ECE352
Spring 07

Dr. Thinh Nguyen

Homework 3 solutions
Due 05/9/07 at the beginning of the class

Problems 4.36, 4.38, 6.28 (f) and (g), 6.31(d) and (e), 6.41 (d) and (f)

4.36. Let $X(e^{j\Omega}) = \frac{\sin(\frac{11\Omega}{2})}{\sin(\frac{\Omega}{2})}$ and define $\tilde{X}[k] = X(e^{jk\Omega_o})$. Find and sketch $\tilde{x}[n]$ where $\tilde{x}[n] \stackrel{DTFS;\Omega_o}{\longleftrightarrow} \tilde{X}[k]$ for the following values of Ω_o:

$$\tilde{X}[k] = X(e^{jk\Omega_o})$$

$$\tilde{X}[k] = \frac{\sin(\frac{11k\Omega_o}{2})}{\sin(\frac{k\Omega_o}{2})} \stackrel{DTFS;\Omega_o}{\longleftrightarrow} \tilde{x}[n] = \begin{cases} N & |n| \le 5 \\ 0 & 5 < |n| < \frac{N}{2}, \quad N \text{ periodic} \end{cases}$$

(a) $\Omega_o = \frac{2\pi}{15}$, $N = 15$

$$\tilde{x}[n] = \begin{cases} 15 & |n| \le 5 \\ 0 & 5 < |n| < 7, \quad 15 \text{ periodic} \end{cases}$$

(b) $\Omega_o = \frac{\pi}{10}$, $N = 20$

$$\tilde{x}[n] = \begin{cases} 20 & |n| \le 5 \\ 0 & 5 < |n| < 10, \quad 20 \text{ periodic} \end{cases}$$

(c) $\Omega_o = \frac{\pi}{3}$, $N = 6$
Overlap occurs

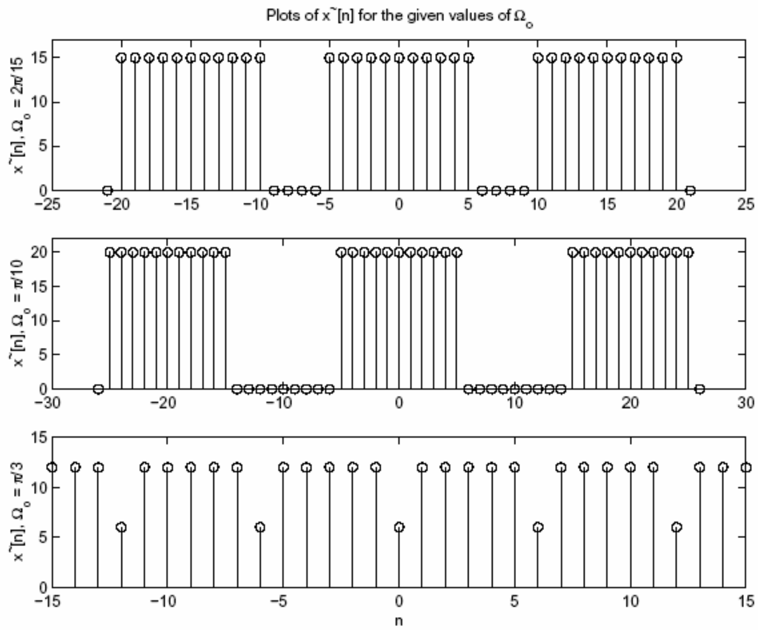

Figure P4.36. Sketch of $\tilde{x}[n]$

4.38. A signal $x(t)$ is sampled at intervals of $T_s = 0.01$ s. One hundred samples are collected and a 200-point DTFS is taken in an attempt to approximate $X(j\omega)$. Assume $|X(j\omega)| \approx 0$ for $|\omega| > 120\pi$ rad/s. Determine the frequency range $-\omega_a < \omega < \omega_a$ over which the DTFS offers a reasonable approximation to

$X(j\omega)$, the effective resolution of this approximation, ω_r, and the frequency interval between each DTFS coefficient, $\Delta\omega$.

$$\begin{array}{rl} x(t) & T_s = 0.01 \\ 100 \text{ samples} & M = 100 \end{array}$$

