

Homework 1
CS 321
Due Date: 10/9/09, 2 PM

Note: The homeworks should be your own work. You can discuss the homeworks orally with your peers, however. When you use the web sources to answer questions, please be sure to cite them, and answer in your own words (don't copy verbatim which would be considered plagiarism.) All questions carry equal weight. Please see the TA and the instructor during the office hours to get more help.

1. Search the web and find 3 different applications of finite state automata other than digital logic design, neural networks, and scanners. Give a brief description or example of each application.
2. Section 1.1. Problem 4. Consider the relation between two sets defined by $S_1 \equiv S_2$ if and only if $|S_1| = |S_2|$. Note $|S|$ is the size of the set S . Show that this is an equivalence relation.
3. Section 1.1. Problem 26. Show by induction on n that

$$\sum_{i=1}^n \frac{1}{i^2} \leq 2 - \frac{1}{n} \tag{1}$$

4. Section 1.1. Problem 31. Show that $\sqrt{3}$ is not a rational number.
Hint: Follow the proof that $\sqrt{2}$ is not a rational number in the book.
5. Section 1.2. Problem 1. Use induction on n to show that $|u^n| = n|u|$ for all strings u and integers $n \geq 0$.
Hint: Use the fact that $u^{i+1} = u^i u$ and $u^0 = \lambda$.
6. Section 1.2. Problem 8. Show that $(L_1 L_2)^R = L_2^R L_1^R$.
Hint: Use the following fact (problem 2) as given. For any strings u, v over some alphabet $(uv)^R = v^R u^R$. Note that u^R is the reverse of string u . $(L)^R$ is the set of reverses of strings in L .
7. Section 1.2. Problem 14(a). Give a grammar to generate the language $L_1 = \{a^n b^m : n \geq 0, m > n\}$.
8. Section 1.2. Problem 14(b). Give a grammar to generate the language $L_2 = \{a^n b^{2n} : n \geq 0\}$.
9. **Extra Credit:** The "game of life" by Conway inspired a class of automata called cellular automata. Although we will not study cellular automata in this course, they are quite interesting. Search the web and write about what these are and what they can be used to model.