Embedded Zero-tree Wavelet
EZW

Thinh Nguyen
Outline

- Introduction
- Concept of EZW
- Algorithm
- Examples
Introduction

- **Embedded Image coding using Zerotrees of Wavelet coefficients** by J. M. Shapiro.

- Uses “parent-child” dependencies between subband coefficients at the same spatial location.

- Bit-plane coding: enables an embedded bitstream wrt distortion
Concepts of EZW

- (1) a discrete wavelet transform or hierarchical subband decomposition,
- (2) prediction of the absence of significant information across scales by exploiting the self-similarity inherent in images,
- (3) entropy-coded successive-approximation quantization, and
- (4) universal lossless data compression which is achieved via adaptive arithmetic coding
FIGURE 14.15  A 10-band wavelet decomposition.
FIGURE 14.16 Data structure used in the EZW coder.
How does it work?
Scanning a zerotree
Terminology

- **sp**: Given a threshold $T$, if a given coefficient has a magnitude greater than $T$, it is called a **significant coefficient** at level $T$.

- **sn**: *negative significant*

- **zr**: If the magnitude of the coefficient is less than $T$ (it is insignificant), and all its descendants have magnitudes less than $T$, then the coefficient is called a **zerotree root**.

- **iz**: it might happen that the coefficient itself is less than $T$ but some of its descendants have a value greater than $T$. Such a coefficient is called an **isolated zero**.
Algorithm Chart:

Input

Is coef. significant?

Yes

Does coef. descend from a Zerotree root?

+ What sign?

- Code Positive symbol

Code Negative symbol

No

Does coef. have significant descendants?

Yes

Code Isolated Zero symbol

No

Code Zerotree Root symbol
EZW Example (1): seven-level decomposition shown below to demonstrate the various steps of EZW

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- Initial threshold

\[ T_0 = 2^{\lfloor \log_2 26 \rfloor} = 16 \]

- 8 bits from bit budget
**EZW Example (1):** seven-level decomposition shown below to demonstrate the various steps of EZW

- **26 > 16 → sp**
- **6 < 16 →**
  - descendants < 16 → **zr**
- **-7 < 16 →**
  - descendants < 16 → **zr**
- **7 < 16 →**
  - descendants < 16 → **zr**
- **labels to be transmitted** *sp zr zr zr*

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- **Initial threshold**

\[
T_0 = 2^{\left\lfloor \log_2 26 \right\rfloor} = 16
\]

- **8 bits from bit budget**
EZW Example (1): Subordinate Pass

- $L_s = \{26\}$
- *The significant coefficient reconstructed value*
  
  $1.5T_0 = 24$
- reconstructed bands

\[
\begin{array}{cccc}
24 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{array}
\]
EZW Example (1): Subordinate Pass

- $L_s = \{26\}$
- The significant coefficient $1.5T_o = 24$
- reconstructed bands

- Difference $26 - 24$
- Using a 2-level quantizer with reconstruction levels $\pm T_o/4$, correction term of 4
- Reconstruction $24 + 4 = 28$
- Transmitting the correction term costs a single bit.

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EZW Example (1):

- \( T1 = \frac{1}{2} * T0 = \frac{1}{2} * 16 = 8 \)
- \( 6 < 8 \rightarrow \)
  - descendants > 8 \( \rightarrow \) iz
- \( -7 < 8 \rightarrow \)
  - descendants < 8 \( \rightarrow \) zr
- \( 7 < 8 \rightarrow \)
  - descendants < 8 \( \rightarrow \) zr
- \( 13 \) no descendants > 8 \( \rightarrow \) sp
- \( 10 \) no descendants > 8 \( \rightarrow \) sp
- \( 6 \) no descendants < 8 \( \rightarrow \) iz
- \( 4 \) no descendants < 8 \( \rightarrow \) iz

\[
\begin{array}{cccc}
* & 6 & 13 & 10 \\
-7 & 7 & 6 & 4 \\
4 & -4 & 4 & -3 \\
2 & -2 & -2 & 0 \\
\end{array}
\]
EZW Example (1):

