ECE352
Spring 07

Dr. Thinh Nguyen

Solution to Homework 6
Due 06/08/07 at the beginning of the class

Problems 7.26, 7.29, 7.32, 7.38, 7.39

7.26. Use the following clues to determine the signal $x[n]$ and rational z-transform $X(z)$.

(a) $X(z)$ has poles at $z = 1/2$ and $z = -1$, $x[1] = 1, x[-1] = 1$, and the ROC includes the point $z = 3/4$.

Since the ROC includes the point $z = 3/4$, the ROC is $\frac{1}{2} < |z| < 1$.

$$\begin{aligned} X(z) &= \frac{A}{1 - \frac{1}{2}z^{-1}} + \frac{B}{1 + z^{-1}} \\ x[n] &= A\left(\frac{1}{2}\right)^n u[n] - B(-1)^n u[-n-1] \\ x[1] = 1 &= A\left(\frac{1}{2}\right) \\ A &= 2 \\ x[-1] = 1 &= -1B(-1) \\ B &= 1 \\ x[n] &= 2\left(\frac{1}{2}\right)^n u[n] - (-1)^n u[-n-1] \end{aligned}$$

(b) $x[n]$ is right-sided, $X(z)$ has a single pole, and $x[0] = 2, x[2] = 1/2$.

$$\begin{aligned} x[n] &= c(p)^n u[n] \quad \text{where } c \text{ and } p \text{ are unknown constants.} \\ x[0] = 2 &= c\,(p)^0 \end{aligned}$$

$$
\begin{aligned}
c &= 2 \\
x[2] = \frac{1}{2} &= 2\,(p)^2 \\
p &= \frac{1}{2} \\
x[n] &= 2\left(\frac{1}{2}\right)^n u[n]
\end{aligned}
$$

(c) $x[n]$ is two-sided, $X(z)$ has one pole at $z = 1/4$, $x[-1] = 1, x[-3] = 1/4$, and $X(1) = 11/3$.

$$
\begin{aligned}
X(z) &= \frac{A}{1-\frac{1}{4}z^{-1}} + \frac{B}{1-cz^{-1}} \\
x[n] &= A\left(\frac{1}{4}\right)^n u[n] - B(c)^n u[-n-1] \\
x[-1] = 1 &= -Bc^{-1} \\
x[-3] = \frac{1}{4} &= -Bc^{-3} \\
c &= 2 \\
B &= -2 \\
X(1) = \frac{11}{3} &= \frac{A}{1-\frac{1}{4}} + \frac{-2}{1-2} \\
A &= \frac{5}{4} \\
x[n] &= \frac{5}{4}\left(\frac{1}{4}\right)^n u[n] + 2(2)^n u[-n-1]
\end{aligned}
$$

7.29. A causal system has input $x[n]$ and output $y[n]$. Use the transfer function to determine the impulse response of this system.

(a) $x[n] = \delta[n] + \frac{1}{4}\delta[n-1] - \frac{1}{8}\delta[n-2], \;\; y[n] = \delta[n] - \frac{3}{4}\delta[n-1]$

$$
\begin{aligned}
X(z) &= 1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-1} \\
Y(z) &= 1 - \frac{3}{4}z^{-1} \\
H(z) &= \frac{Y(z)}{X(z)} \\
&= \frac{-\frac{2}{3}}{1-\frac{1}{4}z^{-1}} + \frac{\frac{5}{3}}{1+\frac{1}{2}z^{-1}} \\
h[n] &= \frac{1}{3}\left[5(-\frac{1}{2})^n - 2(\frac{1}{4})^n\right] u[n]
\end{aligned}
$$

(b) $x[n] = (-3)^n\, u[n], \;\; y[n] = 4(2)^n u[n] - \left(\frac{1}{2}\right)^n u[n]$

$$
\begin{aligned}
X(z) &= \frac{1}{1+3z^{-1}} \\
Y(z) &= \frac{3}{(1-2z^{-1})(1-\frac{1}{2}z^{-1})} \\
H(z) &= \frac{Y(z)}{X(z)} \\
&= \frac{10}{1-2z^{-1}} + \frac{-7}{1-\frac{1}{2}z^{-1}} \\
h[n] &= \left[10(2)^n - 7(\frac{1}{2})^n\right] u[n]
\end{aligned}
$$

7.32. Determine (i) transfer function and (ii) difference-equation representations for the systems with the following impulse responses:

(a) $h[n] = 3\left(\frac{1}{4}\right)^n u[n-1]$

$$\begin{aligned} h[n] &= 3\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)^{n-1} u[n-1] \\ H(z) &= \frac{\frac{3}{4}z^{-1}}{1-\frac{1}{4}z^{-1}} \\ &= \frac{Y(z)}{X(z)} \end{aligned}$$

