

CS321
Theory of Computation
Practice Problems

1. (a) Use induction on the size of the set S to show that $|2^S| = 2^{|S|}$.
 - (b) Show by direct proof that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
 - (c) Consider a DFA $M = (Q, \Sigma, \delta, q_0, F)$.
 - i. What is the domain and range of the transition function δ ? (The domain of a function is the set of possible inputs. The range is the set of possible outputs.)
 - ii. Give a formal definition of $L(M)$ in set notation. (You will want to make use of the extended transition function δ^* .)
 - (d) Consider an NFA $M = (Q, \Sigma, \delta, q_0, F)$.
 - i. What is the domain and range of the transition function δ ?
 - ii. Give a formal definition of $L(M)$ in set notation. (You will want to make use of the extended transition function δ^* .)
 - (e) Given an NFA N and a regular expression r , what does it mean to say that N and r are equivalent?
2. Consider the following language

$$L = \{w_1 w_2 w_3 : w_1 \in \{aa\}^+, w_2 \in \{ba\}^*, w_3 \in \{bb\}^+\}$$

- (a) Give an informal description of L in your own words.
- (b) Underline the strings in the following list that are in L .

λ $aabb$ $aabababb$ $aaabababb$ $abaababb$

- (c) Consider the following language over alphabet $\Sigma = \{a, b\}$

$$L_1 = \{w_1 b a w_2 : w_1 = a^n, w_2 = b^m, n \geq 0, m \geq 0\}$$

Describe L_1^R in set notation.

- (d) Let $L_2 = \{b\}^* \{a\}^*$. Describe the language $L_1 \cap L_2$ in set notation, where L_1 is defined as in part (c).

3. (a) Draw a DFA M such that $L(M) = \{w : w = abaw_1, w_1 \in \{a, b\}^*\}$.
- (b) Draw an NFA N such that

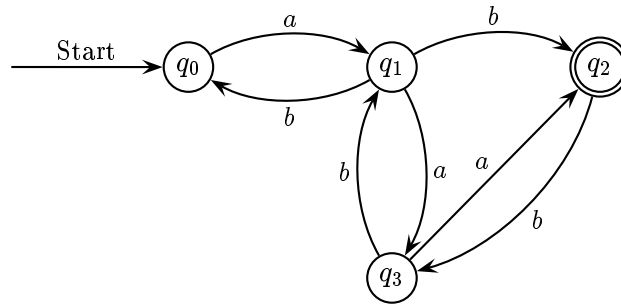
$$L(N) = \{w : w \in \{a, b, c\}^*, w \text{ does not contain all of the characters in } \{a, b, c\}\}$$

. So for example $aa, bb, cc, abba, bbc,$ and ca are in $L(N)$, but abc and $baacc$ are not.

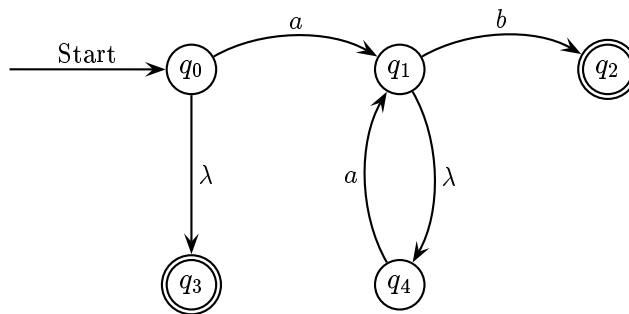
- (c) Write a regular expression r such that

$$L(r) = \{w : w \in \{a, b\}^*, n_a(w) \bmod 2 = 0 \text{ or } n_b(w) \bmod 3 = 0\}$$

4. Consider the following NFA N .



- (a) Draw an equivalent generalized transition graph G_1 that results by removing q_1 from N .
 - (b) Draw an equivalent generalized transition graph G_2 that results by removing q_3 from G_1 .
 - (c) Write the regular expression that is equivalent to the generalized transition graph G_2 .
5. (a) Let DFA_L be the set of languages accepted by DFAs and NFA_L be the set of languages accepted by NFAs. Is it true that $\text{DFA}_L \subseteq \text{NFA}_L$? Give a brief argument in support of your answer.
- (b) Convert the following NFA to an equivalent DFA.



- (a) Give a right linear grammar for the following language: $\{w \mid n_a(w) \text{ is even and } n_b(w) \text{ is odd}\}$.
- (b) Give a left linear grammar for the language $((1 + 0)^* 00(0 + 1)^* 11)^*$.
- (c) What language does the following grammar generate?
 $S \rightarrow Saa \mid B$
 $B \rightarrow Sb \mid \lambda$