- $T_1 = \frac{1}{2} \times T_0 = \frac{1}{2} \times 16 = 8$
- $6 < 8 \Rightarrow$
  - descendants $> 8 \Rightarrow iz$
- $-7 < 8 \Rightarrow$
  - descendants $< 8 \Rightarrow zr$
- $7 < 8 \Rightarrow$
  - descendants $< 8 \Rightarrow zr$
- $13$ no descendants $> 8 \Rightarrow sp$
- $10$ no descendants $> 8 \Rightarrow sp$
- $6$ no descendants $< 8 \Rightarrow iz$
- $4$ no descendants $< 8 \Rightarrow iz$
- \[ \begin{array}{c|c|c|c}
  * & 6 & 13 & 10 \\
  \hline
  -7 & 7 & 6 & 4 \\
  4 & -4 & 4 & -3 \\
  2 & -2 & -2 & 0 \\
\end{array} \]
- labels to be transmitted
  \[ zipp zr zr sp sp zip zip \]
- Requires 14 bits
- Total bits = $9 + 14 = 23$
EZW Example (1): Subordinate Pass

- The significant coefficient
  \[1.5T_1 = 1.5 \times 8 = 12\]
- \(L_s = \{26, 13, 10\}\)
- reconstructed bands

\[
\begin{array}{cccc}
28 & 0 & 12 & 12 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{array}
\]
EZW Example (1): Subordinate Pass

- The significant coefficient: $1.5T_1 = 1.5 \times 8 = 12$
- $L_s = \{26, 13, 10\}$
- reconstructed bands

with a 2-level quantizer with reconstruction levels $\pm T_1 / 4 = \pm 2$

- $26 - 28 = -2$ Correction term = -2
- $13 - 12 = 1$ Correction term = 2
- $10 - 12 = -2$ Correction term = -2

Each correction requires a single bit, the total bits $23 + 3 = 26$.

Reconstruction

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EZW Example (1):

- $T_2 = \frac{1}{2} \times T_1 = \frac{1}{2} \times 8 = 4$
- $6 > 4 \rightarrow \text{sp}$
- $|-7| > 4 \rightarrow \text{sn}$
- $7 > 4 \rightarrow \text{sp}$
- $6 > 4 \rightarrow \text{sp}$
- $4 = 4 \rightarrow \text{sp}$
- $4 = 4 \rightarrow \text{sp}$
- $|-4| = 4 \rightarrow \text{sn}$
- $2, -2$ are coded as iz
- $4 = 4 \rightarrow \text{sp}$
- $-3, -2, 0$ are iz

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EZW Example (1):

- $T_2 = \frac{1}{2} * T_1 = \frac{1}{2} * 8 = 4$
- $6 > 4 \rightarrow \text{sp}$
- $| -7 | > 4 \rightarrow \text{sn}$
- $7 > 4 \rightarrow \text{sp}$
- $6 = 4 \rightarrow \text{sp}$
- $4 = 4 \rightarrow \text{sp}$
- $4 = 4 \rightarrow \text{sp}$
- $| -4 | = 4 \rightarrow \text{sn}$
- $2, -2$ are coded as $\text{iz}$
- $4 = 4 \rightarrow \text{sp}$
- $-3, -2, 0$ are $\text{iz}$

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- $\text{sp ~ sn ~ sp ~ sp ~ sp ~ sp ~ sn ~ iz ~ iz}$
- $\text{sp ~ iz ~ iz ~ iz}$
- Requires 26 bits
- Total bits = $26 + 26 = 52$
EZW Example (1): Subordinate Pass

- The significant coefficient
  \[1.5T2 = 1.5 \times 4 = 6\]
- \(Ls = \{26, 13, 10, 6, -7, 7, 6, 4, 4, -4, 4\}\)
- reconstructed bands

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