Taking the inverse z-transform yields:

$$y[n] - \frac{1}{4}y[n-1] \;=\; \frac{3}{4}x[n-1]$$

(b) $h[n] = \left(\frac{1}{3}\right)^n u[n] + \left(\frac{1}{2}\right)^{n-2} u[n-1]$

$$\begin{aligned} h[n] &= \left(\frac{1}{3}\right)^n u[n] + 2\left(\frac{1}{2}\right)^{n-1} u[n-1] \\ H(z) &= \frac{1+\frac{3}{2}z^{-1}-\frac{2}{3}z^{-2}}{1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}} \\ &= \frac{Y(z)}{X(z)} \end{aligned}$$

Taking the inverse z-transform yields:

$$y[n] - \frac{5}{6}y[n-1] + \frac{1}{6}y[n-2] \;=\; x[n] + \frac{3}{2}x[n-1] - \frac{2}{3}x[n-2]$$

(c) $h[n] = 2\left(\frac{2}{3}\right)^n u[n-1] + \left(\frac{1}{4}\right)^n [\cos(\frac{\pi}{6}n) - 2\sin(\frac{\pi}{6}n)]u[n]$

$$\begin{aligned} H(z) &= \frac{\frac{4}{3}z^{-1}}{1-\frac{2}{3}z^{-1}} + \frac{1-\frac{1}{4}z^{-1}\cos(\frac{\pi}{6})-\frac{1}{2}z^{-1}\sin(\frac{\pi}{6})}{1-z^{-1}\frac{1}{2}\cos(\frac{\pi}{6})+\frac{1}{16}z^{-2}} \\ &= \frac{1+\left(\frac{5}{12}-\frac{1}{8}\sqrt{3}\right)z^{-1}+\left(\frac{1}{6}-\frac{1}{4}\sqrt{3}\right)z^{-2}+\frac{1}{12}z^{-3}}{1-\left(\frac{2}{3}+\frac{1}{4}\sqrt{3}\right)z^{-1}+\left(\frac{1}{16}+\frac{1}{6}\sqrt{3}\right)z^{-2}-\frac{1}{24}z^{-3}} \end{aligned}$$

Taking the z-transform yields:

$y[n] - \left(\frac{2}{3}+\frac{1}{4}\sqrt{3}\right)y[n-1] + \left(\frac{1}{16}+\frac{1}{6}\sqrt{3}\right)y[n-2] - \frac{1}{24}y[n-3]$
$= x[n] + \left(\frac{5}{12}-\frac{1}{8}\sqrt{3}\right)x[n-1] + \left(\frac{1}{6}-\frac{1}{4}\sqrt{3}\right)x[n-2] + \frac{1}{12}x[n-3]$

(d) $h[n] = \delta[n] - \delta[n-5]$

$$H(z) = 1 - z^{-5}$$

Taking the z-transform yields:

$$y[n] = x[n] - x[n-5]$$

7.38. Draw block-diagram implementations of the following systems as a cascade of second-order sections with real-valued coefficients:

(a) $H(z) = \frac{(1-\frac{1}{4}e^{j\frac{\pi}{4}}z^{-1})(1-\frac{1}{4}e^{-j\frac{\pi}{4}}z^{-1})(1+\frac{1}{4}e^{j\frac{\pi}{8}}z^{-1})(1+\frac{1}{4}e^{-j\frac{\pi}{8}}z^{-1})}{(1-\frac{1}{2}e^{j\frac{\pi}{3}}z^{-1})(1-\frac{1}{2}e^{-j\frac{\pi}{3}}z^{-1})(1-\frac{3}{4}e^{j\frac{7\pi}{8}}z^{-1})(1-\frac{3}{4}e^{-j\frac{7\pi}{8}}z^{-1})}$

$$\begin{aligned} H(z) &= H_1(z)H_2(z) \\ H_1(z) &= \frac{1 - \frac{1}{2}\cos(\frac{\pi}{4})z^{-1} + \frac{1}{16}z^{-2}}{1 - \cos(\frac{\pi}{3})z^{-1} + \frac{1}{4}z^{-2}} \\ H_2(z) &= \frac{1 + \frac{1}{2}\cos(\frac{\pi}{8})z^{-1} + \frac{1}{16}z^{-2}}{1 - \frac{3}{2}\cos(\frac{7\pi}{8})z^{-1} + \frac{9}{16}z^{-2}} \end{aligned}$$

