1. Let $w^R$ be the reverse of string $w$. It is formally defined using the following recursive definition:
   Basis: $\epsilon^R = \epsilon$.
   Inductive step: $(aw)^R = w^Ra$.
   Using the above definition, show that $(uv)^R = v^Ru^R$, for any strings $u$ and $v$. Note that $uv$ denotes the concatenation of $u$ and $v$.
   Hint: Use induction on the length of $u$.

2. Show that in every undirected graph with no self-loops and at least two nodes, there are at least two nodes which have the same degree. Use proof by contradiction.

3. Give DFA for $L = \{w | w$ is of even length and contains odd number of 1’s $\}$. Justify your answer.

4. Give a DFA accepting the following language over the alphabet $0,1$: The set of all strings that, when interpreted as a binary integer in reverse, is divisible by 5. Examples of strings in the language are $0, 10011, 0101$.

5. Let $F$ be the language of all strings of 0s and 1s that do not contain a pair of 1’s separated by an odd number of symbols. Give a state diagram for a DFA with 5 states that recognizes $F$.
   Hint: First show that the strings in $F$ do not contain more than two 1’s (use proof by contradiction).

6. Let $B_n = \{a^k | k$ is a multiple of $n\}$. Show that for each $n$, $B_n$ is a regular language.

7. Let $\Sigma = \{0, 1\}$ and $D = \{w | w$ contains equal number of occurrences of 01 and 10$\}$. For example $101 \in D$, but $1010 \not\in D$. Show that $D$ is regular.