ECE352
Spring 07

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Homework 4 Solutions

Problems 6.35, 6.36, 6.37(i) and (f), 6.38(a) and (b), 6.43 (a) and (b)

6.35. Determine the initial value $x(0^+)$ given the following Laplace transforms $X(s)$:
(a) $X(s) = \frac{1}{s^2+5s-2}$

$$x(0^+) = \lim_{s\to\infty} sX(s) = \frac{s}{s^2+5s-2} = 0$$

(b) $X(s) = \frac{s+2}{s^2+2s-3}$

$$x(0^+) = \lim_{s\to\infty} sX(s) = \frac{s^2+2s}{s^2+2s-3} = 1$$

(c) $X(s) = e^{-2s}\frac{6s^2+s}{s^2+2s-2}$

$$x(0^+) = \lim_{s\to\infty} sX(s) = e^{-2s}\frac{6s^3+s^2}{s^2+2s-2} = 0$$

6.36. Determine the final value $x(\infty)$ given the following Laplace transforms $X(s)$:
(a) $X(s) = \frac{2s^2+3}{s^2+5s+1}$

$$x(\infty) = \lim_{s \to 0} sX(s) = \frac{2s^3 + 3s}{s^2 + 5s + 1} = 0$$

(b) $X(s) = \frac{s+2}{s^3+2s^2+s}$

$$x(\infty) = \lim_{s \to 0} sX(s) = \frac{s + 2}{s^2 + 2s + 1} = 2$$

(c) $X(s) = e^{-3s}\frac{2s^2+1}{s(s+2)^2}$

$$x(\infty) = \lim_{s \to 0} sX(s) = e^{-3s}\frac{2s^2 + 1}{(s+2)^2} = \frac{1}{4}$$

6.37. Use the method of partial fractions to find the time signals corresponding to the follow
lateral Laplace transforms:

(f) $X(s) = \frac{3s+2}{s^2+2s+10}$

$$X(s) = \frac{3s + 2}{s^2 + 2s + 10} = \frac{3(s+1) - 1}{(s+1)^2 + 3^2}$$

$$x(t) = \left[3e^{-t}\cos(3t) - \frac{1}{3}e^{-t}\sin(3t)\right]u(t)$$

(i) $X(s) = \frac{2s^2+11s+16+e^{-2s}}{s^2+5s+6}$

$$\begin{aligned} X(s) &= \frac{2s^2+11s+16+e^{-2s}}{s^2+5s+6} = 2 + \frac{s+4}{s^2+5s+6} + \frac{e^{-2s}}{s^2+5s+6} \\ X(s) &= 2 + \frac{-1}{s+3} + \frac{2}{s+2} - e^{-2s}\frac{1}{s+3} + e^{-2s}\frac{1}{s+2} \\ x(t) &= 2\delta(t) + \left[2e^{-2t} - e^{-t}\right]u(t) + \left[e^{-2(t-2)} - e^{-3(t-2)}\right]u(t-2) \end{aligned}$$

6.38. Determine the forced and natural responses for the LTI systems described by the following ferential equations with the specified input and initial conditions:

(a) $\frac{d}{dt}y(t) + 10y(t) = 10x(t), \;\; y(0^-) = 1, \;\; x(t) = u(t)$

$$
\begin{aligned}
X(s) &= \frac{1}{s} \\
Y(s)(s+10) &= 10X(s) + y(0^-) \\
Y^f(s) &= \frac{10X(s)}{s+10} \\
&= \frac{10}{s(s+10)} \\
&= \frac{1}{s} + \frac{-1}{s+10} \\
y^f(t) &= \left[1 - e^{-10t}\right] u(t) \\
Y^n(s) &= \frac{y(0^-)}{s+10} \\
y^n(t) &= e^{-10t}u(t)
\end{aligned}
$$

(b) $\frac{d^2}{dt^2}y(t) + 5\frac{d}{dt}y(t) + 6y(t) = -4x(t) - 3\frac{d}{dt}x(t), \;\; y(0^-) = -1, \; \frac{d}{dt}y(t)\Big|_{t=0^-} = 5, \;\; x(t) = e^{-t}u(t)$

$$
\begin{aligned}
Y(s)(s^2+5s+6) - 5 + s + 5 &= (-4-3s)\frac{1}{s+1} \\
Y(s) &= \frac{-1}{(s+1)(s+2)(s+3)} + \frac{s}{(s+2)(s+3)} \\
&= Y^f(s) + Y^n(s) \\
Y^f(s) &= \frac{-0.5}{s+1} + \frac{-2}{s+2} + \frac{2.5}{s+3} \\
y^f(t) &= \left(-0.5e^{-t} - 2e^{-2t} + 2.5e^{-3t}\right) u(t) \\
Y^n(s) &= \frac{-2}{s+2} + \frac{3}{s+3} \\
y^n(t) &= \left(-2e^{-2t} + 3e^{-3t}\right) u(t)
\end{aligned}
$$

6.43. Use the method of partial fractions to determine the time signals corresponding to the followin
bilateral Laplace transforms:

(a) $X(s) = \frac{-s-4}{s^2+3s+2}$

$$X(s) = \frac{-3}{s+1} + \frac{2}{s+2}$$

(i) with ROC $\text{Re}(s) < -2$

(left-sided)

$$x(t) = \left(3e^{-t} - 2e^{-2t}\right) u(-t)$$

(ii) with ROC $\text{Re}(s) > -1$

(right-sided)

$$x(t) = \left(-3e^{-t} + 2e^{-2t}\right) u(t)$$

(iii) with ROC $-2 < \text{Re}(s) < -1$

(two sided)

$$x(t) = 3e^{-t}u(-t) + 2e^{-2t}u(t)$$

(b) $X(s) = \frac{4s^2+8s+10}{(s+2)(s^2+2s+5)}$

$$X(s) = \frac{2}{s+2} + \frac{2(s+1)}{(s+1)^2+2^2} + \frac{-2}{(s+1)^2+2^2}$$

(i) with ROC $\text{Re}(s) < -2$

(left-sided)

$$x(t) = \left(-2e^{-2t} - 2e^{-t}\cos(2t) + e^{-t}\sin(2t)\right) u(-t)$$

(ii) with ROC $\text{Re}(s) > -1$

(right-sided)

$$x(t) = \left(2e^{-2t} + 2e^{-t}\cos(2t) - e^{-t}\sin(2t)\right) u(t)$$

(iii) with ROC $-2 < \text{Re}(s) < -1$

(two sided)

$$x(t) = 2e^{-2t}u(t) + \left(-2e^{-t}\cos(2t) + e^{-t}\sin(2t)\right)u(-t)$$