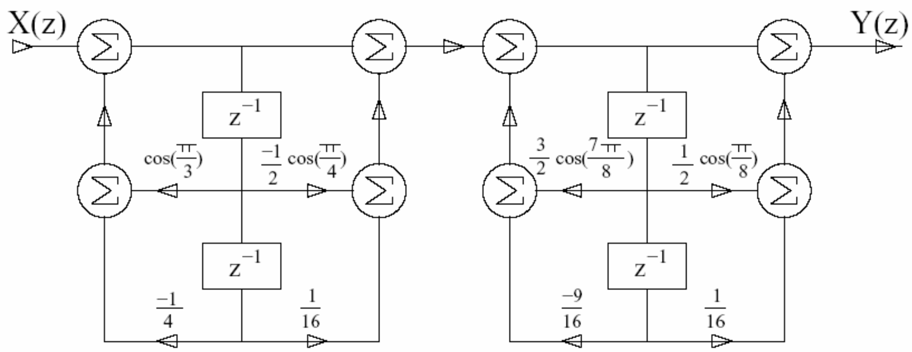

Figure P7.38. (a) Block diagram.

(b) $H(z) = \frac{(1+2z^{-1})^2(1-\frac{1}{2}e^{j\frac{\pi}{2}}z^{-1})(1-\frac{1}{2}e^{-j\frac{\pi}{2}}z^{-1})}{(1-\frac{3}{8}z^{-1})(1-\frac{3}{8}e^{j\frac{\pi}{3}}z^{-1})(1-\frac{3}{8}e^{j\frac{\pi}{3}}z^{-1})(1+\frac{3}{4}z^{-1})}$

$$\begin{aligned} H(z) &= H_1(z)H_2(z) \\ H_1(z) &= \frac{1 + 4z^{-1} + 4z^{-2}}{1 + \frac{3}{8}z^{-1} - \frac{9}{32}z^{-2}} \\ H_2(z) &= \frac{1 + \frac{1}{4}z^{-2}}{1 - \frac{3}{4}\cos(\frac{\pi}{3})z^{-1} + \frac{9}{64}z^{-2}} \end{aligned}$$

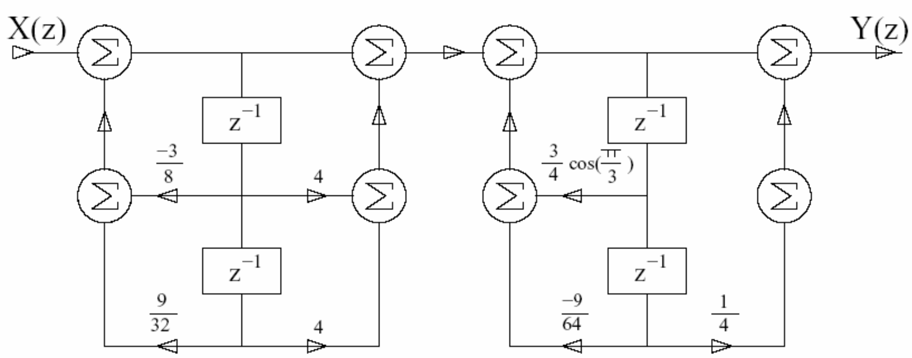

Figure P7.38. (b) Block diagram.

7.39. Draw block diagram implementations of the following systems as a parallel combination of second-order sections with real-valued coefficients:

(a) $h[n] = 2\left(\frac{1}{2}\right)^n u[n] + \left(\frac{j}{2}\right)^n u[n] + \left(\frac{-j}{2}\right)^n u[n] + \left(\frac{-1}{2}\right)^n u[n]$

$$\begin{aligned}
H(z) &= \frac{2}{1-\frac{1}{2}z^{-1}} + \frac{1}{1-\frac{j}{2}z^{-1}} + \frac{1}{1+\frac{j}{2}z^{-1}} + \frac{1}{1+\frac{1}{2}z^{-1}} \\
&= \frac{3+\frac{1}{2}z^{-1}}{1-\frac{1}{4}z^{-2}} + \frac{2}{1+\frac{1}{4}z^{-2}} \\
H(z) &= H_1(z) + H_2(z)
\end{aligned}$$

