of copula model parameters, is provided by Li et al. (2011) and Uzielli and Mayne (2011a; 2011b).

Bivariate distributions of hyperbolic model parameters were evaluated in this study using copula models. Two kinds of copula were evaluated, including the Gaussian-type elliptical copula and the Frank-type Archimedean copula. Copula model parameters were generated in light of the correlation, modeled using Kendall's tau (τ = -0.92), between hyperbolic model parameters for the Gaussian and Frank copulas and resulted in values of -0.98 and -47.63, respectively. Bivariate Gaussian and Frank copulas were produced using the respective copula model parameters, and the beta-distributed marginal distributions were obtained through the inverse transformation of the beta distributions. Figure 6a and 6c presents the load-test derived hyperbolic model parameters and the results of 1,000 random simulations resulting from the Frank and Gaussian copula models, respectively. Compared to the results of the translation model (Figure 5b), the copulas appear to represent the scatter in hyperbolic model parameters quite well. Figures 6b and 6d present the normalized load-displacement curves (Q/Q_{STC} vs η) of the 37 uplift loading tests compared against 1,000 simulated load-displacement curves for each copula model type. The simulated normalized load-displacement curves represent the aleatory uncertainty of loading tests inherent in saturated clay deposits (i.e., the "real" variability in load-displacement behavior) and the transformation error associated with the selected normalization procedure. Due to the selected normalization procedure, uncertainty in load-displacement behavior is medium for small normalized displacements, large for large normalized displacements, and smallest near Q/Q_{STC} = 1.0. However, probabilistic modeling of any load-displacement data will necessarily entail non-zero transformation uncertainty.

