Solve the following problems.

1. Give a CFG over \{a, b\} for the set of strings that have twice as many \( a \)'s as \( b \)'s. Give an inductive proof that your grammar is correct.

2. Give a CFG for the language \{xy | x \neq y and |x| = |y|\} Informally justify your grammar.

3. Let \( E = \{a^ib^j|i \neq j and 2i \neq j\} \). Informally justify your grammar.

4. Convert the following grammar to Chomsky Normal Form.
   \[
   S \rightarrow PQ \\
   P \rightarrow 0P0|1P1|Q \\
   Q \rightarrow 0Q1|1Q0|\epsilon
   \]

5. Give unambiguous CFGs for the following languages. Argue why they are correct and unambiguous.
   (a) \( \{w \mid \text{in every prefix of } w \text{ the number of } a \text{’s is at least the number of } b \text{’s } \}\).
   (b) \( \{w \mid \text{the number of } a \text{’s in } w \text{ is at least the number of } b \text{’s } \}\).