Example

Example 6.10, p_{501}, Fig. 6.7 on p_{493}:

- RC = 0.2
- Input voltage: $x(t) = (3/5)e^{-2t}u(t)$
- Initial condition: $y(0^-) = -2$
- Find $y(t)$, the voltage across the capacitor.

Example (cont.)

The differential equation:

$$\frac{d}{dt}y(t) + \frac{1}{RC}y(t) = \frac{1}{RC}x(t)$$

$$\frac{d}{dt}y(t) + 5y(t) = 5x(t),$$ taking LT of both sides

$$sY(s) - y(0^-) + 5Y(s) = 5X(s)$$

$$Y(s) = \frac{1}{s+5}[5X(s) + y(0^-)]$$

$$x(t) = (3/5)e^{-2t}u(t) \xleftrightarrow{\mathcal{L}_u} X(s) = \frac{3/5}{s+2}$$

$$Y(s) = \frac{3}{(s+2)(s+5)} + \frac{-2}{s+5} = \frac{-2s-1}{(s+2)(s+5)}$$

$$= \frac{1}{s+2} - \frac{3}{s+5}$$

$$y(t) = e^{-2t}u(t) - 3e^{-5t}u(t)$$

Natural and forced responses

- From the example, it can be seen that the output consists of two terms: a term due to input and a term due to initial conditions.
- Let $Y(s) = Y^{(f)}(s) + Y^{(n)}(s)$, where
- $Y^{(f)}(s)$ is entirely associated with the input, called the *forced response* (system initial rest), and
- $Y^{(n)}(s)$ is due entirely to the initial conditions, called the *natural response* (system input=0).

Natural and forced responses: examples

- Read example 6.11, p_{503}.

- Problem 6.9(b), p_{505}: determine the forced and the natural responses of the system described by the following differential equation and initial conditions:

$$
\begin{aligned}
\frac{d^2}{dt^2}y(t) + 4y(t) &= 8x(t) \\
x(t) &= u(t) \\
y(0^-) &= 1 \\
\left.\frac{d}{dt}y(t)\right|_{t=0^-} &= 2.
\end{aligned}
$$

Natural and forced responses: examples (cont.)

- Apply Eq. (6.19) p_{494} and take LT of both sides of the differential equation:

$$
\begin{aligned}
& s^2Y(s) - \left(\frac{d}{dt}y(t)\Big|_{t=0^-} + sy(t)|_{t=0^-} \right) + 4Y(s) \\
= \quad & 8X(s) \\
X(s) \quad = \quad & 1/s \\
Y(s) \quad = \quad & \frac{8}{s(s^2+4)} + \frac{s+2}{s^2+4}, \quad \text{where} \\
Y^{(f)}(s) \quad = \quad & \frac{8}{s(s^2+4)} \\
Y^{(n)}(s) \quad = \quad & \frac{s+2}{s^2+4}
\end{aligned}
$$

Natural and forced responses: examples (cont.)

- Forced response $Y^{(f)}(s)$: three poles at $s = 0$, $s = \pm j2$ (complex pole pair, $\alpha = 0$, $\omega_0 = 2$).

$$
\begin{aligned}
Y^{(f)}(s) &= \frac{A}{s} + \frac{B_1}{s - j2} + \frac{B_2}{s + j2} \\
A &= 2 \\
B_1 &= -1 \\
B_2 &= -1 \\
C_1 &= B_1 + B_2 = -2 \\
C_2 &= j(B_1 - B_2) = 0 \\
y^{(f)}(t) &= 2u(t) + C_1 e^{\alpha t} cos(\omega_0 t) u(t) + C_2 e^{\alpha t} sin(\omega_0 t) \\
&= 2u(t) - 2cos(2t)u(t)
\end{aligned}
$$

Natural and forced responses: examples (cont.)

- Natural response $Y^{(n)}(s)$: two poles at $s = \pm j2$ (complex pole pair, $\alpha = 0$, $\omega_0 = 2$).

$$
\begin{aligned}
Y^{(n)}(s) &= \frac{D_1}{s - j2} + \frac{D_2}{s + j2} \\
D_1 &= \frac{2 + j2}{j4} \\
D_2 &= \frac{-2 + j2}{j4} \\
E_1 &= D_1 + D_2 = 1 \\
E_2 &= j(D_1 - D_2) = 1 \\
y^{(n)}(t) &= E_1 e^{\alpha t} cos(\omega_0 t)u(t) + E_2 e^{\alpha t} sin(\omega_0 t) \\
&= cos(2t)u(t) + sin(2t)u(t)
\end{aligned}
$$

Laplace transform in circuit analysis

- Resistor:

$$
\begin{aligned}
v_R(t) &= Ri_R(t) \\
V_R(s) &= RI_R(s)
\end{aligned}
$$

- Inductor:

$$
\begin{aligned}
v_L(t) &= L\frac{d}{dt}i_L(t) \\
V_L(s) &= sLI_L(s) - Li_L(0^-)
\end{aligned}
$$

- Capacitor:

$$
\begin{aligned}
v_c(t) &= \frac{1}{C}\int_{0^-}^{t} i_C(\tau)d\tau + v_C(0^-) \\
V_c(s) &= \frac{1}{sC}I_C(s) + \frac{v_C(0^-}{s}
\end{aligned}
$$

Laplace transform in circuit analysis (cont.)

