ECE352
Spring 07

Dr. Thinh Nguyen

Homework 2
Due 04/13/07 at the beginning of the class

4.21. Consider the system depicted in Fig. P4.21. The impulse response $h(t)$ is given by

$$h(t) = \frac{\sin(11\pi t)}{\pi t}$$

and we have

$$x(t) = \sum_{k=1}^{\infty} \frac{1}{k^2} \cos(k5\pi t)$$

$$g(t) = \sum_{k=1}^{10} \cos(k8\pi t)$$

Use the FT to determine $y(t)$.

$$\begin{aligned}
y(t) &= [(x(t) * h(t))(g(t) * h(t))] * h(t) \\
&= [x_h(t) g_h(t)] * h(t) \\
&= m(t) * h(t)
\end{aligned}$$

$$h(t) = \frac{\sin(11\pi t)}{\pi t} \quad \xleftrightarrow{FT} \quad H(j\omega) = \begin{cases} 1 & |\omega| \le 11\pi \\ 0 & \text{otherwise} \end{cases}$$

$$x(t) = \sum_{k=1}^{\infty} \frac{1}{k^2} \cos(k5\pi t) \quad \xleftrightarrow{FT} \quad X(j\omega) = \pi \sum_{k=1}^{\infty} \frac{1}{k^2} [\delta(\omega - 5k\pi) + \delta(\omega + 5k\pi)]$$

$$g(t) = \sum_{k=1}^{10} \cos(k8\pi t) \quad = \quad \pi \sum_{k=1}^{10} [\delta(\omega - 8k\pi) + \delta(\omega + 8k\pi)]$$

$$\begin{aligned}
X_h(j\omega) &= X(j\omega)H(j\omega) \\
&= \pi \sum_{k=1}^{2} \frac{1}{k^2} [\delta(\omega - 5k\pi) + \delta(\omega + 5k\pi)]
\end{aligned}$$

$$
\begin{aligned}
G_h(j\omega) &= G(j\omega)H(j\omega) \\
&= \pi\delta(\omega - 8\pi) + \pi\delta(\omega - 8\pi) \\
M(j\omega) &= \frac{1}{2\pi} X_h(j\omega) * G_h(j\omega) \\
&= \frac{1}{2}\left[X_h(j(\omega - 8\pi)) + X_h(j(\omega + 8\pi))\right] \\
&= \pi \sum_{k=1}^{2} \frac{1}{k^2}\left[(\delta(\omega - 8\pi - 5k\pi) + \delta(\omega - 8\pi + 5k\pi)) + (\delta(\omega + 8\pi - 5k\pi) + \delta(\omega + 8\pi + 5k\pi))\right] \\
Y(j\omega) &= M(j\omega)H(j\omega) \\
&= \frac{\pi}{2}\left[\delta(\omega + 3\pi) + \delta(\omega - 3\pi)\right] + \frac{\pi}{8}\left[\delta(\omega - 2\pi) + \delta(\omega + 2\pi)\right] \\
y(t) &= \frac{1}{2}\cos(3\pi t) + \frac{1}{8}\cos(2\pi t)
\end{aligned}
$$

4.26. The continuous-time signal $x(t)$ with FT as depicted in Fig. P4.26 is sampled. Identify in each case if aliasing occurs.

(a) Sketch the FT of the sampled signal for the following sampling intervals:

(i) $T_s = \frac{1}{14}$

No aliasing occurs.

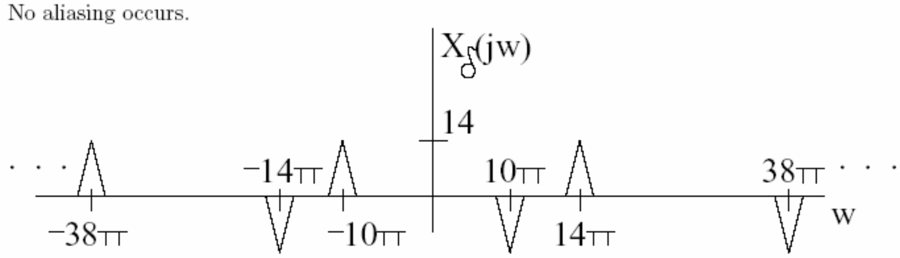

Figure P4.26. (i) FT of the sampled signal

(ii) $T_s = \frac{1}{7}$

Since $T_s > \frac{1}{11}$, aliasing occurs.