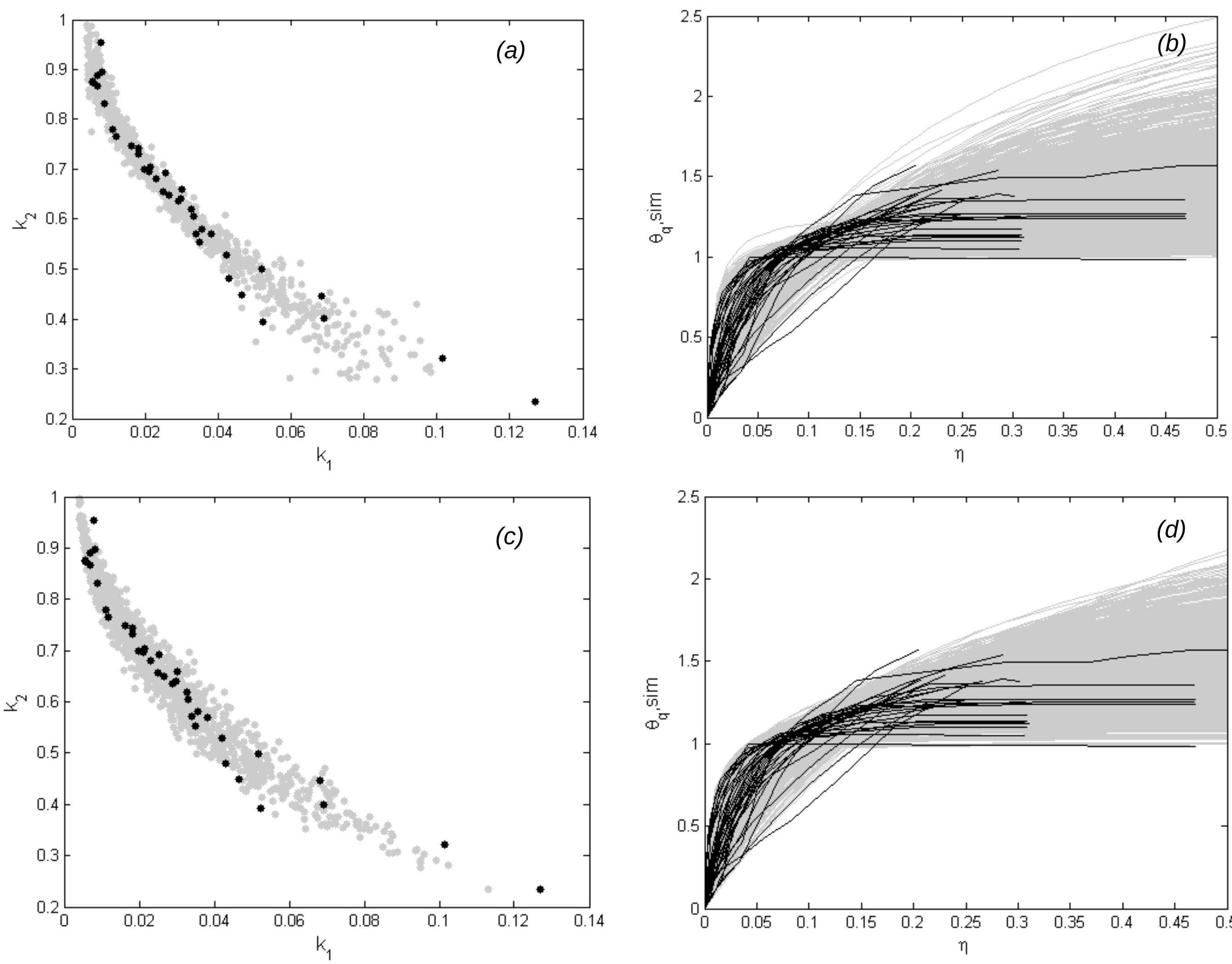

Figure 6. Results of 1,000 copula-based simulations of: (a) empirical and simulated k_1 and k_2 assuming the Frank copula model, (b) comparison of empirical and simulated uplift load-displacement curves resulting from the Frank copula model, (c) empirical and simulated k_1 and k_2 assuming the Gaussian copula model, (d) comparison of empirical and simulated uplift load-displacement curves resulting from the Gaussian copula model. Note, $\theta_{q,sim}$ represents the load ratio Q/Q_{STC}.

In practice, the best-performing copula model should be selected to represent the possible uncertainty in uplift load-displacement performance. The best-fitting copula type was determined by comparing the Euclidean distance between the empirical multi-dimensional CDFs and the simulated CDFs in terms of the discrete norm as proposed by Durrleman et al. (2000). Although it appears that both copula models appear to model the data appropriately, the Gaussian copula model quantitatively yields the best-fit copula. Serviceability limit state design procedures can be generated for uplift of helical anchors in light of the observed and estimated level of uncertainty in soil strength, capacity models, and displacement behavior through probabilistic simulation of the load-displacement performance, as illustrated herein.

CONCLUDING REMARKS

The uncertainty in the uplift load-displacement performance of helical anchors installed within clayey soils was investigated in this paper. A database of thirty-seven uplift loading tests on single and multi-helix anchors was assembled, and the data conditioned such that displacement was normalized with respect to the average plate diameter and load normalized with respect to the slope tangent offset capacity. Hyperbolic and power law model load-displacement curves were fit to the normalized load-displacement data, and their goodness-of-fit quantified. The load-displacement model parameters were found to exhibit a very high degree of correlation, indicating that the stochastic simulation of load-displacement curves would require modeling of bivariate distributions. Based on the observed load-displacement performance, both the hyperbolic and power law models were largely satisfactory; however, the hyperbolic model was selected for simulation purposes due to better goodness-of-fit statistics. Two methodologies available for the simulation of correlated bivariate data were evaluated, including the translation model and copula models. Hyperbolic model parameters simulated using the translation model assuming lognormal marginal distributions were not adequately simulated, as the degree of correlation in the translated normal distribution space could not be modeled. Bivariate distributions of the hyperbolic model parameters simulated using the Frank and Gaussian copula models and appropriate marginal Beta distributions resulted in a satisfactory representation of the observed scatter, and the resulting uncertainty in the load-displacement curves was appropriately bounded. Based on a quantitative comparison of database and simulated load-displacement curves, it was assessed that stochastic simulation of the load-displacement behavior of helical anchors loaded in uplift can be best achieved, for the available dataset, using the Gaussian copula model calibrated as described herein.

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