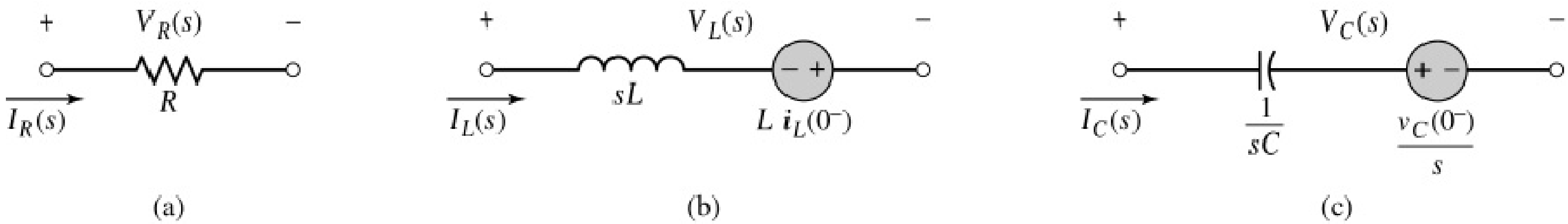


Figure 6.10 (p. 507): Laplace transform circuit models for use with Kirchhoff's voltage law. (a) Resistor. (b) Inductor with initial current $i_L(0^-)$. (c) Capacitor with initial voltage $v_c(0^-)$.

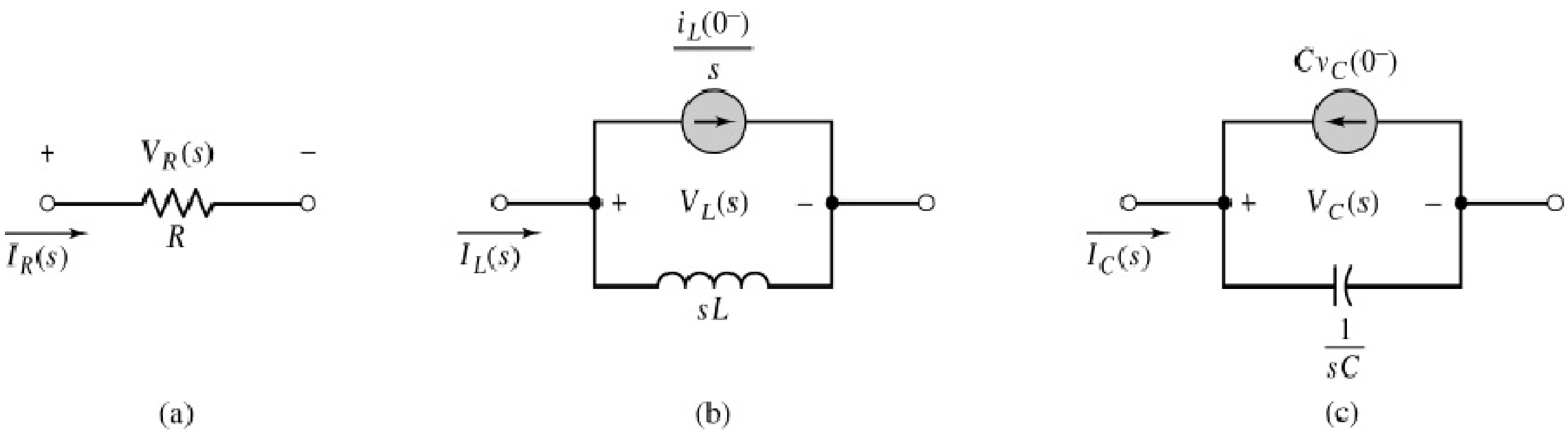


Figure 6.11 (p. 507): Laplace transform circuit models for use with Kirchhoff's current law. (a) Resistor. (b) Inductor with initial current $i_L(0^-)$. (c) Capacitor with initial voltage $v_c(0^-)$.

Laplace transform in circuit analysis: example

Example 6.13, p_{508}: determine output voltage $y(t)$ in the circuit shown in Fig. 6.12, p_{508}. Given $x(t) = 2e^{-10t}u(t)$, $v_C(0^-) = 5$V.