Use $N = 200$ DTFS to approximate $X(j\omega)$, $|X(j\omega)| \approx 0$, $|\omega| > 120\pi$, $\omega_m = 120\pi$

$$\begin{aligned} T_s &< \frac{2\pi}{\omega_m + \omega_a} \\ \omega_a &< \frac{2\pi}{T_s} - \omega_m \end{aligned}$$

Therefore:

$$\begin{aligned} \omega_a &< 80\pi \\ MT_s &> \frac{2\pi}{\omega_r} \end{aligned}$$

Therefore:

$$\begin{aligned} \omega_r &> 2\pi \\ N &> \frac{\omega_s}{\Delta\omega} \\ \Delta\omega &> \frac{\omega_s}{N} \\ \Delta\omega &= \frac{2\pi}{NT_s} \end{aligned}$$

Therefore:

$$\Delta\omega > \pi$$

6.28. Determine the unilateral Laplace transform of the following signals using the defining equation:

(f) $x(t) = u(t) - u(t-2)$

$$\begin{aligned} X(s) &= \int_{0^-}^{2} e^{-st}\, dt \\ &= \frac{1 - e^{-2s}}{s} \end{aligned}$$

(g) $x(t) = \begin{cases} \sin(\pi t), & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$

$$\begin{aligned} X(s) &= \int_{0^-}^{1} \frac{1}{2j}\left(e^{j\pi t} - e^{-j\pi t}\right) e^{-st}\, dt \\ &= \frac{\pi(1 + e^{-s})}{s^2 + \pi^2} \end{aligned}$$

6.31. Given the transform pair $\cos(2t)u(t) \overset{\mathcal{L}_u}{\longleftrightarrow} X(s)$, determine the time signals corresponding to the following Laplace transforms:

(d) $s^{-2}X(s)$

$$\begin{aligned} B(s) = \frac{1}{s}X(s) &\overset{\mathcal{L}_u}{\longleftrightarrow} \int_{-\infty}^{t} x(\tau)d\tau \\ &\overset{\mathcal{L}_u}{\longleftrightarrow} \int_{-\infty}^{t} \cos(2\tau)u(\tau)d\tau \\ &\overset{\mathcal{L}_u}{\longleftrightarrow} \int_{0}^{t} \cos(2\tau)d\tau \end{aligned}$$

$$\begin{aligned} B(s) &\overset{\mathcal{L}_u}{\longleftrightarrow} \frac{1}{2}\sin(2t) \\ \frac{1}{s}B(s) &\overset{\mathcal{L}_u}{\longleftrightarrow} \int_{0}^{t} \frac{1}{2}\sin(2\tau)d\tau \\ &\overset{\mathcal{L}_u}{\longleftrightarrow} \frac{1-\cos(2t)}{4}u(t) \end{aligned}$$

(e) $\frac{d}{ds}\left(e^{-3s}X(s)\right)$

$$\begin{aligned} A(s) = e^{-3s}X(s) &\overset{\mathcal{L}_u}{\longleftrightarrow} a(t) = x(t-3) = \cos(2(t-3))u(t-3) \\ B(s) = \frac{d}{ds}A(s) &\overset{\mathcal{L}_u}{\longleftrightarrow} b(t) = -ta(t) = -t\cos(2(t-3))u(t-3) \end{aligned}$$

6.41. Determine the bilateral Laplace transform and the corresponding ROC for the following signals:

(d) $x(t) = \cos(3t)u(-t) * e^{-t}u(t)$

$$\begin{aligned} a(t) * b(t) &\overset{\mathcal{L}}{\longleftrightarrow} A(s)B(s) \\ X(s) &= -\frac{s}{s^2+9}\left(\frac{1}{s+1}\right) \end{aligned}$$

ROC: -1 < Re(s) < 0

(f) $x(t) = e^t \frac{d}{dt}\left(e^{-2t}u(-t)\right)$

$$\begin{aligned} a(t) = e^{-2t}u(-t) &\overset{\mathcal{L}}{\longleftrightarrow} A(s) = \frac{-1}{s+2} \\ b(t) = \frac{d}{dt}a(t) &\overset{\mathcal{L}}{\longleftrightarrow} B(s) = sA(s) \\ x(t) = e^t b(t) &\overset{\mathcal{L}}{\longleftrightarrow} X(s) = B(s-1) = \frac{1-s}{s+1} \end{aligned}$$

ROC: Re(s) < -1