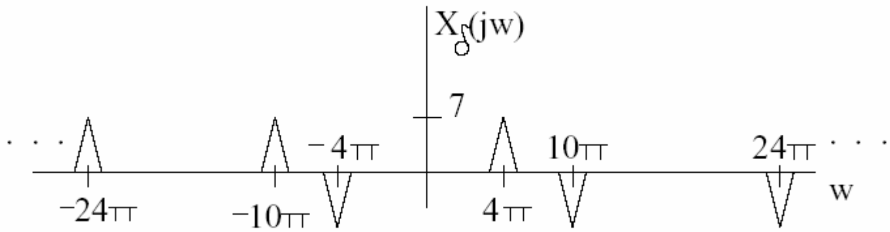

Figure P4.26. (ii) FT of the sampled signal

(iii) $T_s = \frac{1}{5}$

Since $T_s > \frac{1}{11}$, aliasing occurs.

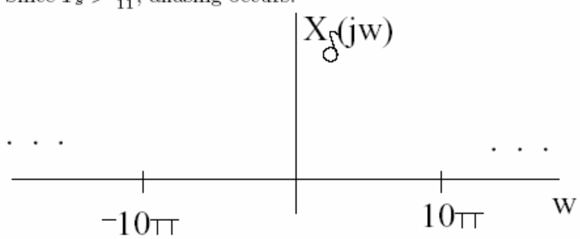

Figure P4.26. (iii) FT of the sampled signal

(b) Let $x[n] = x(nT_s)$. Sketch the DTFT of $x[n]$, $X(e^{j\Omega})$, for each of the sampling intervals given in (a).

The DTFT simple scales the 'x' axis by the sampling rate.