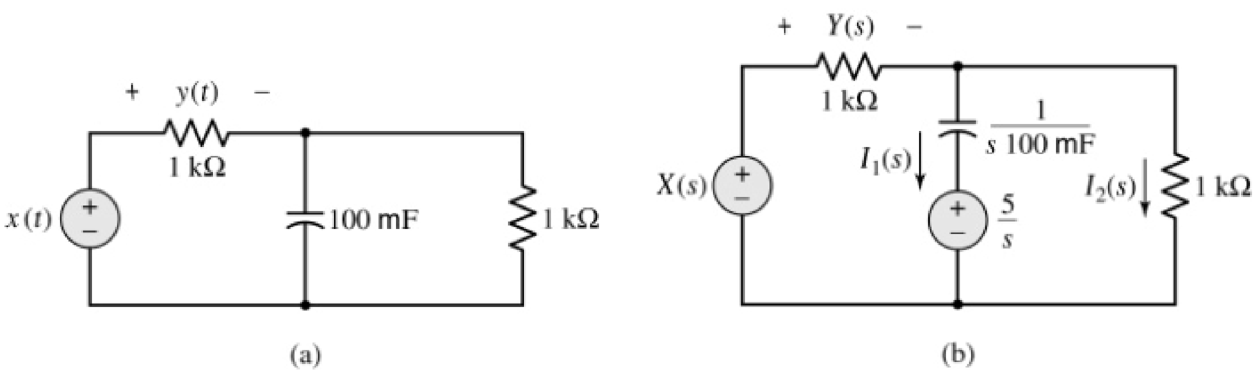


Figure 6.12 (p. 508): Electrical circuit for Example 6.13. (a) Original circuit. (b) Transformed circuit.

Laplace transform in circuit analysis (cont.)

$$
\begin{aligned}
Y(s) &= 1000(I_1(s) + I_2(s)) \\
X(s) &= Y(s) + \frac{5}{s} + \frac{1}{s(10^{-4})} I_1(s) \\
X(s) &= Y(s) + 1000 I_2(s), \quad \text{solving for Y(s) gives} \\
Y(s) &= X(s)\frac{s+10}{s+20} - \frac{5}{s+20} \\
&= \frac{2}{s+10}\frac{s+10}{s+20} - \frac{5}{s+20} = \frac{-3}{s+20} \\
y(t) &= -3e^{-20t}u(t)
\end{aligned}
$$

Laplace transform in circuit analysis (cont.)

- Natural response: setting the voltage or current source associated with input equal to zero.

- Forced response: setting the initial conditions equal to zero, which eliminates the voltage or current sources present in the transformed capacitor and inductor circuit models.

Properties of the bilateral Laplace transform

- Bilateral Laplace transform: $X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$, well suited to problems involving noncausal signals and systems.
- Linearity, scaling (time), s-domain shift, convolution, and differentiation in the s-domain are identical for bilateral and unilateral Laplace transforms.
- The operation of these properties may change the ROC.
- Usually, ROC of a sum of signals are the interactions of the individual signals.
- ROC may be larger than the interaction of the individual ROCs if a pole and zero cancel in the sum.

Properties of the bilateral Laplace transform (cont.)

- Example:

$$x(t) = e^{-2t}u(t) \quad \stackrel{\mathcal{L}}{\longleftrightarrow} \quad X(s) = \frac{1}{s+2}, \ ROC \ Re(s) > -2$$

$$y(t) = e^{-2t}u(t) - e^{-3t}u(t) \quad \stackrel{\mathcal{L}}{\longleftrightarrow} \quad Y(s) = \frac{1}{(s+2)(s+3)},$$

$$ROC \ Re(s) > -3$$

$$x(t) - y(t) \quad \stackrel{\mathcal{L}}{\longleftrightarrow} \quad \frac{1}{s+3}, \ ROC \ Re(s) > -3$$

- If the interactions of the ROCs is the empty set and pole-zero cancellation does not occur, then the Laplace transform of $ax(t) + by(t)$ does not exist.

Properties of the bilateral Laplace transform (cont.)

- The bilateral Laplace transform involving time shifts, differentiation in the time domain, and integration with respect to time differ slightly from their unilateral counterparts.

- Time shift:

$$x(t-\tau) \xleftrightarrow{\mathcal{L}_u} e^{-s\tau}X(s), \text{ restriction :}$$

for all τ such that $x(t-\tau)u(t) = x(t-\tau)u(t-\tau)$.

Shift is always satisfied for causal $x(t)$ with $\tau > 0$

$$x(t-\tau) \xleftrightarrow{\mathcal{L}} e^{-s\tau}X(s) \text{ (restriction removed)}$$

Properties of the bilateral Laplace transform (cont.)

- Differentiation in the time domain:

$$\frac{d}{dt}x(t) \quad \overset{\mathcal{L}_u}{\longleftrightarrow} \quad sX(s) - x(0^-)$$

$$\frac{d}{dt}x(t) \quad \overset{\mathcal{L}}{\longleftrightarrow} \quad sX(s),$$ ROC is at least R_x (R_x: the ROC of $X(s)$).

ROC of $sX(s)$ may be larger than R_x

if $X(s)$ has a single pole at $s = 0$

on the ROC boundary.

Properties of the bilateral Laplace transform (cont.)

