ECE352
Spring 07

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Solution to Homework 5
Due 05/28/07 at the beginning of the class

Problems 6.46, 6.50, 7.18, 7.20 (a) and (b)

6.46. A stable system has input $x(t)$ and output $y(t)$ as given below. Use Laplace transforms to determine the transfer function and impulse response of the system.

(a) $x(t) = e^{-t}u(t),\ y(t) = e^{-2t}\cos(t)u(t)$

$$
\begin{aligned}
X(s) &= \frac{1}{s+1} \\
Y(s) &= \frac{s+2}{(s+1)^2+1} \\
H(s) &= \frac{Y(s)}{X(s)} = \frac{s^2+3s+2}{s^2+4s+5} \\
&= 1 + \frac{-(s+2)}{(s+2)^2+1} + \frac{-1}{(s+2)^2+1} \\
h(t) &= \delta(t) - \left(e^{-2t}\cos(t) + e^{-2t}\sin(t)\right)u(t)
\end{aligned}
$$

(b) $x(t) = e^{-2t}u(t),\ y(t) = -2e^{-t}u(t) + 2e^{-3t}u(t)$

$$
\begin{aligned}
X(s) &= \frac{1}{s+2} \\
Y(s) &= \frac{-2}{s+1} + \frac{2}{s+3} \\
H(s) &= \frac{Y(s)}{X(s)} = \frac{-4(s+2)}{(s+1)(s+3)} \\
&= \frac{-2}{s+1} + \frac{-2}{s+3} \\
h(t) &= \left(-2e^{-t} - 2e^{-3t}\right)u(t)
\end{aligned}
$$

6.50. Determine whether the systems described by the following transfer functions are(i) both stable and causal, and (ii) whether a stable and causal inverse system exists:

(a) $H(s) = \frac{(s+1)(s+2)}{(s+1)(s^2+2s+10)}$

$$\begin{aligned} H(s) &= \frac{s+2}{s^2+2s+10} \\ \text{zero at:} \quad & -2 \\ \text{poles at:} \quad & -1 \pm 3j \end{aligned}$$

(i) All poles are in the LHP, and with ROC: Re(s) > -1, the system is both stable and causal.

(ii) All zeros are in the LHP, so a stable and causal inverse system exists.

(b) $H(s) = \frac{s^2+2s-3}{(s+3)(s^2+2s+5)}$

$$\begin{aligned} H(s) &= \frac{s-1}{s^2+2s+5} \\ \text{zero at:} \quad & 1 \\ \text{poles at:} \quad & -1 \pm 2j \end{aligned}$$

(i) All poles are in the LHP, and with ROC: Re(s) > -1, the system is both stable and causal.

(ii) Not all zeros are in the LHP, so no stable and causal inverse system exists.

(c) $H(s) = \frac{s^2-3s+2}{(s+2)(s^2-2s+8)}$

$$H(s) = \frac{s^2-3s+2}{s^3+4s+16}$$

$$\begin{aligned} \text{zeros at:} \quad & 1, 2 \\ \text{poles at:} \quad & -2, 1 \pm j\sqrt{7} \end{aligned}$$

(i) Not all poles are in the LHP, so the system is not stable and causal.
(ii) No zeros are in the LHP, so no stable and causal inverse system exists.

(d) $H(s) = \frac{s^2+2s}{(s^2+3s-2)(s^2+s+2)}$

$$\begin{aligned} H(s) &= \frac{s^2+2s}{s^4+4s^3+3s^2+4s-4} \\ \text{zeros at:} \quad & 0, -2 \\ \text{poles at:} \quad & \frac{-3 \pm \sqrt{17}}{2}, -0.5 \pm j\sqrt{\frac{7}{4}} \end{aligned}$$

(i) Not all poles are in the LHP, so the system is not stable and causal.
(ii) There is a zero at $s = 0$, so no stable and causal inverse system exists.

7.18. Given the following z-transforms, determine whether the DTFT of the corresponding time signals exists without determining the time signal, and identify the DTFT in those cases where it exists:
(a) $X(z) = \frac{5}{1+\frac{1}{3}z^{-1}}, \quad |z| > \frac{1}{3}$

ROC includes $|z| = 1$, DTFT exists.

$$X(e^{j\Omega}) = \frac{5}{1+\frac{1}{3}e^{-j\Omega}}$$

(b) $X(z) = \frac{5}{1+\frac{1}{3}z^{-1}}, \quad |z| < \frac{1}{3}$

ROC does not include, $|z| = 1$, DTFT does not exist.

(c) $X(z) = \frac{z^{-1}}{(1-\frac{1}{2}z^{-1})(1+3z^{-1})}, \quad |z| < \frac{1}{2}$

ROC does not include, $|z| = 1$, DTFT does not exist.

(d) $X(z) = \frac{z^{-1}}{(1-\frac{1}{2}z^{-1})(1+3z^{-1})}, \quad \frac{1}{2} < |z| < 3$

ROC includes $|z| = 1$, DTFT exists.

$$X(e^{j\Omega}) = \frac{e^{-j\Omega}}{(1-\frac{1}{2}e^{-j\Omega})(1+3e^{-j\Omega})}$$

7.20. Use the tables of z-transforms and z-transform properties given in Appendix E to determine the z-transforms of the following signals:

(a) $x[n] = \left(\frac{1}{2}\right)^n u[n] * 2^n u[-n-1]$

$$\begin{aligned}
a[n] = \left(\frac{1}{2}\right)^n u[n] & \overset{z}{\longleftrightarrow} A(z) = \frac{1}{1-\frac{1}{2}z^{-1}}, \quad |z| > \frac{1}{2} \\
b[n] = 2^n u[-n-1] & \overset{z}{\longleftrightarrow} B(z) = \frac{1}{1-2z^{-1}}, \quad |z| < 2 \\
x[n] = a[n] * b[n] & \overset{z}{\longleftrightarrow} X(z) = A(z)B(z) \\
X(z) & = \left(\frac{1}{1-\frac{1}{2}z^{-1}}\right)\left(\frac{1}{1-2z^{-1}}\right), \quad \frac{1}{2} < |z| < 2
\end{aligned}$$

(b) $x[n] = n\left(\left(\frac{1}{2}\right)^n u[n] * \left(\frac{1}{4}\right)^n u[n-2]\right)$

$$a[n] = \left(\frac{1}{2}\right)^n u[n] \overset{z}{\longleftrightarrow} A(z) = \frac{1}{1-\frac{1}{2}z^{-1}}, \quad |z| > \frac{1}{2}$$

$$\begin{aligned}
b[n] = \left(\frac{1}{4}\right)^n u[n] & \overset{z}{\longleftrightarrow} B(z) = \frac{1}{1-\frac{1}{4}z^{-1}}, \quad |z| > \frac{1}{4} \\
c[n] = b[n-2] & \overset{z}{\longleftrightarrow} C(z) = \frac{z^{-2}}{1-\frac{1}{4}z^{-1}} \\
x[n] = n[a[n] * b[n]] & \overset{z}{\longleftrightarrow} X(z) = -z\frac{d}{dz}A(z)B(z) \\
X(z) & = \frac{2z^{-2} - \frac{3}{4}z^{-3}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2})^2}, \quad |z| > \frac{1}{2}
\end{aligned}$$