Homework 4
CS 321
Due Date: 11/4/09, 2 PM

Note: The homeworks should be your own work. You can discuss the homeworks orally with your peers, however. You should not use any web sources for this assignment. All questions carry equal weight. Please see the TA and the instructor during the office hours to get more help.

1. [4.1 Problem 6] The symmetric difference of two sets S_1 and S_2 is defined as $S_1 \ominus S_2 = \{x : x \in S_1 \text{ or } x \in S_2 \text{ but } x \text{ is not in both } S_1 \text{ and } S_2\}$.

   Show that the family of regular languages is closed under symmetric difference.

2. [4.1 Problem 17] The *TAIL* of a language is defined as all suffixes of its strings, i.e., $\text{TAIL}(L) = \{y : xy \in L \text{ for some } x \in \Sigma^*\}$. Note that a string is also its own suffix. Show that if L is regular, so is $\text{TAIL}(L)$.

3. [4.1 Problem 25] The *MIN* of a langauge L is defined as
   $MIN(L) = \{w \in L : \text{there is no } u \in L, v \in \Sigma^+, \text{ such that } w = uv\}$

   In other words, MIN(L) is the set of strings in L such that no proper prefix of those strings is in L.

   For example, if $L = ab^*$, then $MIN(L) = \{a\}$. If $L = a^*b$, then $\text{MIN}(L) = a^*b$. If $L = a^*b^*$, $\text{MIN}(L) = \lambda$.

   Show that the family of regular languages is closed undet the MIN operation.

4. [4.2 Problem 14] Find an algorithm for determining whether a regular language L contains an infinite number of even length strings.

   (You do not need to write out detailed pseudo-code for an algorithm. Just describe the main steps of your algorithm, possibly refering to one or more of the algorithms introduced in the book section.)

5. Prove that the following languages are not regular.

   (a) [4.3 problem 4c] $L = \{a^n b^l a^k : n = l \text{ or } l \neq k\}$.
   Hint: There are two conditions here, and both should be violated for the contradiction. So pick a string such that the second condition is already violated. Pumping should ensure that it stays violated, while the first condition is also violated.

   (b) [4.3 problem 4d] $L = \{a^n b^l : n \leq l\}$.

   (c) [4.3. problem 5d] $L = \{a^n : n = 2^k \text{ for some } k \geq 0\}$. Hint: It may be useful to use the fact that $m < 2^m$ for $m \geq 0$. In particular $2^m + m < 2^m + 2^m$.

   (d) [4.3 Problem 12] $L = \{a^n b^k c^{(n+k)} : n \geq 0, k \geq 0\}$. You must apply pumping lemma directly and not use the closure properties as was done in Example 4.12.

   (e) [4.3 Problem 16] $L = \{w_1 c w_2 : w_1, w_2 \in \{a, b\}^*, w_1 \neq w_2\}$. Hint: Use closure properties and the pumping lemma.