- Integration with respect to time:

$$\int_{-\infty}^{t} x(\tau)d\tau \quad \overset{\mathcal{L}_u}{\longleftrightarrow} \quad \frac{x^{(-1)}(0^-)}{s} + \frac{X(s)}{s}$$

$$\int_{-\infty}^{t} x(\tau)d\tau \quad \overset{\mathcal{L}}{\longleftrightarrow} \quad \frac{X(s)}{s}, \text{ with ROC } R_x \bigcap Re(s) > 0.$$

- The initial- and final-value theorems apply to the bilateral transform, with the additional restriction that $x(t) = 0$ for $t < 0$.

Properties of the ROC

- Bilateral Laplace transform is not unique, unless the ROC is specified.
- ROC is related to the characteristics of the signal.
- ROC cannot contain any poles.
- *Left-sided signals (LSS)*: a signal for which $x(t) = 0$ for $t > b$.
- *Right-sided signals (RSS)*: a signal for which $x(t) = 0$ for $t < a$.
- *Two-sided signals (TSS)*: a signal that is infinite in extent in both directions.

Properties of the ROC (cont.)

- ROC of an LSS signal is of the form $\sigma < \sigma_n$.
- ROC of an RSS signal is of the form $\sigma > \sigma_p$.
- ROC of a TSS signal is of the form $\sigma_p < \sigma < \sigma_n$.
- Boundaries are determined by pole locations.

Properties of the ROC: example

Example: determine the ROC of $x_1(t) = e^{-2t}u(t) + e^{-t}u(-t)$:

- $e^{-2t}u(t)$: RSS, pole at $s = -2$, ROC: $Re(s) > -2$
- $e^{-t}u(-t)$: LSS, pole at $s = -1$, ROC: $Re(s) < -1$.
- ROC of $x_1(t)$: $-2 < Re(s) < -1$, a strip of the s-plane located between poles.

Properties of the ROC: example (cont.)

Example: determine the ROCs of $x_2(t) = e^{-t}u(t) + e^{-2t}u(-t)$:

- $e^{-t}u(t)$: RSS, pole at $s = -1$, ROC: $Re(s) > -1$
- $e^{-2t}u(-t)$: LSS, pole at $s = -2$, ROC: $Re(s) < -2$.
- ROC of $x_2(t)$ is an empty set. Laplace transform of $x_2(t)$ does not exist.

Properties of the ROC: example (cont.)

Example: determine the ROCs of $x_3(t) = e^{-b|t|}$:

- $x(t) = e^{-bt}u(t) + e^{bt}u(-t)$
- $e^{-bt}u(t)$: RSS, pole at $s = -b$, ROC $Re(s) > -b$. $e^{bt}u(-t)$: LSS, pole at $s = b$, ROC $Re(s) < b$.
- Case 1: $b > 0$.
  - ⋆ ROC of $x_3(t)$: $-b < Re(s) < b$
- Case 2: $b < 0$.
  - ⋆ ROC of $x_3(t)$ is an empty set. Laplace transform of $x_3(t)$ does not exist.

Inversion of the bilateral Laplace transforms

- Primary difference between unilateral and bilateral Laplace transforms is that we must use the ROC to determine a unique inverse transform in the bilateral case.

- $A_k e^{d_k t} u(t) \stackrel{\mathcal{L}}{\longleftrightarrow} \frac{A_k}{s-d_k}$, with ROC $Re(s) > d_k$ (right-sided transform pair).

- $-A_k e^{d_k t} u(-t) \stackrel{\mathcal{L}}{\longleftrightarrow} \frac{A_k}{s-d_k}$, with ROC $Re(s) < d_k$ (left-sided transform pair).

Inversion of the bilateral Laplace transforms: examples

Example 6.17, p_{517}: Find $x(t)$ given

$$X(s) = \frac{-5s-7}{(s+1)(s-1)(s+2)}, \quad \text{with ROC } -1 < Re(s) < 1$$

Partial-fraction expansion of $X(s)$:

$$X(s) = \frac{1}{s+2} + \frac{1}{s+1} - \frac{2}{s-1}$$

Inversion of the bilateral Laplace transforms: examples (cont.)

- Poles at $s = -2$.

$$\text{Right-sided inverse: } e^{-2t}u(t) \quad \stackrel{\mathcal{L}}{\longleftrightarrow} \quad \frac{1}{s+2}$$

$$\text{Left-sided inverse: } -e^{-2t}u(-t) \quad \stackrel{\mathcal{L}}{\longleftrightarrow} \quad \frac{1}{s+2}$$

- Correct choice: the right-sided inverse Laplace transform.

Inversion of the bilateral Laplace transforms: examples (cont.)

- Poles at $s = -1$.

$$\text{Right-sided inverse: } e^{-t}u(t) \quad \stackrel{\mathcal{L}}{\longleftrightarrow} \quad \frac{1}{s+1}$$

$$\text{Left-sided inverse: } -e^{-t}u(-t) \quad \stackrel{\mathcal{L}}{\longleftrightarrow} \quad \frac{1}{s+1}$$

- Correct choice: the right-sided inverse Laplace transform.

Inversion of the bilateral Laplace transforms: examples (cont.)