(i) $T_s = \frac{1}{14}$

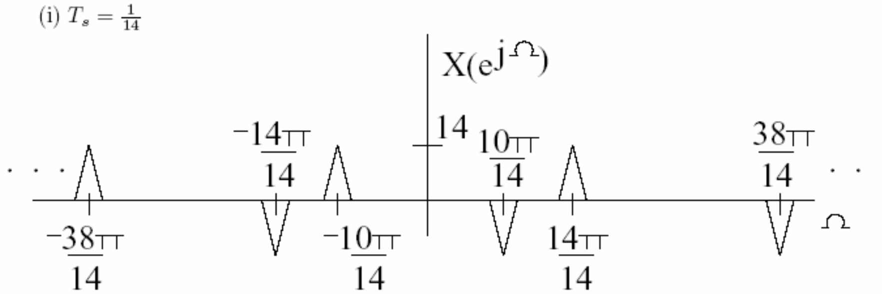

Figure P4.26. (i) DTFT of $x[n]$

(ii) $T_s = \frac{1}{7}$

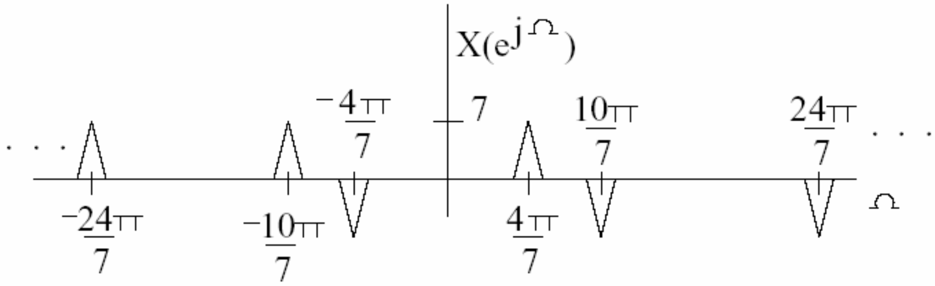

Figure P4.26. (ii) DTFT of $x[n]$

(iii) $T_s = \frac{1}{5}$

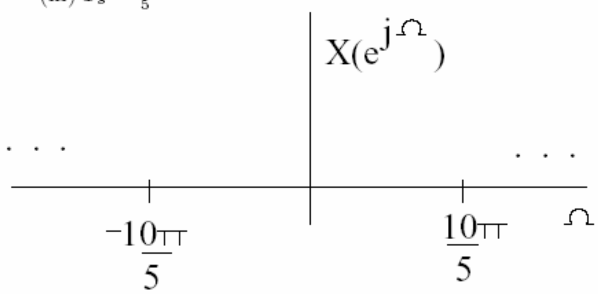

Figure P4.26. (iii) DTFT of $x[n]$

4.29. For each of the following signals sampled with sampling interval T_s, determine the bounds on T_s that gaurantee there will be no aliasing.

(a) $x(t) = \frac{1}{t}\sin 3\pi t + \cos(2\pi t)$

$$\begin{aligned}
\frac{1}{t}\sin(3\pi t) &\stackrel{FT}{\longleftrightarrow} \begin{cases} \frac{1}{\pi} & |\omega| \leq 3\pi \\ 0 & \text{otherwise} \end{cases} \\
\cos(2\pi t) &\stackrel{FT}{\longleftrightarrow} \pi\delta(\Omega - 2\pi) + \pi\delta(\Omega + 2\pi) \\
\omega_{max} &= 3\pi \\
T &< \frac{\pi}{\omega_{max}} \\
T &< \frac{1}{3}
\end{aligned}$$

(b) $x(t) = \cos(12\pi t)\frac{\sin(\pi t)}{2t}$

$$\begin{aligned}
X(j\omega) &= \begin{cases} \frac{1}{4\pi} & |\omega - 12\pi| \leq \pi \\ 0 & \text{otherwise} \end{cases} + \begin{cases} \frac{1}{4\pi} & |\omega + 12\pi| \leq \pi \\ 0 & \text{otherwise} \end{cases} \\
\omega_{max} &= 13\pi \\
T &< \frac{\pi}{\omega_{max}}
\end{aligned}$$

$$T < \frac{1}{13}$$

(c) $x(t) = e^{-6t}u(t) * \frac{\sin(Wt)}{\pi t}$

$$\begin{aligned}
X(j\omega) &= \frac{1}{6 + j\omega}\left[u(\omega + W) - u(\omega - W)\right] \\
\omega_{max} &= W \\
T &< \frac{\pi}{\omega_{max}} \\
T &< \frac{\pi}{W}
\end{aligned}$$

(d) $x(t) = w(t)z(t)$, where the FTs $W(j\omega)$ and $Z(j\omega)$ are depicted in Fig. P4.29.

$$\begin{aligned}
X(j\omega) &= \frac{1}{2\pi}W(j\omega) * G(j\omega) \\
\omega_{max} &= 4\pi + w_a \\
T &< \frac{\pi}{\omega_{max}} \\
T &< \frac{\pi}{4\pi + w_a}
\end{aligned}$$

4.33. The zero-order hold produces a stairstep approximation to the sampled signal $x(t)$ from samples $x[n] = x(nT_s)$. A device termed a first-order hold linearly interpolates between the samples $x[n]$ and thus produces a smoother approximation to $x(t)$. The output of the first- order hold may be described as

$$x_1(t) = \sum_{n=-\infty}^{\infty} x[n]h_1(t - nT_s)$$

where $h_1(t)$ is the triangular pulse shown in Fig. P4.33 (a). The relationship between $x[n]$ and $x_1(t)$ is depicted in Fig. P4.33 (b).

(a) Identify the distortions introduced by the first-order hold and compare them to those introduced by the zero-order hold. *Hint:* $h_1(t) = h_o(t) * h_o(t)$.

$$\begin{aligned} x_1(t) &= \sum_{n=-\infty}^{\infty} x[n]h_1(t - nT_s) \\ &= h_o(t) * h_o(t) * \sum_{n=-\infty}^{\infty} x[n]\delta(t - nT_s) \end{aligned}$$

Thus:

$$X_1(j\omega) = H_o(j\omega)H_o(j\omega)X_\Delta(j\omega)$$

Which implies:

$$H_1(j\omega) = e^{-j\omega T_s}\frac{4\sin^2(\omega\frac{T_s}{2})}{\omega^2}$$

Distortions:

(1) A linear phase shift corresponding to a time delay of T_s seconds (a unit of sampling time).