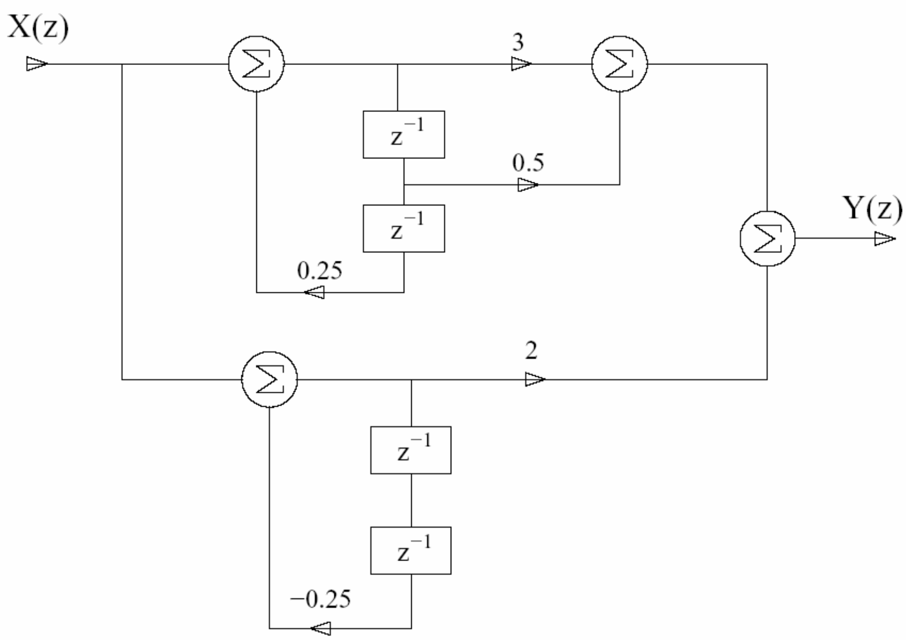

Figure P7.39. (a) Block diagram.

(b) $h[n] = 2\left(\frac{1}{2}e^{j\frac{\pi}{4}}\right)^n u[n] + \left(\frac{1}{4}e^{j\frac{\pi}{3}}\right)^n u[n] + \left(\frac{1}{4}e^{-j\frac{\pi}{3}}\right)^n u[n] + 2\left(\frac{1}{2}e^{-j\frac{\pi}{4}}\right)^n u[n]$

$$\begin{aligned}
H(z) &= \frac{2}{1-\frac{1}{2}e^{j\frac{\pi}{4}}z^{-1}} + \frac{1}{1-\frac{1}{4}e^{j\frac{\pi}{3}}z^{-1}} + \frac{1}{1-\frac{1}{4}e^{-j\frac{\pi}{3}}z^{-1}} + \frac{2}{1-\frac{1}{2}e^{-j\frac{\pi}{4}}z^{-1}} \\
&= \frac{4-\sqrt{2}z^{-1}}{1-\frac{1}{\sqrt{2}}z^{-1}+\frac{1}{4}z^{-2}} + \frac{2-\frac{1}{4}z^{-1}}{1-\frac{1}{4}z^{-1}+\frac{1}{16}z^{-2}} \\
H(z) &= H_1(z) + H_2(z)
\end{aligned}$$

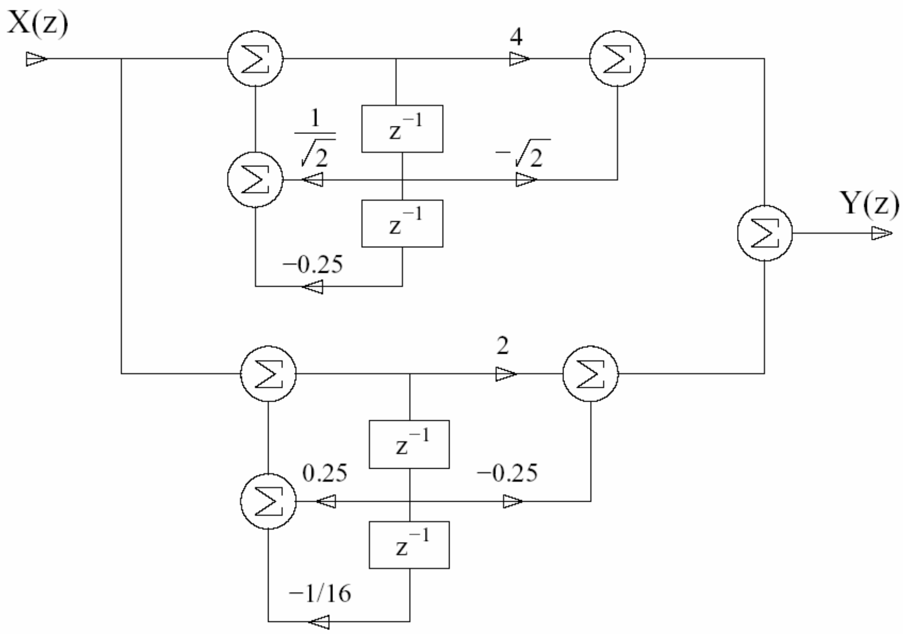

Figure P7.39. (b) Block diagram.