- Poles at $s = 1$.

$$\text{Right-sided inverse:} -2e^{t}u(t) \quad \overset{\mathcal{L}}{\longleftrightarrow} \quad \frac{2}{s-1}$$

$$\text{Left-sided inverse:} 2e^{t}u(-t) \quad \overset{\mathcal{L}}{\longleftrightarrow} \quad \frac{2}{s-1}$$

- Correct choice: the left-sided inverse Laplace transform.

- Thus, $x(t) = e^{-2t}u(t) + e^{-t}u(t) + 2e^{t}u(-t)$.

Inversion of the bilateral Laplace transforms: examples (cont.)

- Read Example 6.18, p_{518}.

- Problem 6.14, p_{518}: find $x(t)$ of

$$X(s) = \frac{s^4 + 3s^3 - 4s^2 + 5s + 5}{s^2 + 3s - 4}, \quad \text{with ROC} \quad -4 < Re(s) < 1.$$

Long division and then partial fraction expansion:

$$X(s) = s^2 + \frac{5s + 5}{(s-1)(s+4)} = s^2 + \frac{2}{s-1} + \frac{3}{s+4}.$$

Inversion of the bilateral Laplace transforms: examples (cont.)

- For pole at $s = 1$, $\frac{2}{s-1}$ corresponds to a left-sided signal with the given ROC. Thus,

$$-2e^{t}u(-t) \stackrel{\mathcal{L}}{\longleftrightarrow} \frac{2}{s-1}$$

- For pole at $s = -4$, $\frac{3}{s+4}$ corresponds to a right-sided signal with the given ROC. Thus,

$$3e^{-4t}u(t) \stackrel{\mathcal{L}}{\longleftrightarrow} \frac{3}{s+4}$$

- Thus, $x(t) = \delta^{(2)}(t) - 2e^{t}u(-t) + 3e^{-4t}u(t)$

The Transfer Function

- For LTI systems: $y(t) = x(t) * h(t)$, $Y(s) = X(s)H(s)$.

$$H(s) = \frac{Y(s)}{X(s)}.$$

- For a system described by the input-output differential equation:

$$\begin{aligned} \sum_{k=0}^{N} a_k \frac{d^k}{dt^k} y(t) &= \sum_{k=0}^{M} b_k \frac{d^k}{dt^k} x(t) \\ H(s) = \frac{Y(s)}{X(s)} &= \frac{\sum_{k=0}^{M} b_k s^k}{\sum_{k=0}^{N} a_k s^k} = \frac{\tilde{b} \prod_{k=0}^{M} (s - c_k)}{\prod_{k=0}^{N} (s - d_k)}, \quad \tilde{b} = \frac{b_M}{a_N} \end{aligned}$$

The Transfer Function: examples

- $H(s)$ is the ratio of two polynomials in s, and is termed a *rational transfer function*.

- Read Example 6.19, p_{521}.

- Problem 6.17(b), p_{521}: find $H(s)$ given

$$\frac{d^3}{dt^3}y(t) - \frac{d^2}{dt^2}y(t) + 3y(t) = 4\frac{d}{dt}x(t)$$

$$s^3Y(s) - s^2Y(s) + 3Y(s) = 4sX(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{4s}{s^3 - s^2 + 3}$$

The Transfer Function: examples (cont.)

- Problem 6.18(b), p_{522}: determine the differential-equation description of the system given

$$\begin{aligned} H(s) &= \frac{2(s+1)(s-1)}{s(s+2)(s+1)} \\ &= \frac{Y(s)}{X(s)} = \frac{2s^2-2}{s^3+3s^2+2s} \end{aligned}$$

Cross multiply:

$$\begin{aligned} s^3Y(s)+3s^2Y(s)+2sY(s) &= 2s^2X(s)-2X(s) \\ \frac{d^3}{dt^3}y(t)+3\frac{d^2}{dt^2}y(t)+2\frac{d}{dt}y(t) &= 2\frac{d^2}{dt^2}x(t)-2x(t) \end{aligned}$$

System causality and stability

- System transfer function $H(s) \stackrel{\mathcal{L}}{\longleftrightarrow} h(t)$, system impulse response.

- In order to uniquely determine $h(t)$, must know ROC or other knowledge of the system characteristics.

- Causal system $\rightarrow h(t) = 0$ for $t < 0 \rightarrow H(s)$ is right-sided Laplace transform.

- Stable system $\rightarrow h(t)$ absolutely integrable $\rightarrow$ FT of $x(t)$ exists $\rightarrow$ ROC includes $j\omega$-axis.

System causality and stability (cont.)

- Assume a pole at $s = d_k$.
  - ⋆ If $\alpha = Re(d_k) < 0$ (pole in the left half plane) $h(t)$ contains a term $e^{\alpha t}$ that is exponentially decaying. ▮
  - ⋆ If $\alpha = Re(d_k) > 0$ (pole in the right half plane) $h(t)$ contains a term $e^{\alpha t}$ that is exponentially increasing. ▮

- Conclusion: If a system is causal and stable, then *all poles of $H(s)$ are in the left half of the s-plane*.