(2) $\sin^2(.)$ term introduces more distortion to the portion of $X_\delta(j\omega)$, especially the higher frequency part is severely attenuated compared to the low frequency part which falls within the mainlobe, between $-\omega_m$ and ω_m.

(3) Distorted and attenuated versions of $X(j\omega)$ still remain at the nonzero multiples of ω_m, yet it is lower than the case of the zero order hold.

(b) Consider a reconstruction system consisting of a first-order hold followed by an anti-imaging filter with frequency response $H_c(j\omega)$. Find $H_c(j\omega)$ so that perfect reconstruction is obtained.

$$\begin{aligned} X_\Delta(j\omega)H_1(j\omega)H_c(j\omega) &= X(j\omega) \\ H_c(j\omega) &= \frac{e^{j\omega T_s}\omega^2}{4\sin^2(\omega\frac{T_s}{2})}T_sH_{LPF}(j\omega) \end{aligned}$$

where $H_{LPF}(j\omega)$ is an ideal low pass filter.

$$H_c(j\omega) = \begin{cases} \frac{e^{j\omega T_s}\omega^2}{4\sin^2(\omega\frac{T_s}{2})}T_s & |\omega| \le \omega_m \\ \text{don't care} & \omega_m \le |\omega| < \frac{2\pi}{T_s} - \omega_m \\ 0 & |\omega| > \frac{2\pi}{T_s} - \omega_m \end{cases}$$

Assuming $X(j\omega) = 0$ for $|\omega| > \omega_m$

(c) Determine the constraints on $|H_c(j\omega)|$ so that the overall magnitude response of this reconstruction system is between 0.99 and 1.01 in the signal passband and less than 10^{-4} to the images of the signal spectrum for the following values. Assume $x(t)$ is bandlimited to 12π, that is, $X(j\omega) = 0$ for $|\omega| > 12\pi$. Constraints:

(1) In the pass band:

$$0.99 < |H_1(j\omega)||H_c(j\omega)| < 1.01$$

$$\frac{0.99\omega^2}{4\sin^2(\omega\frac{T_s}{2})} < |H_c(j\omega)| < \frac{1.01\omega^2}{4\sin^2(\omega\frac{T_s}{2})}$$

(2) In the image region: $\omega = \frac{2\pi}{T_s} - \omega_m$

$$\begin{aligned} |H_1(j\omega)||H_c(j\omega)| &< 10^{-4} \\ |H_c(j\omega)| &< \frac{10^{-4}\omega^2}{4\sin^2(\omega\frac{T_s}{2})} \end{aligned}$$

(i) $T_s = .05$

$$\begin{aligned} \omega &= \frac{2\pi}{T_s} - \omega_m \\ &= \frac{2\pi}{0.05} - 12\pi = 28\pi \\ |H_c(j\omega)| &< \left|\frac{10^{-4}(28\pi)^2}{4\sin^2(28\pi(\frac{0.05}{2}))}\right| \\ &\approx 0.2956 \end{aligned}$$

$$\begin{aligned} \frac{0.99\omega^2}{4\sin^2(\omega\frac{T_s}{2})} < &\ |H_c(j\omega)| < \frac{1.01\omega^2}{4\sin^2(\omega\frac{T_s}{2})} \\ 2926.01 < &\ |H_c(j\omega)| < 2985.12 \end{aligned}$$

(ii) $T_s = .02$

$$\begin{aligned}\omega &= \frac{2\pi}{0.02} - 12\pi = 88\pi \\ |H_c(j\omega)| &< \left|\frac{10^{-4}(88\pi)^2}{4\sin^2(88\pi(\frac{0.02}{2}))}\right| \\ &\approx 14.1\end{aligned}$$

$$\begin{aligned}\frac{0.99\omega^2}{4\sin^2(\omega\frac{T_s}{2})} < \quad & |H_c(j\omega)| \quad < \frac{1.01\omega^2}{4\sin^2(\omega\frac{T_s}{2})} \\ 139589 < \quad & |H_c(j\omega)| \quad < 142409\end{aligned}$$