System causality and stability: examples

- Example 6.21, p_{525}: given

$$H(s) = \frac{2}{s+3} + \frac{1}{s-2}.$$

  Determine $h(t)$ assuming

  - ⋆ the system is stable
  - ⋆ the system is causal
  - ⋆ can the system be both causal and stable?

System causality and stability: examples

Poles at $s = -3$ and $s = 2$.

- If the system is stable, then pole at $s = -3$ contributes to a right-sided term $2e^{-3t}u(t)$, and pole at $s = 2$ contributes to a left-sided term $-e^{2t}u(-t)$ (otherwise this term is not absolutely integrable). Thus

$$h(t) = 2e^{-3t}u(t) - e^{2t}u(-t).$$

- If the system is causal, then both poles must contribute to right-sided terms, thus

$$h(t) = 2e^{-3t}u(t) + e^{2t}u(t).$$

- The system cannot be both causal and stable because pole at $s = 2$ is in the right half of the s-plane.

System causality and stability: examples (cont.)

- Problem 6.19(a), p_{526}: given

$$\frac{d^2}{dt^2}y(t) + 5\frac{d}{dt}y(t) + 6y(t) = \frac{d^2}{dt^2}x(t) + 8\frac{d}{dt}x(t) + 13x(t)$$

  Determine $h(t)$ assuming

  - ⋆ the system is stable
  - ⋆ the system is causal

System causality and stability: examples (cont.)

Taking Laplace transform of both sides of the diff. equation gives

$$\begin{aligned} s^2Y(s) + 5sY(s) + 6Y(s) &= s^2X(s) + 8sX(s) + 13X(s) \\ H(s) &= \frac{s^2 + 8s + 13}{s^2 + 5s + 6} = 1 + \frac{3s + 7}{s^2 + 5s + 6} \\ &= 1 + \frac{1}{s+2} + \frac{2}{s+3} \end{aligned}$$

Poles at $s = -2$ and $s = -3$ are in the left half of the s-plane. For both causal and stable systems, these poles contributed to right-sided terms. Thus, for both cases,

$$h(t) = \delta(t) + 2e^{-3t}u(t) + e^{-2t}u(t).$$

Freq. response from poles and zeros

- If ROC includes the $j\omega$-axis, frequency response can be obtained as $H(j\omega) = H(s)|_{s=j\omega}$.
- We examine both the magnitude and phase responses of $H(j\omega)$ using the *Bode diagram* approach.

For rational transfer function, the freq. response is obtained as

$$\begin{aligned} H(j\omega) &= \frac{\tilde{b}\prod_{k=1}^{M}(j\omega - c_k)}{\prod_{k=1}^{N}(j\omega - d_k)} \\ &= \frac{K\prod_{k=1}^{M}(1 - \frac{j\omega}{c_k})}{\prod_{k=1}^{N}(1 - \frac{j\omega}{d_k})}, \quad \text{where} \quad K = \frac{\tilde{b}\prod_{k=1}^{M}(-c_k)}{\prod_{k=1}^{N}(-d_k)} \end{aligned}$$

Freq. response from poles and zeros-*Bode diagram*

Magnitude and phase responses:

$$|H(j\omega)|_{dB} = 20log_{10}|K| + \sum_{k=1}^{M} 20log_{10}\left|1 - \frac{j\omega}{c_k}\right| - \sum_{k=1}^{N} 20log_{10}\left|1 - \frac{j\omega}{d_k}\right|$$

$$arg\{H(j\omega)\} = arg\{K\} + \sum_{k=1}^{M} arg\left(1 - \frac{j\omega}{c_k}\right) - \sum_{k=1}^{N} arg\left(1 - \frac{j\omega}{d_k}\right)$$

Freq. response from poles and zeros-*Bode diagram* (cont.)

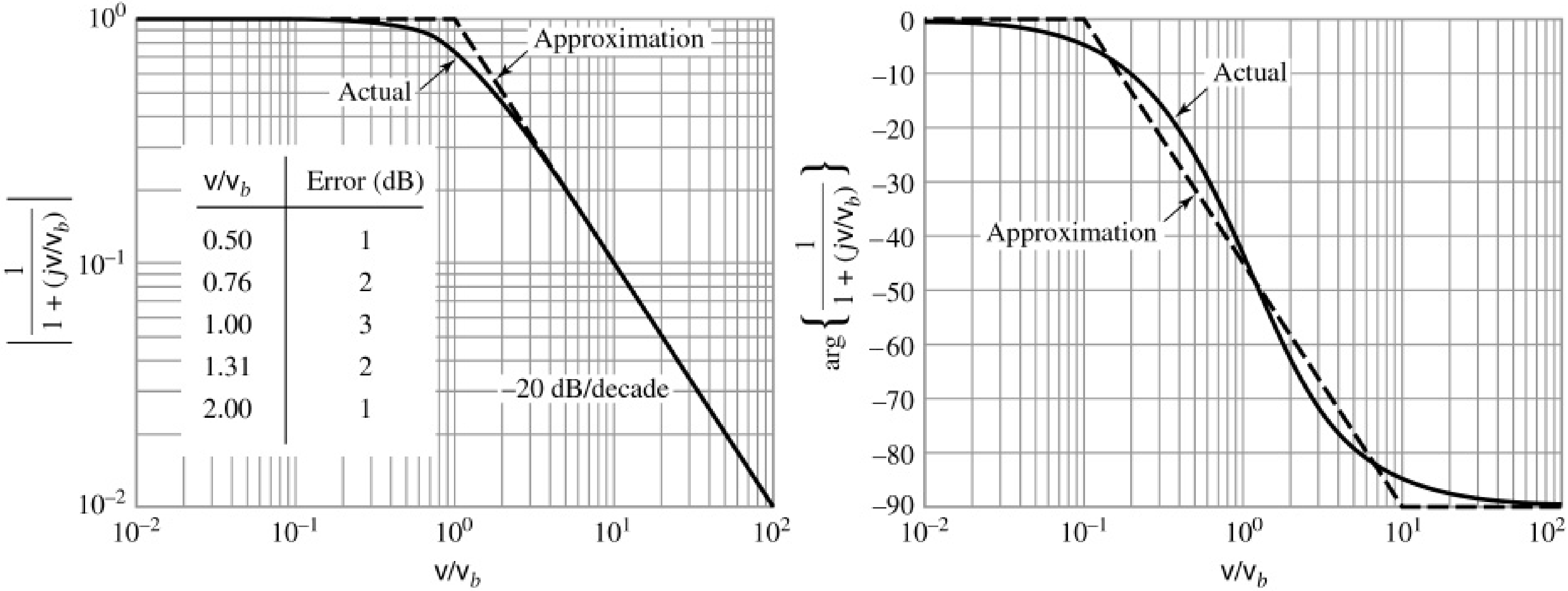
Consider a pole factor $(1 - j\omega/d_0)$ for which $d_0 = -\omega_b$ where ω_b is a real number.

- Approximate gain response:
  $-20log_{10}\left|1 + \frac{j\omega}{\omega_b}\right| = -10log_{10}\left(1 + \frac{\omega^2}{\omega_b^2}\right)$
  - ⋆ *Low-frequency asymptote:* $\omega \ll \omega_b$,
    $-10log_{10}\left(1 + \frac{\omega^2}{\omega_b^2}\right) \approx -10log_{10}(1) = 0dB$
  - ⋆ *High-frequency asymptote:* $\omega \gg \omega_b$,
    $-10log_{10}\left(1 + \frac{\omega^2}{\omega_b^2}\right) \approx -20log_{10}\left|\frac{\omega}{\omega_b}\right|$, a straight line with a slope of -20 dB/decade.
  - ⋆ The intersection frequency ω_b: *corner frequency or break frequency of the Bode diagram*.

Freq. response from poles and zeros-*Bode diagram* (cont.)

- Approximate phase response:
  $-arg\{1 + j\omega/\omega_b\} = -arctan\left(\frac{\omega}{\omega_b}\right)$
  - ⋆ $\omega < \omega_b/10$: 0°
  - ⋆ $\omega_b/10 < \omega < 10\omega_b$: linearly decreases from 0° to -90°.
  - ⋆ $10\omega_b < \omega$: -90°

Freq. response from poles and zeros-*Bode diagram* (cont.)



(a) (b)

Bode diagram - example

Example 6.25, p_{535}: sketch the magnitude and phase response as a Bode diagram for the LTI system described by transfer function:

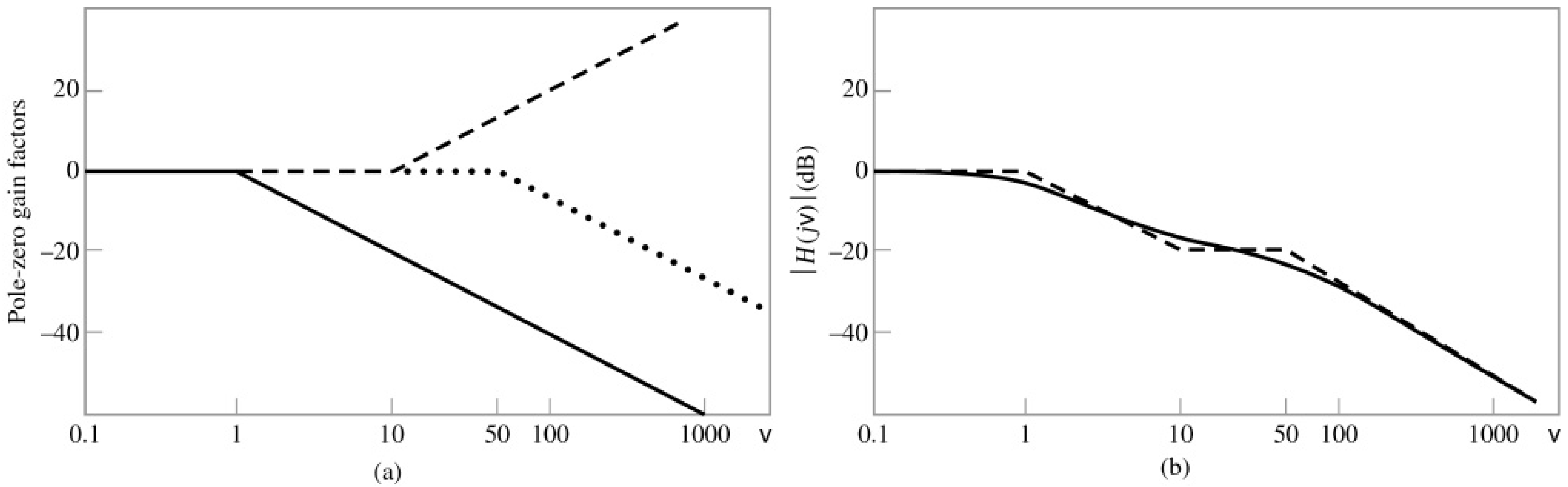
$$H(s) = \frac{5(s+10)}{(s+1)(s+50)}$$

Frequency response:

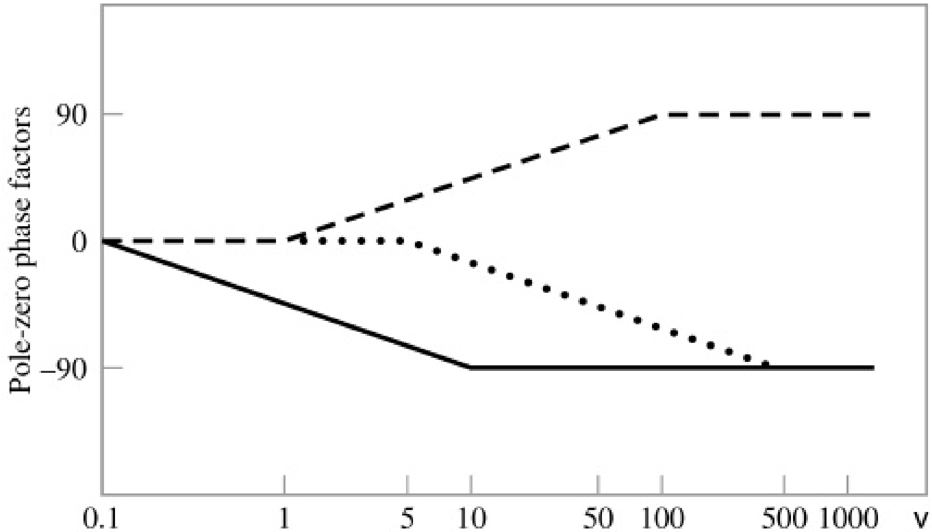
$$H(j\omega) = \frac{1 + \frac{j\omega}{10}}{(1 + j\omega)\left(1 + \frac{j\omega}{50}\right)}$$

- Two pole corner frequencies: $\omega = 1$ and $\omega = 50$
- Single zero corner frequencies: $\omega = 10$

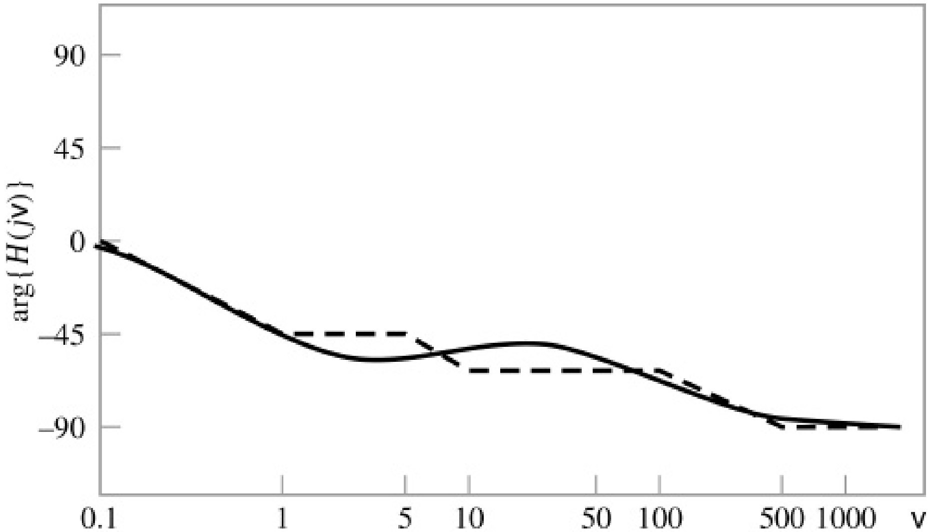
Bode diagram - example (cont.)



Bode diagram - example (cont.)



(c)



(d)